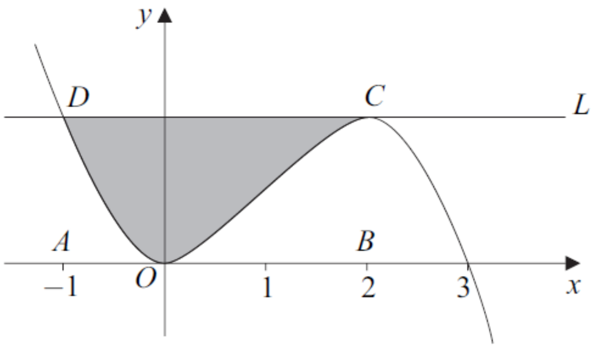
Core 1 Integration Questions

8 The diagram shows the curve with equation $y = 3x^2 - x^3$ and the line L.



The points A and B have coordinates $(-1, 0)$ and $(2, 0)$ respectively. The curve touches the x-axis at the origin O and crosses the x-axis at the point $(3, 0)$. The line L cuts the curve at the point D where $x = -1$ and touches the curve at C where $x = 2$.

(a) Find the area of the rectangle $ABCD$. *(2 marks)*

(b) (i) Find $\int (3x^2 - x^3)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line L. *(4 marks)*

(c) For the curve above with equation $y = 3x^2 - x^3$:

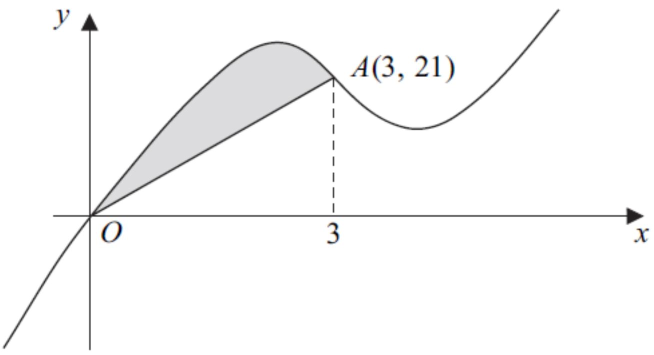
(i) find $\dfrac{dy}{dx}$; *(2 marks)*

(ii) hence find an equation of the tangent at the point on the curve where $x = 1$; *(3 marks)*

(iii) show that y is decreasing when $x^2 - 2x > 0$. *(2 marks)*

(d) Solve the inequality $x^2 - 2x > 0$. *(2 marks)*

5 The curve with equation $y = x^3 - 10x^2 + 28x$ is sketched below.



The curve crosses the x-axis at the origin O and the point $A(3, 21)$ lies on the curve.

(b) (i) Find $\int (x^3 - 10x^2 + 28x)\,dx$. *(3 marks)*

(ii) Hence show that $\int_0^3 (x^3 - 10x^2 + 28x)\,dx = 56\frac{1}{4}$. *(2 marks)*

(iii) Hence determine the area of the shaded region bounded by the curve and the line OA. *(3 marks)*

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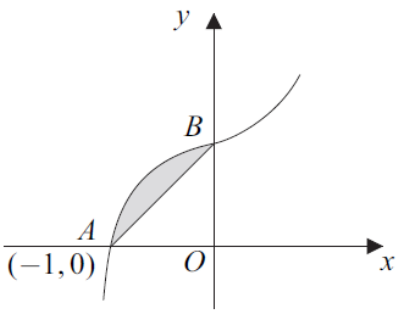
6 The curve with equation $y = 3x^5 + 2x + 5$ is sketched below.



The curve cuts the x-axis at the point $A(-1, 0)$ and cuts the y-axis at the point B.

(a) (i) State the coordinates of the point B and hence find the area of the triangle AOB, where O is the origin. *(3 marks)*

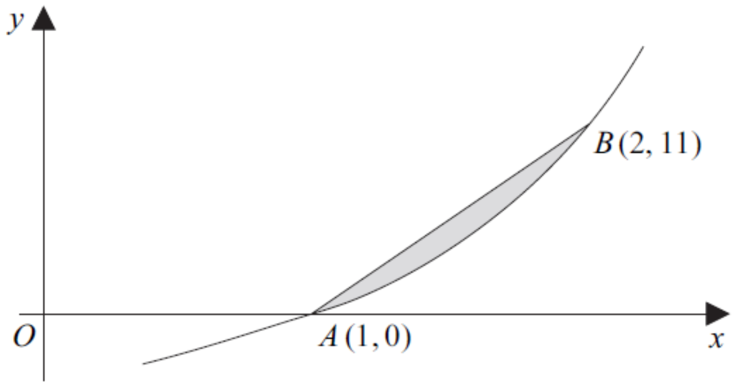
(ii) Find $\int (3x^5 + 2x + 5)\,dx$. *(3 marks)*

(iii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

(b) (i) Find the gradient of the curve with equation $y = 3x^5 + 2x + 5$ at the point $A(-1, 0)$. *(3 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

(b) The curve with equation $y = x^3 + 4x - 5$ is sketched below.



The curve cuts the x-axis at the point $A(1, 0)$ and the point $B(2, 11)$ lies on the curve.

(i) Find $\int (x^3 + 4x - 5)\,dx$. *(3 marks)*

(ii) Hence find the area of the shaded region bounded by the curve and the line AB. *(4 marks)*

Core 1 Integration Answers

8(a)	$y_D = 3+1 = 4$ or $y_C = 12-8=4$	M1		Attempt at either y coordinate
	Area $ABCD$ $= 3\times 4 = 12$	A1	2	
(b)(i)	$x^3 - \dfrac{x^4}{4}$ $(+C)$	M1 A1 A1	3	Increase one power by 1 One term correct unsimplified All correct unsimplified (condone no +C)
(ii)	Sub limits -1 and 2 into their (b) (i) ans $[8-4]-\left[-1-\dfrac{1}{4}\right]$ $= 5\dfrac{1}{4}$	M1 A1		May use both $-1, 0$ and $0, 2$ instead
	Shaded area = "their" (rectangle– integral)	M1		Alt method: difference of two integrals
	$=12-5\dfrac{1}{4}=6\dfrac{3}{4}$	A1	4	**CSO**. Attempted M2, A2
(c)(i)	$\dfrac{dy}{dx}=6x-3x^2$	M1 A1	2	One term correct All correct (no +C etc)
(ii)	When $x=1$, $y=2$ when $x=1$,	B1		May be implied by correct tgt equation
	$\dfrac{dy}{dx}=3$ as 'their' grad of tgt	M1√		Ft their derivative when $x=1$
	Tangent is $y-2=3(x-1)$	A1	3	Any correct form $y=3x-1$ etc

(iii)	Decreasing when $\dfrac{dy}{dx}=6x-3x^2 < 0$	M1		Watch no fudging here!! May work backwards in proof.
	$3(2x-x^2)<0 \Rightarrow x^2-2x>0$	A1	2	**AG** (be convinced no step incorrect)
(d)	Two critical points 0 and 2	M1		Marked on diagram or in solution or M1 A0 for $0 < x < 2$ or $0 > x > 2$
	$x > 2$, $x < 0$ ONLY	A1	2	**SC** B1 for $x > 2$ (or $x < 0$)
	Total		**18**	

(b)(i)	$\dfrac{x^4}{4}-\dfrac{10x^3}{3}+14x^2$ $(+c)$	M1 A1 A1	3	One term correct unsimplified Another term correct unsimplified All correct unsimplified (condone missing + c)
(ii)	$\left[\dfrac{81}{4}-90+126\right]$ (-0)	M1		Attempt to sub limit 3 into their (b)(i)
	$=56\dfrac{1}{4}$	A1	2	AG Integration, limit sub'n all correct
(iii)	Area of triangle = $31\dfrac{1}{2}$	B1		Correct unsimplified $\dfrac{1}{2}\times 21\times 3$
	Shaded Area = $56\dfrac{1}{4} - \textit{triangle area}$	M1		
	$=24\dfrac{3}{4}$	A1	3	Or equivalent such as $\dfrac{99}{4}$

6(a)(i)	B (0,5)	B1		
	Area $AOB = \frac{1}{2}\times 1\times 5$	M1		Condone slip in number or a minus sign
	$= 2\frac{1}{2}$	A1	3	
(ii)	$\frac{3x^6}{6}+\frac{2x^2}{2}+5x$ or $\frac{x^6}{2}+x^2+5x$	M1 A1		Raise one power by 1 One term correct
	(may have $+\,c$ or not)	A1	3	All correct unsimplified
(iii)	Area under curve = $\int_{-1}^{0} \mathrm{f}(x)\,\mathrm{d}x$	B1		Correctly written or F(0) – F(–1) correct
	$[0]-\left[\frac{1}{2}+1-5\right]$	M1		Attempt to sub limit(s) of –1 (and 0) Must have integrated
	Area under curve = $3\frac{1}{2}$	A1		**CSO** (no fudging)
	Area of shaded region = $3\frac{1}{2}-2\frac{1}{2}=1$	B1√	4	**FT** their integral and triangle (very generous)

(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=15x^4+2$	M1 A1		One term correct All correct (no +c etc)
	when $x=-1$, gradient = 17	A1	3	**cso**
(ii)	$y=$"their gradient"$(x+1)$	B1√	1	Must be finding **tangent** – not normal any form e.g. $y=17x+17$
	Total		**14**	

(b)(i)	$\int \ldots\,\mathrm{d}x=\frac{x^4}{4}+2x^2-5x\,(+c)$	M1 A1 A1	3	one term correct unsimplified second term correct unsimplified all correct unsimplified
(ii)	$[4+8-10]-\left[\frac{1}{4}+2-5\right]$	M1		correct use of limits 1 and 2; $\mathrm{F}(2)-\mathrm{F}(1)$ attempted
	$=4\frac{3}{4}$	A1		
	Area of $\Delta=\frac{1}{2}\times 11=5\frac{1}{2}$	B1		correct unsimplified
	$\Rightarrow$ shaded area $=5\frac{1}{2}-4\frac{3}{4}$			combined integral of $7x-6-x^3$ scores M1 for limits correctly used then
	$=\frac{3}{4}$	A1	4	A3 correct answer with all working correct