[Ch. 11]

Applications of Differentiation [Ch. 12]

1 It is given that $f(x) = 5x^3 - 15x^2$.

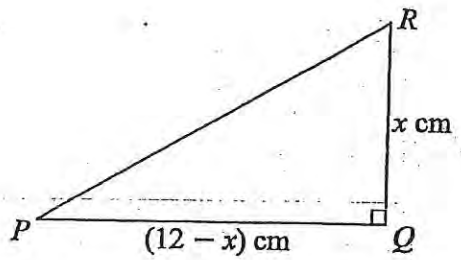
(i) Find $f'(x)$. [2]

(ii) Find $f''(x)$. [1]

(iii) Find the x-coordinates of the stationary points on the curve $y = 5x^3 - 15x^2$. [2]

(iv) Determine whether each stationary point is a maximum point or a minimum point. [2]

2



The diagram shows the triangle PQR with $PQ = (12 - x)$ cm, $QR = x$ cm and angle $PQR = 90°$. The area of the triangle is A cm^2.

(i) Show that $A = 6x - \frac{1}{2}x^2$. [1]

(ii) Find $\frac{dA}{dx}$. [1]

(iii) Hence find the greatest possible area of triangle PQR, showing that it is the greatest and not the least. [5]

3 (i) Given that $y = x^3 - 6x^2 + 9x + 2$, find $\frac{dy}{dx}$. [2]

(ii) Hence find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x + 2$ and determine whether each stationary point is a maximum or a minimum. [5]

(iii) The tangent to the curve at (a, b) is parallel to the tangent to the curve at $(-1, -14)$. Show that the distance between (a, b) and $(-1, -14)$ is $6\sqrt{37}$. [6]

4 It is given that $f(x) = x^3 - 4x^2 - 3x + 1$.

(i) Find $f'(x)$. [2]

(ii) Find $f''(x)$. [1]

(iii) Find the x-coordinates of the stationary points on the curve $y = x^3 - 4x^2 - 3x + 1$. [4]

(iv) Determine whether each stationary point is a maximum or a minimum point. [2]

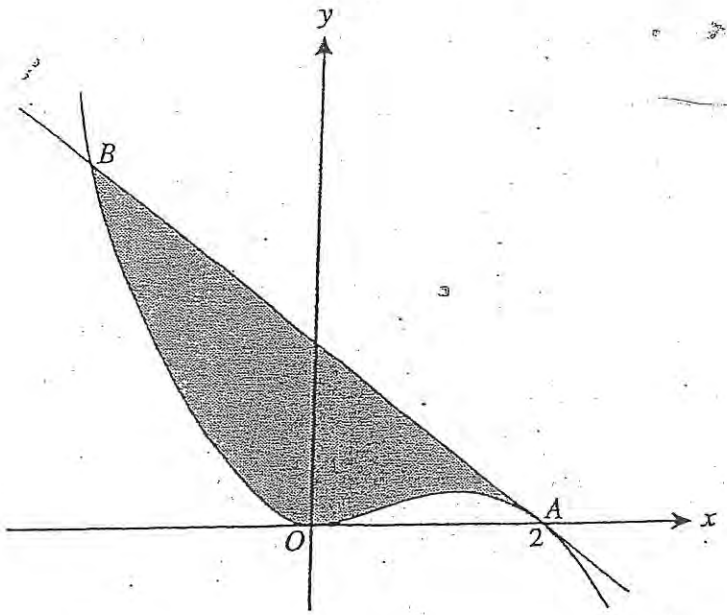
[Ch. 11]

Applications of Differentiation [Ch. 12]

5 (i) Show that there is only one point P on the curve $y = 6x^3 - 12x^2 + 8x - 1$ where the gradient of the curve is zero. [5]

(ii) Find the equation of the tangent to the curve at the point P. [2]

6



The diagram shows the curve $y = 2x^2 - x^3$ and the tangent to the curve at the point A (2, 0).

(i) Find $\frac{dy}{dx}$ and hence find the equation of the tangent at A. [4]

(ii) The tangent at A meets the curve again at the point B. Verify that the coordinates of B are $(-2, 16)$. [2]

7 (i) Find the coordinates of the stationary points on the curve

$$y = 2x^3 - 3x^2 - 12x - 7.$$ [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [2]

(iii) It is given that

$$2x^3 - 3x^2 - 12x - 7 \equiv (x + 1)^2(2x - 7).$$

Sketch the curve $y = (x + 1)^2(2x - 7)$. [3]

(iv) Write down the set of values of the constant k for which the equation $2x^3 - 3x^2 - 12x - 7 = k$ has exactly one real solution. [2]

Applications of Differentiation [Ch. 12]

8 (i) Given that $y = \frac{1}{3}x^3 - 9x$, find $\frac{dy}{dx}$. [2]

(ii) Find the coordinates of the stationary points on the curve $y = \frac{1}{3}x^3 - 9x$. [3]

(iii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iv) Given that $24x + 3y + 2 = 0$ is the equation of the tangent to the curve at the point (p, q), find p and q. [5]