CORE MATHEMATICS (C) UNIT 1 TEST PAPER 8

1. Express in its simplest form without brackets: $(x - 2y)^2(x^2 - 4y^2)$ [4]

2. Find the coordinates of the minimum point on the graph of $y = 8x + \dfrac{1}{2x^2}$. [5]

3. The lines l_1 and l_2 have equations $3y = 2x - 4$ and $2y = 3 - 3x$.
   (i) Find the coordinates of the point where l_1 and l_2 intersect. [4]
   (ii) Show, by calculation, that l_1 and l_2 are perpendicular. [2]

4. (i) Find the exact value of $\left(\sqrt{2} + \sqrt{18}\right)^2$. [3]
   (ii) Express in its simplest form with a rational denominator: $\dfrac{1}{5 - \sqrt{15}}$. [3]

5. The line $y + x = 23$ cuts the curve $y = 25 - (x - 4)^2$ at the points P and Q.
   The x-coordinate of P is less than that of Q.
   Find the coordinates of P and of Q. [6]

6. The line with equation $x + y = k$ meets the circle with equation $x^2 + y^2 = k$ in two distinct points.
   Find the range of possible values of k. [7]

7. The equation of a curve is $y = x^3 - 5x^2 + 4x + 2$.
   (i) Find an equation of the tangent to the curve at the point $(2, -2)$. [4]
   (ii) Find the x-coordinates of the points on the curve where the tangent has gradient -3. [4]

8. Sketch graphs of each of the following, showing clearly the behaviour of the graphs as they intersect or approach the coordinate axes.
   (i) $y = (x + 1)^2(x - 1)$, [3]
   (ii) $y = -2\sqrt{x}$, $x > 0$, [3]
   (iii) $y = -\dfrac{1}{x^2}$, $x \neq 0$. [3]

9. Express in the form 9^y

(i) $\frac{1}{9^{1-x}}$, (ii) 81^{x-2}, (iii) 3^{4x+6}. [5]

Hence, or otherwise, find the value of x for which $\frac{1}{9^{1-x}} = \frac{81^{x-2}}{3^{4x+6}}$. [4]

10. The circle C has centre $(-2, t)$ and radius $2\sqrt{2}$.

(i) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where b and c are to be expressed in terms of t. [3]

Given that C passes through the point P (0, 2),

(ii) find the possible values of t. [3]

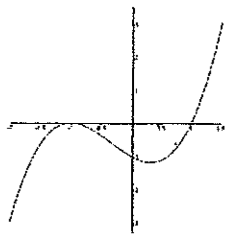
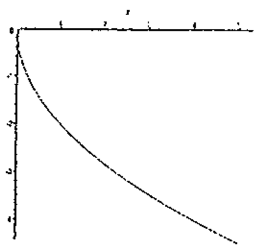
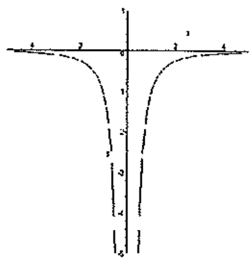
Given also that $t > 0$,

(iii) find an equation of the tangent to C at P. [4]

(iv) Find the area of the triangle formed by the tangent at P, the x-axis and the y-axis. [2]

CORE MATHS 1 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	$(x^2 + 4y^2 - 4xy)(x^2 - 4y^2) = x^4 - 4x^3y + 16xy^3 - 16y^4$		M1 A1 M1 A1	4
2.	$\dfrac{dy}{dx} = 8 - \dfrac{1}{x^3} = 0$ when $x = 1/2$	Point is $(1/2, 6)$	M1 A1 M1 A1 A1	5
3.	(i) $2x - 3y = 4,\ 3x + 2y = 3$	$6x - 9y = 12,\ 6x + 4y = 6$	M1	
	$13y = -6 \quad y = -6/13$	Point is $(17/13, -6/13)$	M1 A1 A1	
	(ii) Gradients are $2/3$ and $-3/2$	Product $= -1$, so perpendicular	M1 A1	6
4.	(i) $2 + 18 + 2\sqrt{2}\sqrt{18} = 20 + 2(6) = 32$		M1 A1 A1	
	(ii) $\dfrac{1}{5-\sqrt{15}} = \dfrac{(5+\sqrt{15})}{(5-\sqrt{15})(5+\sqrt{15})} = \dfrac{5+\sqrt{15}}{10}$		M1 A1 A1	6
5.	$23 - x = 9 + 8x - x^2 \quad x^2 - 9x + 14 = 0$	$x = 2, x = 7$	M1 A1 M1 A1	
	P is $(2, 21)$, Q is $(7, 16)$		A1 A1	6
6.	$x^2 + (k - x)^2 = k$	$2x^2 - 2kx + (k^2 - k) = 0$	M1 A1 A1	
	For 2 real roots, $4k^2 - 8(k^2 - k) > 0 \quad 4k(k-2) < 0$	$0 < k < 2$	M1 A1 M1 A1	7
7.	(i) $dy/dx = 3x^2 - 10x + 4 = -4$ when $x = 2$	$y + 2 = -4(x - 2)$	M1 A1 M1 A1	
	(ii) When $dy/dx = -3$, $3x^2 - 10x + 7 = 0$	$(3x - 7)(x - 1) = 0$	B1 M1	
	$x = 1$ or $x = 7/3$		A1 A1	8

8. (i)  (ii)  (iii) 

B3 B3 B3 — 9

9.	(i) 9^{x-1} (ii) $(9^2)^{x-2} = 9^{2x-4}$	(iii) $(9^{1/2})^{4x+6} = 9^{2x+3}$	B1 M1 A1 M1 A1	
	$x - 1 = 2x - 4 - (2x + 3) \quad x - 1 = -7$	$x = -6$	M1 A1 A1 A1	9
10.	(i) $(x + 2)^2 + (y - t)^2 = 8$	$x^2 + y^2 + 4x - 2ty + (t^2 - 4) = 0$	M1 A1 A1	
	(ii) $4 - 4t + t^2 - 4 = 0$	$t = 0$ or $t = 4$	M1 A1 A1	
	(iii) When $t = 4$, gradient of radius $= -1$ so gradient of tangent $= 1$		M1 A1	
	Tangent is $y = x + 2$		M1 A1	
	(iv) Tangent cuts axes at $(-2, 0)$, $(0, 2)$ so area of triangle $= 2$ units^2		M1 A1	12