CORE MATHEMATICS (C) UNIT 1 TEST PAPER 4

1. Simplify as far as possible: (i) $\left(3^{\frac{1}{2}}+3^{-\frac{1}{2}}\right)\left(3^{\frac{3}{2}}-3^{\frac{1}{2}}\right)$, (ii) $\left(4^{-3}\right)^{\frac{1}{3}}$. [4]

2. A ball is thrown from a point O. After t seconds its distance from O is x m, where $x = 40t - 5t^2$. Find the rate of change of x with time after 3 seconds. [4]

3. Differentiate with respect to x:
   (i) $\frac{1}{2x} - \sqrt{x}$, (ii) $\frac{x^2+3}{2x^2}$. [7]

4. A rectangular garden is to have length x m, where $x > 0$. The width of the garden must be 4 m less than its length.
   The perimeter of the garden cannot be more than 36 m and the area must be at least 60 m^2.
   (i) Form a linear inequality and a quadratic inequality in x. [4]
   (ii) Solve your inequalities to find the range of allowable values of x. [5]

5. (i) Sketch on one diagram the straight line $y = 4 - x$ and the curve $y = \frac{1}{2x}$. [5]
   The line intersects the curve at the points P and Q.
   (ii) Show that the x-coordinates of P and Q are $2 + a$ and $2 - a$, where a is an irrational number to be found. [5]

6. The straight line $4y + 3x = 7$ is the tangent at the point P (1, 1) to a circle with centre C.
   (i) Find an equation of the straight line which passes through P and C. [4]
   (ii) Given that the x-coordinate of C is 4, find the y-coordinate of C. [2]
   (iii) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [5]

7. (i) Express $x^2 - 4kx + 9$ in the form $(x + p)^2 + q$ where p and q are given in terms of k. [4]
   (ii) Find the roots of the equation $x^2 - 4kx + 9 = 0$ in terms of k and deduce the set of values of k for which these roots are real and distinct. [5]
   (iii) Find the exact roots of the equation $x^2 - 4kx + 9 = 0$ in the case $k = \sqrt{3}$. [3]

8. The curve C has equation $y = x^2 - 5x + 7$.

(i) Find the coordinates of the stationary point on C. [3]

(ii) Find an equation of the normal to C at the point where $x = 1$. [5]

(iii) Calculate the coordinates of the points where this normal intersects C again. [7]

CORE MATHS 1 (C) TEST PAPER 4 : ANSWERS AND MARK SCHEME

1. (i) $9 - 1 = 8$ (ii) $4^{-1} = 1/4$ M1 A1 M1 A1 4

2. $dx/dt = 40 - 10t$ When $t = 3$, rate of change = 10 m/s M1 A1 M1 A1 4

3. (i) $\frac{d}{dx}\left(\frac{1}{2}x^{-1} - x^{1/2}\right) = -\frac{1}{2}x^{-2} - \frac{1}{2}x^{-1/2} = -\frac{1}{2}\left(\frac{1}{x^2} + \frac{1}{\sqrt{x}}\right)$ B1 B1 M1 A1

(ii) $\frac{d}{dx}\left(\frac{1}{2} + \frac{3}{2}x^{-2}\right) = -3x^{-3} = -\frac{3}{x^3}$ M1 A1 A1 7

4. (i) Width $= x - 4$ $4x - 8 \le 36$ $x(x-4) \ge 60$ M1 A1 M1 A1

(ii) $4x \le 44$ so $x \le 11$ $x^2 - 4x - 60 \ge 0$ B1 M1 A1

$(x + 6)(x - 10) \ge 0$, so $x \ge 10$ Hence $10 \le x \le 11$ M1 A1 9

5. (i) Line sketched, and curve with asymptotes $x = 0$, $y = 0$ B2 B3

(ii) Intersect where $2x(4 - x) = 1$ $2x^2 - 8x + 1 = 0$ M1 A1

$2(x-2)^2 - 7 = 0$ $x = 2 \pm \sqrt{7/2}$, so $a = \sqrt{7/2}$ M1 A1 A1 10

6. (i) PC is perp. to tangent so has gradient 4/3 $y - 1 = 4/3\ (x - 1)$ M1 A1 M1 A1

(ii) $y - 1 = 4/3\ (3) = 4$ $y = 5$ at C M1 A1

(iii) Radius = distance from (1, 1) to (4, 5) = 5 M1 A1

$(x-4)^2 + (y-5)^2 = 25$ $x^2 + y^2 - 8x - 10y + 16 = 0$ M1 A1 A1 11

7. (i) $x^2 - 4kx + 9 = (x - 2k)^2 + (9 - 4k^2)$ M1 M1 A1 A1

(ii) $x = 2k \pm \sqrt{4k^2 - 9}$, real and distinct for $4k^2 > 9$ $k < -3/2$, $k > 3/2$ M1 A1 M1 A1 A1

(iii) When $k = \sqrt{3}$, $x = 2\sqrt{3} \pm \sqrt{3} = \sqrt{3}$ or $3\sqrt{3}$ M1 A1 A1 12

8. (i) $2x - 5 = 0$ when $x = 2.5$ Stat. point (2.5, 0.75) M1 A1 A1

(ii) At (1, 3), gradient = –3 Normal is $y - 3 = 1/3\ (x - 1)$ B1 M1 A1 M1 A1

(iii) $3y = x + 8$ $3(x^2 - 5x + 7) = x + 8$ $3x^2 - 16x + 13 = 0$ M1 A1

$(3x - 13)(x - 1) = 0$ $x = 1, x = 13/3$ M1 A1 A1

When $x = 13/3$, $y = (13/3 + 8)/3 = 37/9$ Point is (13/3, 37/9) M1 A1 I5