⟨**C3**⟩ **FUNCTIONS**   *Answers - Worksheet E*

Note: For this worksheet especially, there may be alternative correct answers

1　**a**　translated 3 units in negative *x*-direction and translated 2 units in positive *y*-direction

　　b　reflected in the *y*-axis and stretched by a factor of 2 in *y*-direction

　　c　translated 1 unit in positive *x*-direction and stretched by a factor of 3 in *y*-direction

　　d　reflected in the *x*-axis and then translated 4 units in positive *y*-direction

2　**a**　$= (x + 3)^2 - 9 + 2 = (x + 3)^2 - 7$

　　b　translation by 3 units in negative *x*-direction and translation by 7 units in negative *y*-direction

3　**a**　$y = 2[2(x - 3) + 7] \quad \Rightarrow \quad y = 4x + 2$

　　b　$y = 2[3e^{(x - 3)}] \quad\quad \Rightarrow \quad y = 6e^{x - 3}$

　　c　$y = 2[(x - 3)^2 - 3(x - 3) + 1] \quad \Rightarrow \quad y = 2x^2 - 18x + 38$

　　d　$y = 2[\dfrac{1}{(x - 3)}] \quad\quad \Rightarrow \quad y = \dfrac{2}{x - 3}$

4　**a**　stretch by a factor of $\frac{1}{3}$ in *x*-direction and reflection in the *x*-axis (either first)

　　b　reflection in the *y*-axis and translation by 5 units in positive *y*-direction (either first)

　　c　translation by 4 units in negative *x*-direction and stretch by a factor of 3 in *y*-direction (either first)

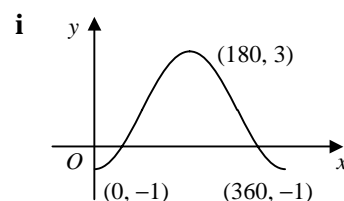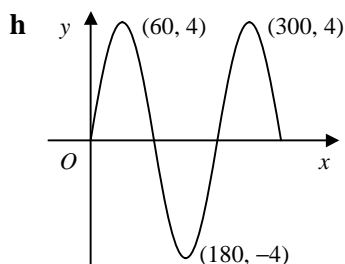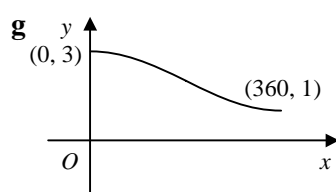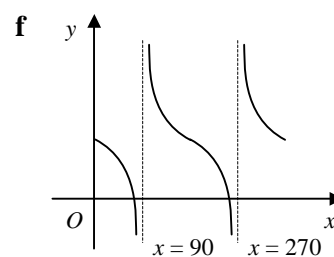　　**d**　stretch by a factor of 3 in *y*-direction, then translation by 2 units in positive *y*-direction

5　**a**　　**b**　　**c**　　**d**　
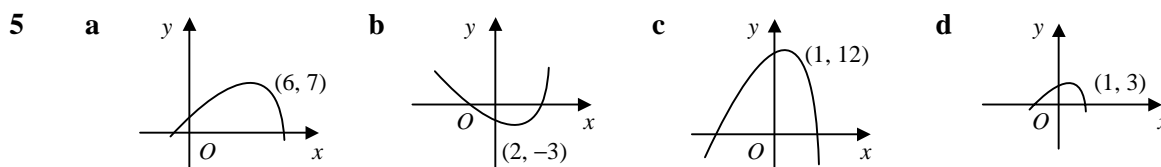
6　first　　$\Rightarrow \quad y = (x + 2)^2 + 4(x + 2) - 2 \quad \Rightarrow \quad y = x^2 + 8x + 10$
　　second　$\Rightarrow \quad y = 3[x^2 + 8x + 10] \quad\quad\quad \Rightarrow \quad y = 3x^2 + 24x + 30$
　　third　　$\Rightarrow \quad y = 3(-x)^2 + 24(-x) + 30 \quad \Rightarrow \quad y = 3x^2 - 24x + 30$

7　**a**　$= 2[x^2 - 2x] + 7 = 2[(x - 1)^2 - 1] + 7 = 2(x - 1)^2 + 5$

　　b　translation by 5 units in negative *y*-direction,
　　　　then stretch by a factor of $\frac{1}{2}$ in *y*-direction,
　　　　then translation by 1 unit in negative *x*-direction

8　**a**　$f'(x) = 3x^2 - 6x$
　　　　SP:　$3x^2 - 6x = 0$
　　　　　　$3x(x - 2) = 0$
　　　　　　$x = 0, 2$
　　　　$\therefore \ (0, 4) \text{ and } (2, 0)$

　　b i　$(0, -8)$ and $(2, 0)$　　　**ii**　$(0, 7)$ and $(4, 3)$　　　**iii**　$(2, 1)$ and $(4, 0)$

9   **a**   stretch by factor of 3 in *y*-direction,   **10**   **a**   180°
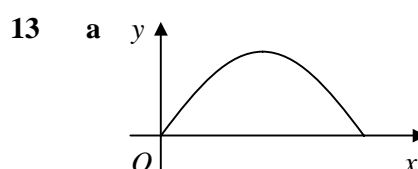   then reflection in *x*-axis,   **b**   (0, 1)
              then translation by 2 units in +ve *y*-dir'n   **c**   (90, 3) and (270, 3)

     b
              (0, 2), $(\frac{4}{9}, 0)$

11   **a**
              (0, 3), (180, 3), (360, 3), (90, −3), (270, −3)

     b
              x = 135, *x* = 315, *x* = 45, *x* = 225

     c
              (90, 3), (270, −1)

     d
              (210, 1), (30, −1)

     e
              (45, 2), (225, −2)

     f
              x = 90, *x* = 270

     g
              (0, 3), (360, 1)

     h
              (60, 4), (300, 4), (180, −4)

     i
              (180, 3), (0, −1), (360, −1)

12   **a**   60°

     b $\dfrac{360°}{k}$

13   **a**

     b   (π, 2)

     c $2 \sin \frac{1}{2} x = \sqrt{2}$

              $\sin \frac{1}{2} x = \dfrac{1}{\sqrt{2}}$

              $\frac{1}{2} x = \frac{\pi}{4}, \pi - \frac{\pi}{4}$

              $= \frac{\pi}{4}, \frac{3\pi}{4}$
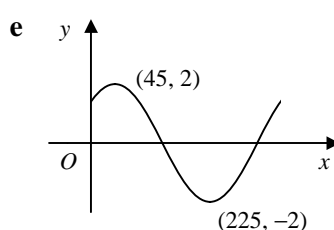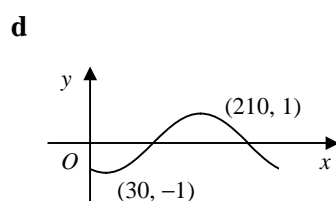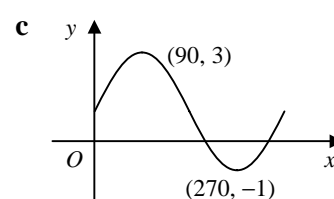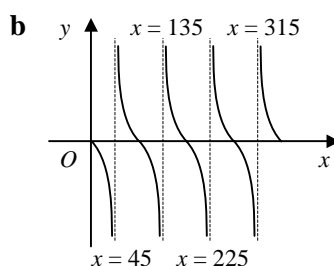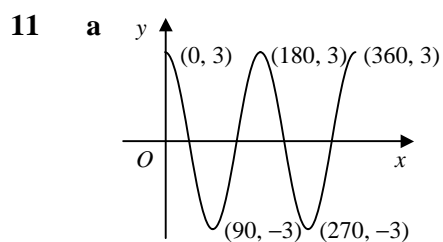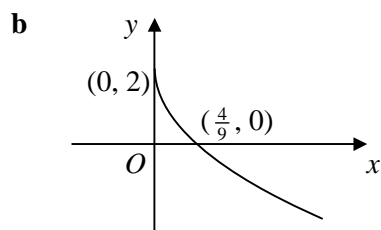
              $x = \frac{\pi}{2}, \frac{3\pi}{2}$

14   **a**   max. value 4   ∴ *a* = 4

              max. occurs at *x* = 45   ∴ *b* = 2

     b   (135, −4)