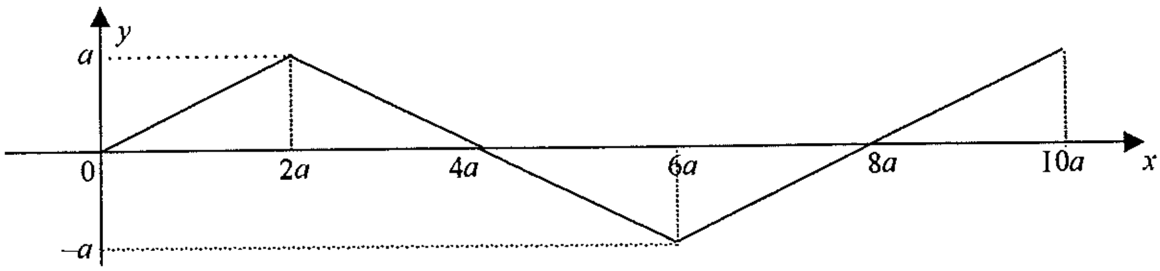
CORE MATHEMATICS (C) UNIT 1 TEST PAPER 6

1. The equations of two straight lines are $2x = y + 3$ and $2y = x + 3$.
   (i) Determine by calculation whether the lines are perpendicular. [2]
   (ii) Calculate the coordinates of the point where the lines intersect. [2]

2. (i) Express $(1+\sqrt{2})^2$ in the form $a + b\sqrt{2}$, where a and b are integers to be found. [2]
   (ii) By substituting $x = y^{\frac{1}{2}}$, or otherwise, solve the equation $6y^{\frac{1}{2}} = y + 9$. [4]

3. The diagram shows the graph of $y = \text{f}(x)$.



   Sketch, for $0 \leq x \leq 8a$, graphs of (i) $y = \text{f}(x + a)$, (ii) $y = \text{f}(x) + a$.
   In each case, indicate clearly the coordinates of any points of intersection with the axes. [6]

4. Solve for x the equations
   (i) $5^{2x+1} = \dfrac{1}{25}$, (ii) $\sqrt{x+3} = x - 3$. [7]

5. The circle C has equation $x^2 + y^2 - 2x + 5y + 7 = 0$.
   (i) Find the coordinates of the centre of C and show that the radius of C is $\dfrac{1}{2}$. [4]
   (ii) Show that the point A $(1, -2)$ lies on C and find the coordinates of the point B such that AB is a diameter of C. [3]

6. Given that $W = 18v + \dfrac{2500}{v}$, where $v > 0$,
   (i) find $\dfrac{\text{d}W}{\text{d}v}$ and $\dfrac{\text{d}^2W}{\text{d}v^2}$ in terms of v. [4]
   (ii) Find the value of v when $\dfrac{\text{d}W}{\text{d}v} = 2$. [3]

7. In this question, $f(x) \equiv 1 - 5x$ and $g(x) \equiv \dfrac{16}{1-5x}$.

   Solve for x the equations

   (i) $f(x) = g(x)$, [4]

   (ii) $f\left(\dfrac{1}{x}\right) = g(x)$. [6]

8. The point $(5, k)$ lies on the curve $y = x(8 - x)$.

   (i) Find the value of k. [1]

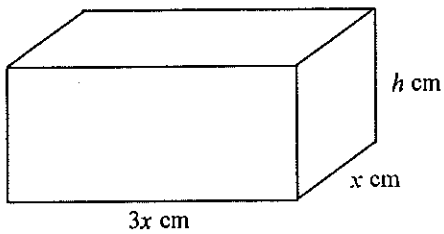
   The tangent to the curve $y = x(8 - x)$ at $(5, k)$ meets the x-axis at P. This tangent also meets the line of symmetry of the curve at Q.

   (ii) Find the coordinates of P and Q. [7]

   R is the point where the curve cuts the x-axis, for $x > 0$.

   (iii) Find the area of the triangle PQR. [3]

9. A closed rectangular box has width x cm, length $3x$ cm and height h cm, as shown.



   (i) Show that the total surface area of the box is $2x(3x + 4h)$ cm^2. [4]

   (ii) Find an expression in terms of x and h for the volume of the box. [2]

   (iii) Given that the surface area has a fixed value of 450 cm^2, show that $h = \dfrac{225 - 3x^2}{4x}$. [2]

   (iv) Use differentiation to find the value of x for which the volume is maximum subject to the surface area being 450 cm^2, and state this maximum volume. [6]

CORE MATHS 1 (C) TEST PAPER 6 : ANSWERS AND MARK SCHEME

1.	(i) Gradients are 2 and ½, so not perpendicular	M1 A1	
	(ii) $4y - 6 = y + 3$ $y = 3$ Point is (3, 3)	M1 A1	4
2.	(i) $1 + 2 + 2\sqrt{2}$ $a = 3, b = 2$	M1 A1	
	(ii) $6x = x^2 + 9$ $x^2 - 6x + 9 = 0$ $x = 3$ $y = 9$	M1 A1 M1 A1	6
3.	(i) Translated a units to the left, through $(-a, 0)$, $(0, a/2)$, $(3a, 0)$, $(7a, 0)$	B3	
	(ii) Translated a units upwards, through $(0, a)$, $(6a, 0)$	B3	6
4.	(i) $2x + 1 = -2$ $x = -3/2$	M1 A1	
	(ii) $x + 3 = x^2 - 6x + 9$ $x^2 - 7x + 6 = 0$ $(x - 1)(x - 6) = 0$	M1 A1 A1	
	$x = 1$ or 6 Must have $x - 3 > 0$, so $x = 6$	A1 A1	7
5.	(i) $(x - 1)^2 + (y + 5/2)^2 - ¼ = 0$ Centre = $(1, -5/2)$, radius = ½	M1 M1 A1 A1	
	(ii) $1 + 4 - 2 - 10 + 7 = 0$ $B = (1, -3)$	B1 M1 A1	7
6.	(i) $\dfrac{dW}{dv} = 18 - \dfrac{2500}{v^2}$, $\dfrac{d^2W}{dv^2} = \dfrac{5000}{v^3}$	M1 A1 M1 A1	
	(ii) $18 - \dfrac{2500}{v^2} = 2$ $v^2 = 2500/16$ $v > 0$ so $v = 50/4 = 12.5$	M1 A1 A1	7
7.	(i) $(1 - 5x)^2 = 16$ $1 - 5x = 4$ or -4 $x = -3/5$ or $x = 1$	B1 M1 A1 A1	
	(ii) $1 - \dfrac{5}{x} = \dfrac{16}{1 - 5x}$ $x(1 - 5x) - 5(1 - 5x) = 16x$	M1 A1 A1	
	$5x^2 - 10x + 5 = 0$ $5(x - 1)^2 = 0$ $x = 1$	M1 A1 A1	10
8.	(i) $k = 5(3) = 15$	B1	
	(ii) Gradient = $8 - 2x = -2$ at (5, 15) Tangent is $y - 15 = -2(x - 5)$	M1 A1 M1 A1	
	P is (12.5, 0) At Q, $x = 4$ so Q is (4, 17)	A1 M1 A1	
	(iii) R is (8, 0) Area = $9/4 \times 17 = 153/4$	B1 M1 A1	11
9.	(i) $A = 2(xh) + 2(3xh) + 2(3x^2) = 6x^2 + 8xh = 2x(3x + 4h)$	M1 A1 M1 A1	
	(ii) $V = 3x^2h$ cm^3	M1 A1	
	(iii) If $A = 450$, $3x^2 + 4xh = 225$ $h = (225 - 3x^2)/4x$	M1 A1	
	(iv) Then $V = 675x/4 - 9x^3/4$ $dV/dx = 675/4 - 27x^2/4 = 0$ when $x = 5$	M1 A1 M1 A1	
	$h = 15/2$ $V_{max} = 562.5$	M1 A1	14

