Core 2 - May 2007

1a) i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}} = x^{\frac{3}{2}+\frac{1}{2}}$

$= \underline{\underline{x^2}}$

ii) $x^{\frac{3}{2}} \div x^1 = x^{\frac{3}{2}-1}$

$= \underline{\underline{x^{1/2}}}$

iii) $(x^{\frac{3}{2}})^2 = x^{3/2 \times 2}$

$= \underline{\underline{x^3}}$

b) i) $\int 3x^{1/2}\,dx = \dfrac{3x^{\frac{3}{2}}}{3/2} + c$

$= \underline{\underline{2x^{3/2} + c}}$

ii) $\int_1^9 3x^{1/2}\,dx = \left[2x^{3/2}\right]_1^9$

$= \left(2(9)^{3/2}\right) - \left(2(1)^{3/2}\right)$

$= 54 - 2 = \underline{\underline{52}}$

2a) $U_n = 3 \times 4^n$

$U_1 = 3 \times 4^1 = \underline{\underline{12}}$

$U_2 = 3 \times 4^2 = \underline{\underline{48}}$

b) $r = \dfrac{48}{12} = \underline{\underline{4}}$

c) i) $S_{12} = \dfrac{a(1-r^n)}{1-r}$ $\quad a = 12,\ r = 4,\ n = 12$

$= \dfrac{12(1-4^{12})}{1-4} = \dfrac{12(1-4^{12})}{-3} = -4(1-4^{12})$

$= -4 + 4^{13} = \underline{\underline{4^{13} - 4}}$

ii) $\sum_{n=2}^{12} U_n = S_{12} - S_1$

$= 4^{13} - 4 - 12$

$= \underline{\underline{67108848}}$

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3a) arc length $= r\theta$ $\quad r = 20, \quad$ arc length $= 28$

$$28 = 20 \times \theta \quad (\div 20)$$

$$\theta = \frac{28}{20} = 1.4 \text{ (as req)}$$

b) $\text{area} = \frac{1}{2} r^2 \theta$

$$= \frac{1}{2} \times 20^2 \times 1.4$$

$$= 280\text{ cm}^2$$

c) i) shaded area = area of sector − area of triangle

Area of triangle $= \frac{1}{2} ab\sin C \qquad a = 15, \; b = 20 \quad \hat{C} = 1.4$

$$= \frac{1}{2} \times 15 \times 20 \times \sin 1.4$$

* RADIANS MODE *

$$= 147.8174595$$

shaded area $= 280 - 147.817\ldots$

$$= 132.1825\ldots$$

$$= 132\text{ cm}^2 \text{ (3sf)}$$

ii) $BD^2 = 20^2 + 15^2 - (2 \times 20 \times 15 \times \cos 1.4)$

$$BD^2 = 523.019\ldots$$

$$BD = \sqrt{523.019\ldots}$$

$$= 22.8696\ldots$$

$$= 22.9\text{ cm (3sf)}$$

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4) $S_{29} = 1102$ $S_n = \frac{n}{2}(2a + (n-1)d$

a) $\frac{29}{2}(2a + 28d) = 1102$

$29(a + 14d) = 1102$ $(\div 29)$

$a + 14d = 38$ (as required)

b) $U_2 + U_7 = 13$ $U_n = a + (n-1)d$

$a + d + a + 6d = 13$ $U_2 = a + d$, $U_7 = a + 6d$

$2a + 7d = 13$ ① $(\times 2)$

$a + 14d = 38$ ②

$4a + 14d = 26$ ③ → ③ - ②

$-$

$a + 14d = 38$

$3a = -12$

$a = -4$ → sub in to find d into ②

$-4 + 14d = 38$

$14d = 42$

$d = 3$

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5) $y = \left(1+\frac{2}{x}\right)^2$

a) at P, $x = 2$:-

$y = \left(1+\frac{2}{2}\right)^2 = \underline{4}$

b) $\left(1+\frac{2}{x}\right)^2 = \left(1+\frac{2}{x}\right)\left(1+\frac{2}{x}\right)$

$= 1 + \frac{2}{x} + \frac{2}{x} + \frac{4}{x^2}$

$= \underline{1 + \frac{4}{x} + \frac{4}{x^2}}$

c) $y = 1 + 4x^{-1} + 4x^{-2}$

$\frac{dy}{dx} = \underline{-4x^{-2} - 8x^{-3}}$

d) when $x = 2$ (P) :-

$\frac{dy}{dx} = -4(2)^{-2} - 8(2)^{-3}$

$= -1 - 1 = \underline{-2}$ (as req)

e) gradient of tangent is -2

gradient of normal is $\frac{1}{2}$, $P(2, 4)$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{1}{2}(x - 2)$

$2y - 8 = x - 2$

$\underline{x - 2y + 6 = 0}$ (as req)

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⑥ $y = 3(2^x + 1)$

a) at A, $x = 0$:-

$y = 3(2^0 + 1)$

$y = 6$

b) $\int_0^6 3(2^x + 1)\,dx$ $n = 3$ $h = \frac{6-0}{3} = 2$

x	0	2	4	6
y	6	15	51	195

$\int_0^6 3(2^x + 1)\,dx \approx \frac{2}{2}\left(6 + 2(15 + 51) + 195\right)$

$= 333$

c) $y = 21$ intersects $y = 3(2^x + 1)$ at P

i) $\therefore 3(2^x + 1) = 21$ $(\div 3)$

$2^x + 1 = 7$ (-1)

$2^x = 6$ (as req)

ii) $2^x = 6$

$x \log 2 = \log 6$

$x = \frac{\log 6}{\log 2}$

$x = 2.584962501$

$x = 2.58$ (3sf)

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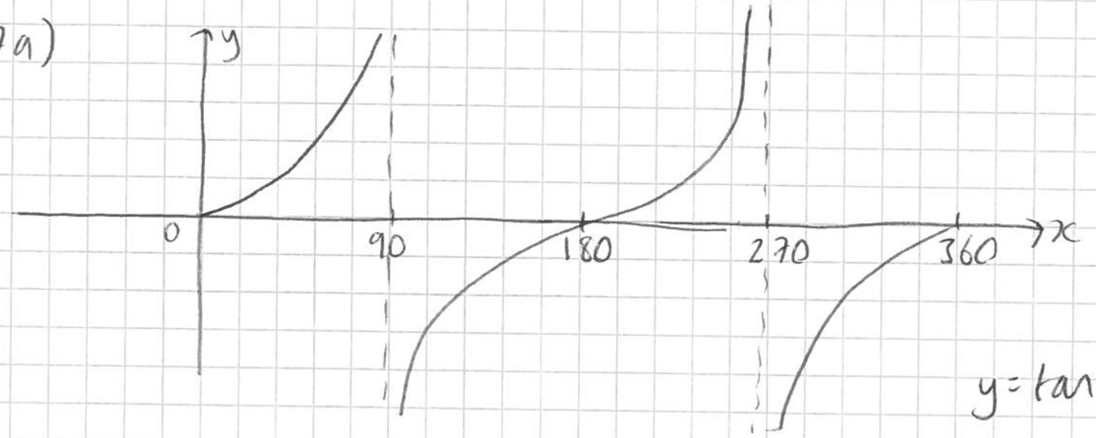
7a)



b) $\tan x = \tan 61$ $\quad 0 \leq x \leq 360$

$x = \underline{61}$, $x = 180 + 61$ (graph or CAST)

$x = \underline{241}$

ci) $\sin\theta + \cos\theta = 0$ $\quad (-\cos\theta)$

$\sin\theta = -\cos\theta$ $\quad (\div \cos\theta)$

$\frac{\sin\theta}{\cos\theta} = \frac{-\cos\theta}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\underline{\tan\theta = -1}$ (as req)

ii) $\sin(x-20) + \cos(x-20) = 0$

* DEGREES MODE *

$\therefore \tan(x-20) = -1$

let $\theta = x - 20$ * adjust range

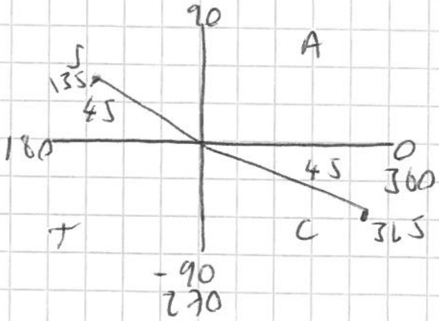
$-20 \leq x - 20 \leq 340$

$x - 20 = \tan^{-1}(-1)$

$x - 20 = -45°$

$x - 20 = 135, 315$

$x = \underline{155°}, \underline{335°}$



d) $y = \tan x \rightarrow y = \tan(x - 20°)$

translation $\begin{pmatrix} 20° \\ 0 \end{pmatrix}$

e) stretch scale factor $\frac{1}{4}$ in x direction → replace 'x' with '$4x$'

$\underline{f(x) = \tan 4x}$

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8) $\log_a n = \log_a 3 + \log_a (2n-1)$

a) $\log_a n = \log_a 3(2n-1)$

$n = 3(2n-1)$

$n = 6n - 3$

$5n = 3$

$n = \frac{3}{5}$

b) $\log_a x = 3$ $\quad\quad$ $\log_a y - 3\log_a 2 = 4$

i) $x = a^3$

ii) $\log_a y - 3\log_a 2 = 4$

$\log_a y = 3\log_a 2 + 4$

$\log_a y = \log_a 2^3 + 4$

$\log_a y = \log_a 8 + 4$ $\quad \rightarrow \quad$ $\log_a y - \log_a 8 = 4$

$\frac{y}{8} = a^4$ $\quad\quad$ $\log_a \left(\frac{y}{8}\right) = 4$

$y = 8a^4$

$xy = a^3 \times 8a^4$

$xy = 8a^7$