C3 FUNCTIONS — Answers - Worksheet C

1 **a** $y = 10x + 3$

swap $x = 10y + 3$

$y = \frac{x-3}{10}$

$f^{-1}(x) = \frac{x-3}{10},\ x \in \mathbb{R}$

b $y = 9 + 2x$

swap $x = 9 + 2y$

$y = \frac{x-9}{2}$

$f^{-1}(x) = \frac{x-9}{2},\ x \in \mathbb{R}$

c $y = 5 - 6x$

swap $x = 5 - 6y$

$y = \frac{5-x}{6}$

$f^{-1}(x) = \frac{5-x}{6},\ x \in \mathbb{R}$

d $y = \frac{x+3}{4}$

swap $x = \frac{y+3}{4}$

$y = 4x - 3$

$f^{-1}(x) = 4x - 3,\ x \in \mathbb{R}$

e $y = \frac{1}{3}(2x - 5)$

swap $x = \frac{1}{3}(2y - 5)$

$y = \frac{3x+5}{2}$

$f^{-1}(x) = \frac{3x+5}{2},\ x \in \mathbb{R}$

f $y = 8 - \frac{3}{5}x$

swap $x = 8 - \frac{3}{5}y$

$y = \frac{40-5x}{3}$

$f^{-1}(x) = \frac{40-5x}{3},\ x \in \mathbb{R}$

2 **a** $y = \ln x$

swap $x = \ln y$

$y = e^x$

$f^{-1}(x) = e^x,\ x \in \mathbb{R}$

b $y = \frac{1}{x}$

swap $x = \frac{1}{y}$

$y = \frac{1}{x}$

$f^{-1}(x) = \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

c $y = \sqrt[4]{x}$

swap $x = \sqrt[4]{y}$

$y = x^4$

$f^{-1}(x) = x^4,\ x \in \mathbb{R},\ x > 0$

d $y = 3x - 4$

swap $x = 3y - 4$

$y = \frac{x+4}{3}$

$f(0) = -4,\ f(3) = 5$

$f^{-1}(x) = \frac{x+4}{3},\ x \in \mathbb{R},\ -4 \leq x < 5$

e $y = \frac{1}{x-5}$

swap $x = \frac{1}{y-5}$

$y = \frac{1}{x} + 5$

$f^{-1}(x) = \frac{1}{x} + 5,\ x \in \mathbb{R},\ x \neq 0$

f $y = 2 + \frac{1}{x}$

swap $x = 2 + \frac{1}{y}$

$y = \frac{1}{x-2}$

$f^{-1}(x) = \frac{1}{x-2},\ x \in \mathbb{R},\ x \neq 2$

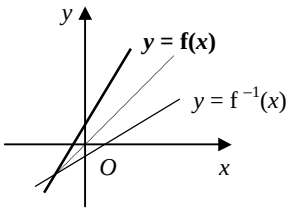
3 **a** **i** $y = 2x + 1$

swap $x = 2y + 1$

$y = \frac{x-1}{2}$

$f^{-1} : x \to \frac{x-1}{2},\ x \in \mathbb{R}$

ii



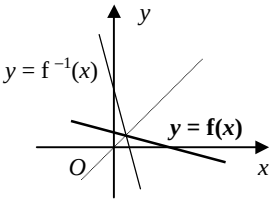
b **i** $y = \frac{1-x}{5}$

swap $x = \frac{1-y}{5}$

$y = 1 - 5x$

$f^{-1} : x \to 1 - 5x,\ x \in \mathbb{R}$

ii



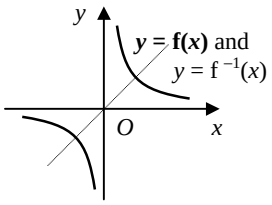
c **i** $y = \frac{10}{x}$

swap $x = \frac{10}{y}$

$y = \frac{10}{x}$

$f^{-1} : x \to \frac{10}{x}, x \in \mathbb{R}, x \neq 0$

ii



d **i** $y = x^2$

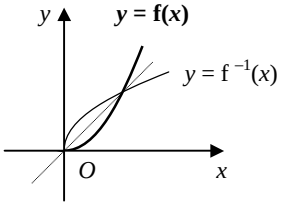
swap $x = y^2$

$y = \pm\sqrt{x}$

(domain of $f \Rightarrow +$)

$f^{-1} : x \to \sqrt{x}, x \in \mathbb{R}, x > 0$

ii



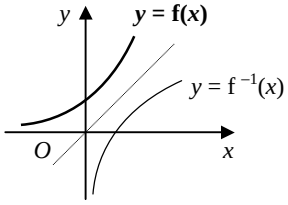
e **i** $y = e^x$

swap $x = e^y$

$y = \ln x$

$f^{-1} : x \to \ln x, x \in \mathbb{R}, x > 0$

ii



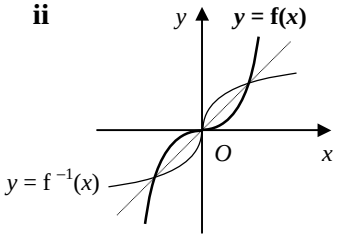
f **i** $y = x^3$

swap $x = y^3$

$y = \sqrt[3]{x}$

$f^{-1} : x \to \sqrt[3]{x},\ x \in \mathbb{R}$

ii



4 **a** $y = 5x + 1$

swap $x = 5y + 1$

$f^{-1}(x) = y = \frac{x-1}{5}$

$\frac{x-1}{5} = 2$

$x - 1 = 10$

$x = 11$

b $y = \frac{2x-4}{3}$

swap $x = \frac{2y-4}{3}$

$f^{-1}(x) = y = \frac{3x+4}{2}$

$\frac{3x+4}{2} = 7 - x$

$3x + 4 = 14 - 2x$

$x = 2$

c $y = e^x + 2$

swap $x = e^y + 2$

$f^{-1}(x) = y = \ln(x - 2)$

$\ln(x - 2) = \ln(3x - 8)$

$x - 2 = 3x - 8$

$x = 3$

d $y = \sqrt{x+2}$

swap $x = \sqrt{y+2}$

$f^{-1}(x) = y = x^2 - 2$

$x^2 - 2 = 3x - 4$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1, 2$

e $y = \frac{4}{x+3}$

swap $x = \frac{4}{y+3}$

$f^{-1}(x) = y = \frac{4}{x} - 3$

$\frac{4}{x} - 3 = 5(x + 1)$

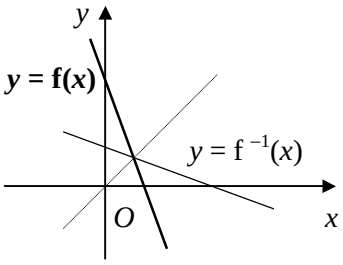
$4 - 3x = 5x(x + 1)$

$5x^2 + 8x - 4 = 0$

$(5x - 2)(x + 2) = 0$

$x = -2, \frac{2}{5}$

5 **a**



b $4 - 2x = x$

$x = \frac{4}{3}$

$\therefore (\frac{4}{3}, \frac{4}{3})$

6 **a** g $\Rightarrow$ $y = \frac{1}{2x+4}$

swap $x = \frac{1}{2y+4}$

$y = \frac{1}{2}(\frac{1}{x} - 4) = \frac{1}{2x} - 2$

$g^{-1}(x) = \frac{1}{2x} - 2,\ x \in \mathbb{R},\ x \neq 0$

range: $g^{-1}(x) \in \mathbb{R},\ g^{-1}(x) \neq -2$

b $= g(3 - 2x)$

$= \frac{1}{2(3-2x)+4} = \frac{1}{10-4x}$

$gf(x) = \frac{1}{10-4x},\ x \in \mathbb{R},\ x \neq \frac{5}{2}$

c f $\Rightarrow$ $y = 3 - 2x$

swap $x = 3 - 2y$

$f^{-1}(x) = y = \frac{3-x}{2}$

$\therefore \frac{1}{10-4x} = \frac{3-x}{2}$

$2 = (3 - x)(10 - 4x)$

$2x^2 - 11x + 14 = 0$

$(2x - 7)(x - 2) = 0$

$x = 2, \frac{7}{2}$

7 **a** **i** $y = 5x + 2$

swap $x = 5y + 2$

$y = \frac{x-2}{5}$

$f^{-1} : x \to \frac{x-2}{5},\ x \in \mathbb{R}$

ii $= f(\frac{1}{x})$

$= \frac{5}{x} + 2$

$fg : x \to \frac{5}{x} + 2,\ x \in \mathbb{R},\ x \neq 0$

iii $y = \frac{5}{x} + 2$

swap $x = \frac{5}{y} + 2$

$y = \frac{5}{x-2}$

$(fg)^{-1} : x \to \frac{5}{x-2},\ x \in \mathbb{R},\ x \neq 2$

b $\frac{x-2}{5} = \frac{5}{x} + 2$

$x(x - 2) = 25 + 10x$

$x^2 - 12x - 25 = 0$

$x = \frac{12 \pm \sqrt{144+100}}{2} = 6 \pm \sqrt{61} = -1.81, 13.81$

8 **a** $y = \frac{1}{2}\ln(4x - 9)$

swap $x = \frac{1}{2}\ln(4y - 9)$

$4y - 9 = e^{2x}$

$y = \frac{1}{4}(e^{2x} + 9)$

$f^{-1} : x \to \frac{1}{4}(e^{2x} + 9),\ x \in \mathbb{R}$

b $y = \frac{x-2}{x+5}$

swap $x = \frac{y-2}{y+5}$

$xy + 5x = y - 2$

$y(1 - x) = 5x + 2$

$y = \frac{5x+2}{1-x}$

$f^{-1} : x \to \frac{5x+2}{1-x},\ x \in \mathbb{R},\ x \neq 1$

c $y = e^{0.4x - 2}$

swap $x = e^{0.4y - 2}$

$0.4y - 2 = \ln x$

$y = \frac{5}{2}(2 + \ln x)$

$f^{-1} : x \to 5 + \frac{5}{2}\ln x,\ x \in \mathbb{R},\ x > 0$

d $y = \sqrt[3]{x^5 - 3}$

swap $x = \sqrt[3]{y^5 - 3}$

$y^5 - 3 = x^3$

$y = \sqrt[5]{x^3 + 3}$

$f^{-1} : x \to \sqrt[5]{x^3 + 3},\ x \in \mathbb{R}$

e $y = \log_{10}(2 - 7x)$

swap $x = \log_{10}(2 - 7y)$

$2 - 7y = 10^x$

$y = \frac{1}{7}(2 - 10^x)$

$f^{-1} : x \to \frac{1}{7}(2 - 10^x),\ x \in \mathbb{R}$

f $y = \frac{4-x}{3x+2}$

swap $x = \frac{4-y}{3y+2}$

$3xy + 2x = 4 - y$

$y(3x + 1) = 4 - 2x$

$y = \frac{4-2x}{3x+1}$

$f^{-1} : x \to \frac{4-2x}{3x+1},\ x \in \mathbb{R},\ x \neq -\frac{1}{3}$

9 **a** **i** $y = e^{2x}$

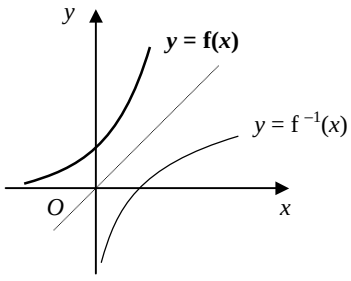
swap $x = e^{2y}$

$2y = \ln x$

$y = \frac{1}{2}\ln x$

$f^{-1} : x \to \frac{1}{2}\ln x,\ x \in \mathbb{R},\ x > 0$

ii



b **i** $y = x^2 + 4$

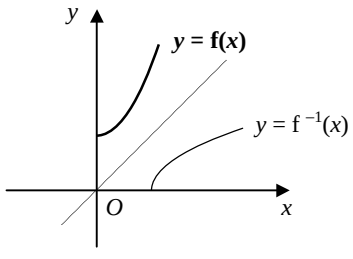
swap $x = y^2 + 4$

$y = \pm\sqrt{x - 4}$

(domain of $f \Rightarrow +$)

$f^{-1} : x \to \sqrt{x - 4},\ x \in \mathbb{R},\ x > 4$

ii



c **i** $y = \ln(x - 3)$

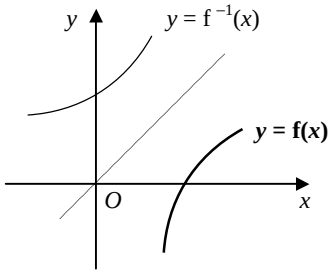
swap $x = \ln(y - 3)$

$y - 3 = e^x$

$y = e^x + 3$

$f^{-1} : x \to e^x + 3,\ x \in \mathbb{R}$

ii



d **i** $y = x^2 + 6x + 9 = (x + 3)^2$

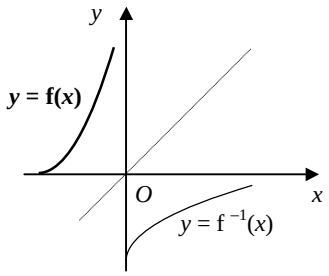
swap $x = (y + 3)^2$

$y = -3 \pm \sqrt{x}$

(domain of $f \Rightarrow +$)

$f^{-1} : x \to -3 + \sqrt{x},\ x \in \mathbb{R},\ x > 0$

ii



10 **a** **i** $f(x) = (x + 3)^2 - 6$

$x < -3$ ∴ range: $f(x) > -6$

ii $y = (x + 3)^2 - 6$

swap $x = (y + 3)^2 - 6$

$y = -3 \pm \sqrt{x + 6}$

(domain of $f \Rightarrow -$)

$f^{-1}(x) = -3 - \sqrt{x + 6},\ x \in \mathbb{R},\ x > -6$

b **i** $f(x) = (x - 2)^2 + 1$

$x \geq 2$ ∴ range: $f(x) \geq 1$

ii $y = (x - 2)^2 + 1$

swap $x = (y - 2)^2 + 1$

$y = 2 \pm \sqrt{x - 1}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = 2 + \sqrt{x - 1},\ x \in \mathbb{R},\ x \geq 1$

c **i** $f(x) = (x + \frac{5}{2})^2 - \frac{33}{4}$

$x < -\frac{5}{2}$ ∴ range: $f(x) > -8\frac{1}{4}$

ii $y = (x + \frac{5}{2})^2 - \frac{33}{4}$

swap $x = (y + \frac{5}{2})^2 - \frac{33}{4}$

$y = -\frac{5}{2} \pm \sqrt{x + \frac{33}{4}}$

(domain of $f \Rightarrow -$)

$f^{-1}(x) = -\frac{5}{2} - \sqrt{x + \frac{33}{4}},\ x \in \mathbb{R},\ x > -8\frac{1}{4}$

d **i** $f(x) = (x - \frac{3}{2})^2 + \frac{11}{4}$

$2 < x < 4,\ f(2) = 3,\ f(4) = 9$

∴ range: $3 < f(x) < 9$

ii $y = (x - \frac{3}{2})^2 + \frac{11}{4}$

swap $x = (y - \frac{3}{2})^2 + \frac{11}{4}$

$y = \frac{3}{2} \pm \sqrt{x - \frac{11}{4}}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = \frac{3}{2} + \sqrt{x - \frac{11}{4}},\ x \in \mathbb{R},\ 3 < x < 9$

e **i** $f(x) = 8 - 2x - x^2 = 9 - (x + 1)^2$

$x \geq -1$ ∴ range: $f(x) \leq 9$

ii $y = 9 - (x + 1)^2$

swap $x = 9 - (y + 1)^2$

$y = -1 \pm \sqrt{9 - x}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = -1 + \sqrt{9 - x},\ x \in \mathbb{R},\ x \leq 9$

f **i** $f(x) = -5(x^2 - 4x) = 20 - 5(x - 2)^2$

$x > 2$ ∴ range: $f(x) < 20$

ii $y = 20 - 5(x - 2)^2$

swap $x = 20 - 5(y - 2)^2$

$y = 2 \pm \sqrt{\frac{20 - x}{5}}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = 2 + \sqrt{\frac{20 - x}{5}},\ x \in \mathbb{R},\ x < 20$

11 **a** $y = \frac{1}{3}(2x - 5)$

swap $x = \frac{1}{3}(2y - 5)$

$f^{-1}(x) = y = \frac{3x+5}{2}$

$\therefore \frac{3x+5}{2} = \frac{4}{2-x}$

$(3x + 5)(2 - x) = 8$

$3x^2 - x - 2 = 0$

$(3x + 2)(x - 1) = 0$

$x = -\frac{2}{3}, 1$

b $y = \ln \frac{x+3}{5}$

swap $x = \ln \frac{y+3}{5}$

$f^{-1}(x) = y = 5e^x - 3$

$\therefore 5e^x - 3 = 10 - 6e^{-x}$

$5e^{2x} - 13e^x + 6 = 0$

$(5e^x - 3)(e^x - 2) = 0$

$e^x = \frac{3}{5}, 2$

$x = \ln \frac{3}{5}, \ln 2$

$x = -0.511, 0.693$ (3sf)

c $y = x^2 - 4$

swap $x = y^2 - 4$

$y = \pm\sqrt{x+4}$

(domain of $f \Rightarrow +$)

$f^{-1}(x) = \sqrt{x+4}$

$\therefore \sqrt{x+4} = \frac{x+6}{3}$

$x + 4 = \frac{(x+6)^2}{9}$

$9(x + 4) = x^2 + 12x + 36$

$x^2 + 3x = 0$

$x(x + 3) = 0$

$x = -3, 0$

12 **a** -2

b $\frac{6+b}{6-2} = 4$

$6 + b = 16$

$b = 10$

c $y = \frac{x+10}{x-2}$

swap $x = \frac{y+10}{y-2}$

$xy - 2x = y + 10$

$y(x - 1) = 2x + 10$

$y = \frac{2x+10}{x-1}$

$f^{-1}(x) = \frac{2x+10}{x-1}, \ x \in \mathbb{R}, \ x \neq 1$

13 **a** $y = x^2 - 3x = (x - \frac{3}{2})^2 - \frac{9}{4}$

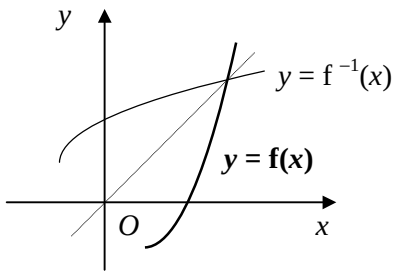
swap $x = (y - \frac{3}{2})^2 - \frac{9}{4}$

$y = \frac{3}{2} \pm \sqrt{x+\frac{9}{4}}$

(domain of $f \Rightarrow +$)

$f^{-1} : x \to \frac{3}{2} + \sqrt{x+\frac{9}{4}}, \ x \in \mathbb{R}, \ x \geq -\frac{9}{4}$

b



c $g \Rightarrow y = 2x + 3$

swap $x = 2y + 3$

$g^{-1}(x) = y = \frac{x-3}{2}$

$g^{-1}(12) = \frac{9}{2}$

$f^{-1}g^{-1}(12) = f^{-1}(\frac{9}{2})$

$= \frac{3}{2} + \sqrt{\frac{27}{4}}$

$= \frac{3}{2} + \frac{3}{2}\sqrt{3}$

$= \frac{3}{2}(1 + \sqrt{3})$

$\therefore a = 1\frac{1}{2}$