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Edexcel Core 1 Help Guide

Content

Formulae Given in Exam

Mensuration

Surface area of sphere = $4\pi r^2$
Area of curved surface of cone = πrl (where l is the slant height)

Arithmetic series

$$a_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + l)$$

Indices

The rules of indices

The index is simply the power a base is raised to. In the example $2^3 = 8$, 2 is the base, 3 is the index, exponent or power and 8 is the numeric value obtained if we raise 2 to the 3rd power.
Most questions will ask you to simplify expression or find numerical values and use one of the rules below.

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$\left(a^m\right)^n = a^{mn}$

$a^0 = 1$

$a^{-m} = \dfrac{1}{a^m}$

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$

Most exam questions will be 1 or 2 marks. If they are worth 2 marks show workings e.g. *Find* $\left(\dfrac{36}{25}\right)^{-\frac{3}{2}}$

$$\left(\frac{36}{25}\right)^{-\frac{3}{2}} = \left(\frac{25}{36}\right)^{\frac{3}{2}} = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$$

Another typical question might ask us to *Find the value of* x *given* $2^{3x} = 4^{x+1}$.
I know 4 is a power of 2 (or vice versa) so I can rewrite the equation such that I have the same base (2):
$2^{3x} = (2^2)^{x+1}$
Using the rules of indices:
$2^{3x} = 2^{2(x+1)}$
If the bases (2) are the same we can simply equate the powers and solve for x.

$3x = 2(x+1)$

$3x = 2x + 2$

$x = 2$

To check my answer I can substitute $x = 2$ back in to the equation see if it makes the equation true. We know $2^6 = 4^3 = 64$ and therefore we have the correct answer. Do check your answer!

We may have to expand brackets and simplify. An exam question could be:
Expand and simplify $\left(x^{\frac{1}{2}} - 3\right)\left(x^{\frac{3}{2}} + 5\right)$ *giving your answers in descending powers of* x *:*

Showing full workings and using the rules of indices:

$$\left(x^{\frac{1}{2}}\right)\left(x^{\frac{3}{2}}\right) + \left(x^{\frac{1}{2}}\right)(5) + \left(x^{\frac{3}{2}}\right)(-3) + (-3)(5)$$

$$x^2 + 5x^{\frac{1}{2}} - 3x^{\frac{3}{2}} - 15$$

Finally, putting my answer in descending powers of x, the answer is:

$$x^2 - 3x^{\frac{3}{2}} + 5x^{\frac{1}{2}} - 15$$

If we have $16^{\frac{1}{4}}$ we don't divide by 4, we take the 4th root. This is a common error. Another common error is with negative powers. Remember $a^{-m} = \dfrac{1}{a^m}$. This will not give a negative value. $2^{-3} = \dfrac{1}{2^3} = \dfrac{1}{8}$

One important rule to remember is $\sqrt{a} = a^{\frac{1}{2}}$. You can use this when working with surds and indices... This will be very important when it comes to differentiation and integration in the section on calculus.

Surds

Surds are irrational numbers and are said to be 'exact values'. Don't be tempted to try and write a truncated or rounded decimal answer. Leave your answer as a surd. Calculations in this form will be more accurate & easy to perform.
Generally you will either have to simplify surds, carry out basic calculations with the 4 operations or rationalise the denominator of a fraction with a surd in.

Some Basic Surd Laws

$\sqrt{a} \times \sqrt{a} = a$

$a\sqrt{c} \times b\sqrt{d} = ab\sqrt{cd}$

$\sqrt{a} + \sqrt{a} = 2\sqrt{a}$

Just try these with numeric values. You will see many break down into integer values

Simplifying Surds

Breaking surds down (simplifying) is just a case of prime factorising e.g. $\sqrt{8} = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2}$

Adding and Subtracting Surds

You can only add and subtract 'like surds' BUT some will simplify to allow you to do that e.g.
$\sqrt{3} + 4\sqrt{3} = 5\sqrt{3}$ or $7\sqrt{2} - 3\sqrt{2} = 4\sqrt{2}$ are examples of surd adding/subtracting without prior simplification.
Sometimes you will have to simplify first
e.g. $\sqrt{50} + \sqrt{8} = (\sqrt{5} \times \sqrt{5} \times \sqrt{2}) + (\sqrt{2} \times \sqrt{2} \times \sqrt{2}) = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$

Expanding Brackets

You may be expected to expand single or double brackets. Most of these questions require full workings e.g.
Write $(2+\sqrt{3})(5-\sqrt{27})$ *in the form* $a + b\sqrt{3}$

$$(2+\sqrt{3})(5-\sqrt{27}) = (2+\sqrt{3})(5-3\sqrt{3}) = (2)(5) + (2)(-3\sqrt{3}) + (\sqrt{3})(5) + (\sqrt{3})(-3\sqrt{3}) = 10 - 6\sqrt{3} + 5\sqrt{3} - 27 = -17 - \sqrt{3}$$

$\therefore a = -17, b = -1$ Some exam questions will not expect that level of working. Do check first!

Rationalising the Denominator

Mathematicians hate having an irrational denominator. A surd is an irrational number. You will come across two types of fractions where you will have to rationalise the denominator:

1 When the denominator is a single surd value such as $\frac{1}{\sqrt{3}}$ or $\frac{4}{5\sqrt{7}}$ or $\frac{2+\sqrt{3}}{\sqrt{5}}$. With this type simply multiply the numerator and the denominator by the surd value in the denominator and simplify. For the first one multiply numerator and denominator by $\sqrt{3}$, the second $\sqrt{7}$ and the third $\sqrt{5}$. Simplify your using the surd rules.

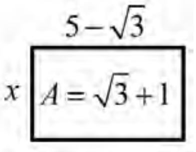
An example could be $\frac{4}{5\sqrt{7}} = \frac{4 \times \sqrt{7}}{5\sqrt{7} \times \sqrt{7}} = \frac{4\sqrt{7}}{5(7)} = \frac{4\sqrt{7}}{35}$

2 When the denominator has two values and an additional or subtraction sign e.g. $\frac{4}{2+\sqrt{3}}$ or $\frac{9}{3-2\sqrt{7}}$ or $\frac{2-\sqrt{3}}{3+\sqrt{5}}$.

With this type we simply multiply the numerator and the denominator is the same values in the denominator but with the opposite sign. This will create a 'difference of squares' and allow you to simplify to give a rational, denominator. Here is an example in the form $a\sqrt{3} + b$.

$$\frac{3}{\sqrt{3}-2} = \frac{3(\sqrt{3}+2)}{(\sqrt{3}-2)(\sqrt{3}+2)} = \frac{3(\sqrt{3}+2)}{3+2\sqrt{3}-2\sqrt{3}-4} = \frac{3(\sqrt{3}+2)}{-1} = -3(\sqrt{3}+2) = -3\sqrt{3} - 6$$

There may be some practical applications of surds such as areas or lengths. Here is a typical question: *Find the value of x given the area of*	length ×width = area ∴ rearranging, rationalising & simplifying

the rectangle below is $\sqrt{3}+1$ *sq units* 	$x\left(5-\sqrt{3}\right)=\sqrt{3}+1$ $x=\dfrac{\left(\sqrt{3}+1\right)}{\left(5-\sqrt{3}\right)}$ $x=\dfrac{\left(\sqrt{3}+1\right)\left(5+\sqrt{3}\right)}{\left(5-\sqrt{3}\right)\left(5+\sqrt{3}\right)}=\dfrac{5\sqrt{3}+3+5+\sqrt{3}}{25+5\sqrt{3}-5\sqrt{3}-3}=\dfrac{6\sqrt{3}+8}{22}=\dfrac{3\sqrt{3}+4}{11}$

Quadratic Equations and Expression

Solving Quadratic Equations

Quadratic equations and expression will have a highest power of the variable as a squared term e.g. $y = x^2 - 3x + 10$ or $2p^2 + 5p - 9 = 0$ where each power is an integer value. An expression is a collection of terms, an equation will have an equals (=) sign in and we could look to use a range of techniques to solve. When solving a quadratic equation consider two possible solutions e.g. $x^2 = 25$ will have two 'real' solutions, $x = \pm 5$. A common error is to simply give the answer of 5. Check the validity of your solutions as the algebra may work but one solution may not be applicable. An example would be a 'negative length' in an area of a rectangle question. Sometimes a quadratic equation will have 'no real solutions'.

When trying to solve a quadratic equation go through a checklist of methods you can use to solve an equation.

<u>Type 1</u> $x^2 = a$.
Simply square root both sides.

$$p^2 = 12$$
$$p = \pm\sqrt{12}$$
$$p = \pm 2\sqrt{3}$$

<u>Type 2</u> $ax^2 + bx = 0$
Set the RHS = 0, factor and solve

$$3x^2 - 4x = 0$$
$$x(3x - 4) = 0$$
$$x = 0 \text{ or } x = \frac{4}{3}$$

<u>Type 3</u> $ax^2 + bx + c = 0$, $a = 1$ that **<u>DO</u>** factor. Set the RHS side to 0, solve.

$$x^2 - 2x - 8 = 0$$
$$(x + 2)(x - 4) = 0$$
$$x = -2 \text{ or } x = 4$$

<u>Type 4</u> $ax^2 + bx + c = 0$, $a \neq 1$ that **<u>DO</u>** factor Set RHS side to 0, solve.

$$6x^2 + 13x = 5$$
$$6x^2 + 13x - 5 = 0$$
$$(3x - 1)(2x + 5) = 0$$
$$x = \frac{1}{3} \text{ or } x = \frac{-5}{2}$$

Remember, not all equations will initially $= 0$. Simply rearrange the equation into the form $ax^2 + bx + c = 0$ and look to solve.

$$-6x^2 = 4 - 5x$$
$$6x^2 + 5x - 4 = 0$$
$$(3x + 4)(2x - 1) = 0$$
$$x = \frac{-3}{4} \text{ or } x = \frac{1}{2}$$

<u>Type 5</u> $ax^2 + bx + c = 0$ that **<u>DON'T</u>** factor. You can use either the formula or you can complete the square. When a quadratic equation is in the form $ax^2 + bx + c = 0$ the solutions to the equation will be

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

example:

$$2x^2 + 5x - 9 = 0$$
$$x = \frac{-5 \pm \sqrt{(+5)^2 - 4(2)(-9)}}{2(2)}$$
$$x = \frac{-5 \pm \sqrt{97}}{4}$$

Check the validity of both solutions when answering the question. The question might, for example, state $x < 3$ which rules one solution out.

<u>Type 5</u> $ax^2 + bx + c = 0$ that **<u>DON'T</u>** factor. Completing the square is an option. The coefficient of the term in x^2 must be 1.

$$2x^2 + 5x - 9 = 0$$
$$x^2 + \frac{5}{2}x - \frac{9}{2} = 0$$
$$\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} - \frac{9}{2} = 0$$
$$\left(x + \frac{5}{4}\right)^2 = \frac{97}{16}$$
$$x = -\frac{5}{4} \pm \sqrt{\frac{97}{16}}$$
$$x = \frac{-5 \pm \sqrt{97}}{4}$$

Completing the Square

Completing the square can be used to solve quadratic equations, find the maximum or minimum point and allow easy graph transformations if you need to sketch a quadratic function. In the exam you might be asked to write a quadratic expression the form $a(x + b)^2 + c$. This is in 'completed square form'.

Completing the square when the quadratic expression is in the form $ax^2 + bx + c$, $a = 1$
If you want an algorithm: *"take half the coefficient of the term in x into the bracket with an x, square the bracket, subtract the squared value away"*

$$x^2 + 4x - 5$$
$$(x + 2)^2 - 4 - 5$$
$$(x + 2)^2 - 9$$

This is a positive quadratic expression. If this was an equation where $y = x^2 + 4x - 5$ the maximum would have the coordinates $(-2, 9)$ and the graph translation of $y = x^2$ of 2 units in the negative x direction and 9 units in the positive y direction.

Completing the square when the quadratic expression is in the form ax^2+bx+c **,** $a \neq 1$

In order the complete the square the coefficient of the term in x^2 must be 1. If it's not you will need to factor out the value of a . If you are simply solving an equation divide through the equation by a . Here is a typical exam style question:

Express $3x^2-5x+4$ *in the form* $a(x+b)^2+c$.

$$3x^2-5x+4$$

$$3\left(x^2-\frac{5}{3}x\right)+4$$

$$3\left[\left(x-\frac{5}{6}\right)^2-\frac{25}{36}\right]+4$$

$$3\left(x-\frac{5}{6}\right)^2-\frac{35}{12}+4$$

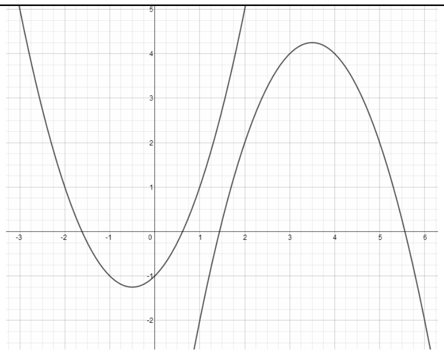
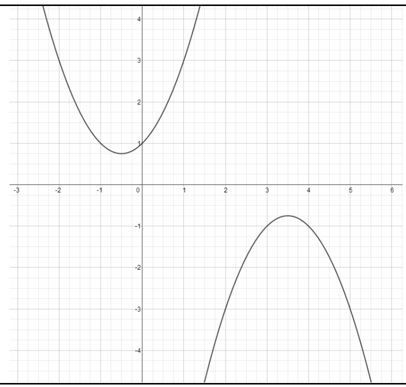
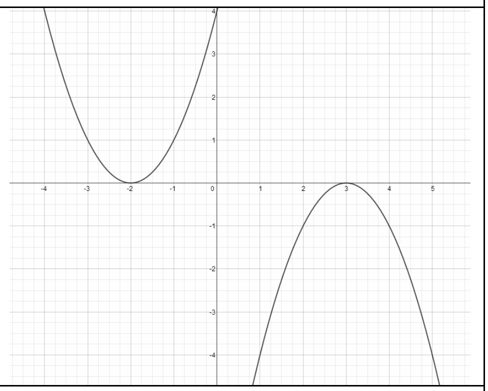
$$3\left(x-\frac{5}{6}\right)^2-\frac{19}{12}$$

If you graphed this the parabola would open upwards (as it's a positive value of a) and the minimum point will have coordinates $\left(\frac{5}{6},-\frac{19}{12}\right)$. If I was **solving** the equation $3x^2-5x+4=0$,I could write $x^2-\frac{5}{3}x+\frac{4}{3}=0$ & solve

If you had to graph $y=-(x+3)^2-5$ then it would be a maximum at $(-3,-5)$ as the value of a is negative.

The Discriminant

The Discriminant determines the nature of the roots of a quadratic equation. Remember 'math error' when using the quadratic equation on a calculator? That was because $b^2-4ac<0$. The 3 scenarios are listed below:

2 Distinct Real Roots	**No Real Roots**	**Repeated or Equal Roots**
$ax^2+bx+c=0$ $b^2-4ac>0$	$ax^2+bx+c=0$ $b^2-4ac<0$	$ax^2+bx+c=0$ $b^2-4ac=0$
		

An example could be find the set of values of k for which the quadratic equation $x^2+kx-2k=0$ has two distinct real roots:

$$x^2+kx-2k=0$$

$$a=1, b=k, c=-2k$$

$b^2-4ac>0$ for real roots

$$k^2-4(1)(-2k)>0$$

$$k(k+8)>0$$

$$k>0, k>-8$$

You can use a sketch to help on the last part (covered in the next section on quadratic inequalities)

A quick tip! – A tangent touches a curve. Some more subtle questions involving tangents may require the use of the Discriminant and in particular $b^2-4ac=0$ for a repeated root. Sketching a quick diagram should make sense of this!

Simultaneous Equations

Linear Simultaneous Equations

Simultaneous equations are simply 2 or more equations that share common solutions. In C1 these will be two equations generally in x and y. You will need to be able deal with 3 different scenarios in the module. (1) Where there are two linear equations (2) Where one equation is linear and one is not and (3) Where both are non linear.

So what does linear mean? The powers of x and y will both be one and there will be no terms in x and y that are multiplied or divided by one another.

Two Linear Equations

You have two choices

(1) Elimination by adding or subtracting the two equations to eliminate one variable
(2) Rearranging and substitution to eliminate one variable

Here is a straight forward example from GCSE maths: *Solve the simultaneous equations:*

$$2x - y = 4$$
$$3x + 2y = 13$$

Method 1

I need either the terms in x or y to be the same.

Labelling the equations

(1) $2x - y = 4$

(2) $3x + 2y = 13$

I am going to multiply equation (1) by 2

(1) $4x - 2y = 8$

(2) $3x + 2y = 13$

Adding the two equations will eliminate y leading to:

$7x = 21$, which gives $x = 3$.

This value of x can now be substituted back into either equation (1) or (2) to solve for y.

I am going to use equation (2)

(2) $3(3) + 2y = 13$

$2y = 4$

$y = 2$

Check your answers for **both** x and y satisfy **both** equations (1) and (2).

This wasn't the only way I could have done the question. I could have multiplied equation (1) by 3 and equation (2) by 2 to make the terms in x the same and then subtracted equation (2) from (1).

Method 2

Labelling the equations

(1) $2x - y = 4$

(2) $3x + 2y = 13$

I am going to write equation (1) to make y the subject such that (1) $y = 2x - 4$

I can now substitute this into equation (2) to eliminate y from the equation such that

(2) $3x + 2(2x - 4) = 13$

If I simplify this I have a linear equation in x such that (2) $7x = 21$. Solving for x we can see $x = 3$.

I am now going to use this value in equation (1) to solve for y.

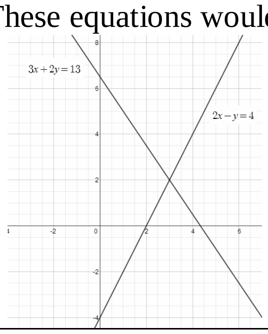
(1) $y = 2(3) - 4$

Again from this we can see $y = 2$.

Check your answers for **both** x and y satisfy **both** equations (1) and (2).

This particular equation lends itself well to this method as y could be written in terms of x with no fractions on the RHS. Some equations will have fractions in!

A graphical representation is shown below. These equations would, in the main, be solve algebraically in C1



One Linear Equation, One Non Linear

Typical examples will include a circle and a line, a parabola and a line or a line and another type of equation where y is implicitly defined as a function of x (usually where there is a product of terms in x and y) such as $xy + y^2 - 3y = 4x$ and $3x - y = 6$.

Let's look at a circle and a line.

Find the coordinates of points of intersection of the equations $3y = 4x$ *and* $x^2 + y^2 = 25$.

This is a relatively straight forward example, others can become quite messy. The first equation is the equation a straight line and the second equation is a circle, centred at the origin and has a radius of 5 units.

Labelling the equations:

(1) $x^2 + y^2 = 25$

(2) $3y = 4x$

I am going to divide both sides of equation (2) by 3 and then substitute this into equation (1) to eliminate y

(2) $y = \frac{4}{3}x$ Substituting into (2) equation (1) will give me a quadratic equation in x.

(1) $x^2 + \left(\frac{4}{3}x\right)^2 = 25$

(1) $x^2 + \frac{16}{9}x^2 = 25$

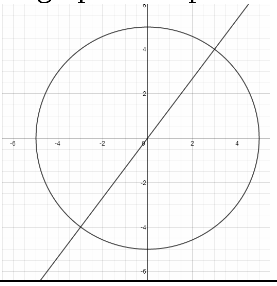
(1) $\frac{25}{9}x^2 = 25$

(1) $x^2 = 9$

Solving for x we can see $x = \pm 3$. At this stage we simply substitute our values of x back into the linear equation (2) to solve for y. (2) $y = \frac{4}{3}x$

When $x = 3, y = 4$ and when $x = -3, y = -4$. This gives us the two points $(3,4)$ and $(-3,-4)$.

You can of course make x the subject of equation (2) at the start and solve the quadratic equation in y THEN find the values of x from the linear equation. A graphical representation is shown below:



Two Non - Linear Equations

You might need to solve simultaneous equations where we have two quadratic functions. These are relatively straight forward and we will work though a typical question: *Solve the simultaneous equations:*

$y = x^2 + 5x - 3$

$y = 2x^2 + 4x + 3$

We have expressions for y in terms of x for both equations so we can simply either set the equations equal or subtract one equation from the other to eliminate y.

If $y = x^2 + 5x - 3$ and $y = 2x^2 + 4x + 3$ it follows that $2x^2 + 4x + 3 = x^2 + 5x - 3$

Setting the RHS to 0, $x^2 - x - 6 = 0$

We can factor and solve for x

$(x-3)(x+2) = 0$ giving the solutions $x = 3$ and $x = -2$.

At this stage some students finish the question without solving for y. We need to find y too!

Using $y = x^2 + 5x - 3$ we can solve to find the two values of y:

When $x = 3$, $y = 9 + 15 - 3$ which gives $y = 21$

When $x = -2$, $y = 4 - 10 - 3$ which gives $y = -9$

The solutions are $x = 3$ and $x = -2$, $y = 21$ and $y = -9$.

Inequalities

Linear Inequalities

Inequalities tell us about the relative size of two values. The variable x for example (which could represent displacement) might be more than 4m in a given situation. Mathematically we could write $x > 4$ and we would read this as "x is greater than 4". This gives us the range of values satisfying the inequality. In C1 you will be expected to solve linear and quadratic inequalities and sometimes state the set of values that satisfies both a linear and quadratic inequality.

Linear Inequalities are solved in a similar manner to linear equations **BUT** If you are multiplying or dividing the inequality by a negative number you must change the inequality sign round.

An example of a linear inequality may be "Find the set of values of x that for which $2x-1>4x-5$"

$2x-1>5x-4$

$-1>3x-4$

$3>3x$

$1>x$

Quadratic Inequalities

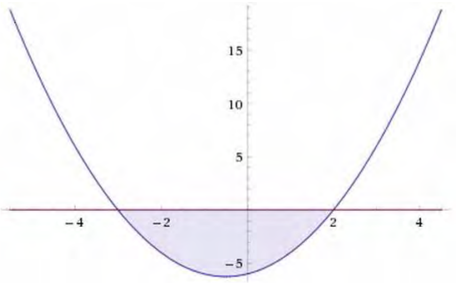
Quadratic inequalities are dealt with in a similar manner to quadratic equations (and will usually factor when written on the form $y = ax^2+bx+c$) and solved often with aid of a sketch.

Some examples are shown below. Remember! The x axis is the line $y=0$

$x^2+x-6<0$

$(x-2)(x+3)<0$

The critical values are 2 and -3

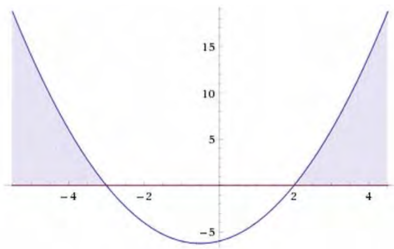


$\therefore -3<x<2$

$x^2+x-6>0$

$(x-2)(x+3)>0$

The critical values are 2 and -3

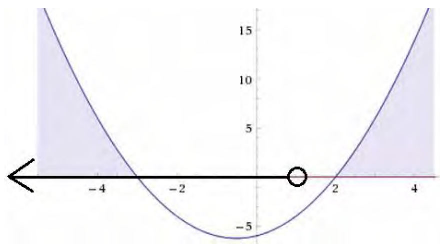


$\therefore x<-3, x>2$

If we were asked to find the set of values that satisfy BOTH inequalities below we could add the linear inequality to our sketch.

$x^2+x-6>0$

$2x-1>5x-4$



We are interest in the set of values where there is both a line and shading. In this case it would be $x<-3$

I have simply used the information from previous questions to graph the inequalities above

Be careful with strict and inclusive inequalities and their respective notation, $</>, \le/\ge$. Many marks are lost on exam papers with incorrect notation or pupils not considering the statement they have written down.

$x>a$ can be read "x is greater than a" An open dot would be used on a number line.

$3>q$ can be read "3 is greater than q" or "q is less than 3". The values that satisfy this inequality are all those strictly less than 3. An open dot would be used on a number line.

$p \ge 2$ can be read "p is equal to or greater than 2". The set of values that satisfy this inequality are all values 2 or more. A closed dot would be used on a number line.

$b \le y$ can be read "b is equal to or less than y" or "y is equal to or greater than b". A closed dot would be used on a number line.

Be warned! - Inequalities can appear on question involving the Discriminant!

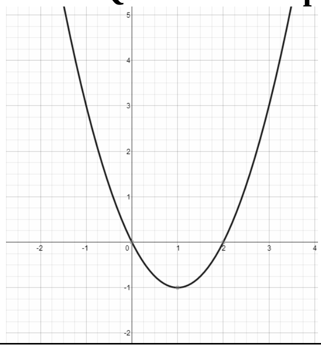
Sketching Curves

You will be expected to sketch the graphs of 3 different functions. Quadratic graphs (parabolas), cubic graphs and reciprocal graphs. A sketch is **not** a plot from a table of values. The examiner is looking for a basic understanding of the shape, key features, any asymptotes and points of intersection. Do not try and write out a table of values.

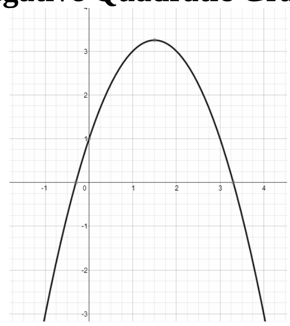
Quadratic Graphs

Quadratic equations can be written in the form $y = ax^2 + bx + c$ and their graphs are symmetric parabolas. Positive (when $a > 0$) quadratic graphs with open upwards and have a minimum. Negative (when $a < 0$) will open downwards and have a maximum. The graphs will cross the y axis when $x = 0$ and the x axis when $y = 0$. These solutions, or roots, can be found using the techniques above & max/min from completing the square.

Positive Quadratic Graph



Negative Quadratic Graph

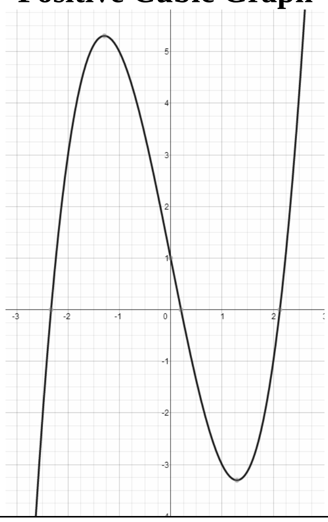


The graph of $y = x^2$ has its vertex (which is a minimum) at the origin and its axis of symmetry is the line $x = 0$. The completed square form $y = a(x+b)^2 + c$ can help sketch the main features of a quadratic graph e.g. $y = (x-3)^2 + 2$ will have a minimum point at (3, 2), open upwards (as it's positive) and the axis of symmetry will be the line $x = 3$. The y intercept will be (0, 11) as when $x = 0$, $y = 11$. The sketch should be smooth, not a collection of straight lines. Any solutions in surd form should be left as exact values.

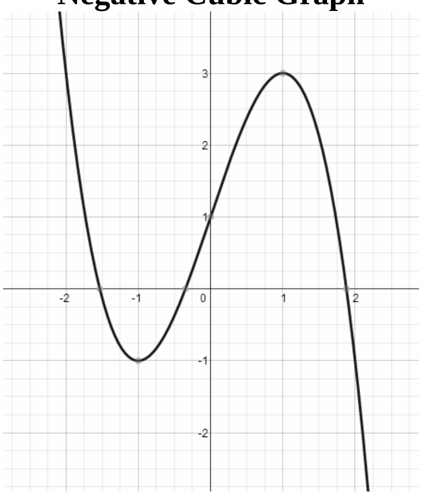
Cubic Graphs

Cubic equations can be written in the form $y = ax^3 + bx^2 + cx + d$. In C1 they equations will either be factored e.g. $y = (x-1)(x-4)(x+5)$ or will have a common factor of x e.g. $y = x^3 + x^2 - 6x$ such that the equation can be factored to give $y = x(x-2)(x+3)$. If the equation was such that $y = 0$ it will have the solutions $x = 0, x = 2$ or $x = -3$ which will assist in a sketch.
Positive cubic graphs (when $a > 0$) will start in the 4th quadrant and leave in the 1st. Negative cubic graphs (when $a < 0$) will start in the 2nd quadrant and leave in the 4th.
An example of a negative cubic graph could be $y = (x+2)(x-5)(1-x)$. We can see if we expanded the brackets the term in x^3 would be negative. If the equation was such that $y = 0$ the roots, or solutions, to the equation would be $x = -2, x = 5$ or $x = 1$. These points would be plotted on the x axis. When $x = 0$ $y = (2)(5)(1)$, which gives the (0, 10) as the point of intersection on the y axis.

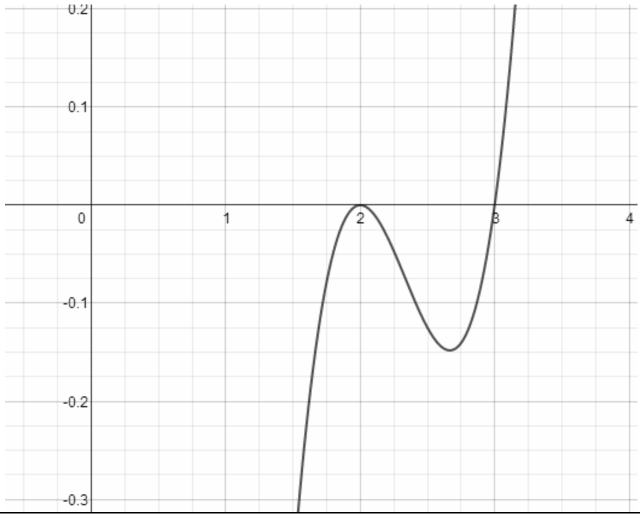
Positive Cubic Graph



Negative Cubic Graph

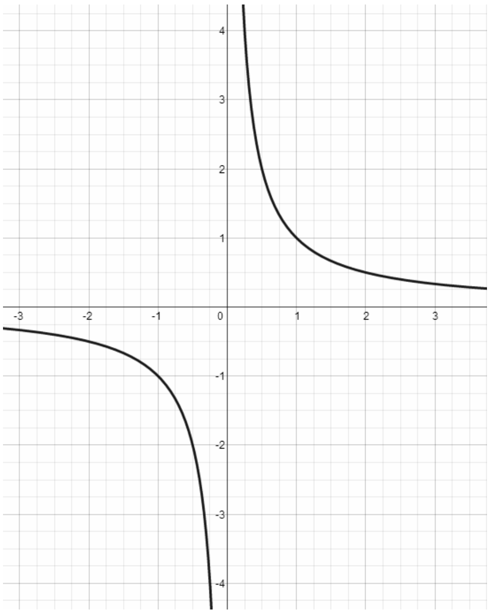


Some cubic graphs have repeated roots. The graph $y=(x-2)(x-3)^2$ will pass through the x axis at 2 and touch the x axis at 3. If the equation had been such that $y=(x-2)^2(x-3)$ then it would touch at 2 and pass through at 3. Below is the graph of $y=(x-2)^2(x-3)$



Reciprocal Graphs

The basic reciprocal function has the equation $y=\frac{1}{x}, x\neq 0$ as division by 0 is undefined. The graph will have two asymptotes, the x axis (or $y=0$) and the y axis (or $x=0$). In C1 you could be asked to apply graph transformations such that a reciprocal function would be written in the form $y=\frac{a}{(x-b)}+c, x\neq b$. These will be covered in the section on transformations below. The standard $y=\frac{1}{x}, x\neq 0$ graph is sketched below. As x gets large y gets small both in the positive and negative direction. As x gets small y gets large both in the positive and negative direction such that $y\rightarrow\pm\infty$ (y tends to positive and negative infinity)



Graph Transformations

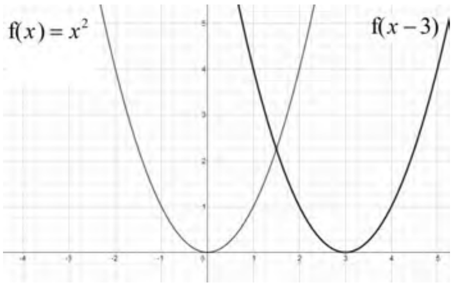
There are 3 types of graph transformations you may be asked to perform. Translations, Reflections and Stretches. Most are fairly intuitive but a way to remember them is “If it’s on the outside of the bracket it changes the y coordinate, if it’s on the inside of the bracket it changes the x coordinate”. Using numeric values are often a good way of confirming this. In the exam you will be given marks for the correct shape of the graph and the coordinates of given points **after** each transformation has been applied. The sketches don’t have to be perfect and you may even find describing what you have done may help out if your sketch is as bad as mine! For example “Scale factor stretch 2 in the x direction” or “Scale factor stretch of ½ in the negative y direction”

Translations in the x direction.

$\mathrm{f}(x+a)$

The graph is translated (translated simply means ‘moved’) a units in the negative x direction or, if you like, move left by a units such that the vector is $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

An example could be $\mathrm{f}(x)=x^2$ and you may be asked to sketch $\mathrm{f}(x-3)$.

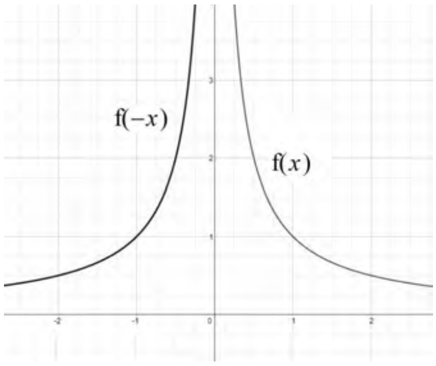


The graph has moved 3 units to the right. Whilst this may seem counterintuitive, using numeric values may help you see why.

Reflections in the y axis.

$\mathrm{f}(-x)$

The sign of the x coordinate changes and the result is a reflection in the y axis.



If you try this with $\mathrm{f}(x)=x^2$ you may be a little disappointed!

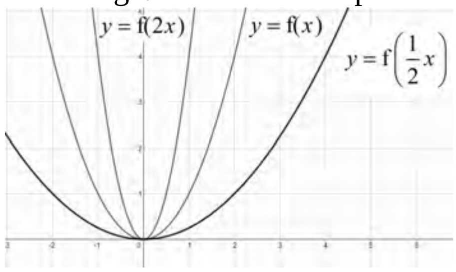
Stretches in the x direction.

$\mathrm{f}(ax)$

This is a $\frac{1}{a}$ scale factor stretch in the x direction. An easier way to think about this is to divide the x coordinates by a. If you are given $\mathrm{f}(2x)$ for example, the x coordinates are divided by 2. As a result the graph is ‘squashed’ towards the y axis.

$\mathrm{f}\left(\frac{1}{2}x\right)$ would see the x divided by ½, or multiplied by 2, such that the graph looks more stretched out.

Using $a=2$ as an example

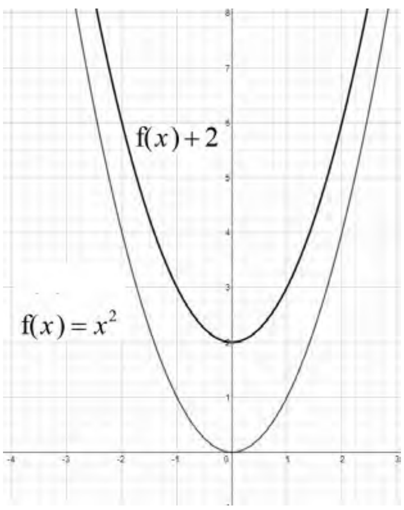


Translations in the y direction.

$\mathrm{f}(x)+a$

The graph is translated a units in the positive y direction such that the vector is $\begin{pmatrix} 0 \\ a \end{pmatrix}$.

An example could be $\mathrm{f}(x)=x^2$ and you may be asked to sketch $\mathrm{f}(x)+2$

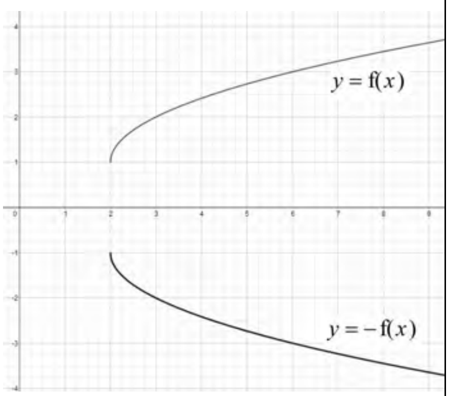


The graph has moved 2 units upwards.

Reflections in the x axis.

$-\mathrm{f}(x)$

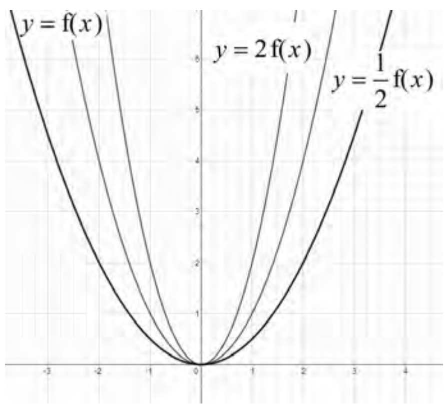
The sign of the y coordinate changes and the result is a reflection in the x axis.



Stretches in the y direction.

$a\,\mathrm{f}(x)$

This is a scale factor stretch of a in the y direction. You can simply multiply the y coordinates by a. The example below shows $a=2$ and $a=\frac{1}{2}$ being applied to $\mathrm{f}(x)$ where $\mathrm{f}(x)=x^2$



A stretch in the y direction will come BEFORE a translation in y

Coordinate Geometry

Before we start…One tip! *"If in doubt, sketch it out"*. Drawing can really help with questions on coordinate geometry no matter how straightforward they may seem. The whole topic is simply about straight lines in the x, y plane. Exam questions range from basic examples of finding an equation of a straight line to more challenging questions on the area of shapes and distances between the vertices of shapes.

The Gradients of a Line or Line Segment

The gradient is *the change in y over the change in* x such that the gradient m is given as $m = \dfrac{y_1 - y_2}{x_1 - x_2}$.

The gradient of the line passing through the points $A(4,3)$ and $B(2,-1)$, for example, is $\dfrac{3-(-1)}{4-2} = \dfrac{4}{2} = 2$.

Often errors are made with signs. You can of course write $\dfrac{-1-3}{2-4} = \dfrac{-4}{-2} = 2$ a sketch will show this is positive.

The Equation of a Straight Line

For the equation of a straight line you need 2 things. (1) A gradient and (2) A point the line passes through. A typical question might be: *Find an equation of the straight line passing through the points* $A(4,3)$ *and* $B(2,-1)$.

We know the gradient, m, is 2 from the previous section. We can now choose either $A(4,3)$ or $B(2,-1)$ as a point the line passes through. I am going to choose $A(4,3)$. If I had chosen $B(2,-1)$ my final answer would be the same.

At this stage I can take either of the approaches below and simply substitute the values in to find an equation.

$y - y_1 = m(x - x_1)$ Using $A(4,3)$ and $m = 2$, $y - 3 = 2(x-4)$ $y - 3 = 2x - 8$ $y = 2x - 5$	$y = mx + c$ Using $A(4,3)$ and $m = 2$, $3 = 2(4) + c$ $3 = 8 + c$ $-5 = c$ $\therefore y = 2x - 5$

I have written the line in the form $y = mx + c$. We may be asked to write the equation in the form $ax + by + c = 0$. The example above would be $2x - y - 5 = 0$.

Let's look at a typical exam style question on straight lines:

Find an equation of the straight line that passes through the point $P(-1,-2)$ *that is parallel to the line* $y = 6 + 3x$ *in the form* $ax + by + c = 0$ *where* a, b *and* c *are integers.*

We need a gradient and a point the line passes through. We have the point $P(-1,-2)$ and we can see the gradient will be 3 from the line parallel to it (as the gradient will be the same).

Using the first method $y - y_1 = m(x - x_1)$, where $P(-1,-2)$ and $m = 3$

$y - (-2) = 3(x - (-2))$

$y + 2 = 3(x + 2)$

$y + 2 = 3x + 6$

$3x - y + 4 = 0$

We could of course write the equation in the form $y = mx + c$ such that $y = 3x + 4$.

A Reminder! A straight line crosses the y axis when $x = 0$ and crosses the x when $y = 0$. These are important basic facts often overlooked or forgotten by students and may be important parts of exam questions. The point of intersection of two lines can be solved by linear simultaneous equations as shown in a previous section. Not all solutions will have integer values so being confident with fractions is very important.

Parallel and Perpendicular Lines

2 or more parallel lines have the same gradient. Perpendicular lines are at right angles and the product of the gradients of two perpendicular lines $= -1$ such that $m_1 \times m_2 = -1$ where m_1 and m_2 are the gradients of the 2 lines.

An easier way to think about it could be to consider the gradient of a perpendicular line is the negative reciprocal of the gradient of the original line. An example could be $m_1 = \frac{-2}{3}, \therefore m_2 = \frac{3}{2}$. You will generally have to state the fact $m_1 \times m_2 = -1$ in an exam when working with perpendicular lines. A basic exam question could be: *The line l_1 passes through the points $A(4,3)$ and $B(2,-1)$. Find an equation for the line l_2 which is perpendicular to l_1 and passes through the point $C(5,-3)$*.

For the equation of a straight line we need a gradient and a point the line passes through. We have the point $C(5,-3)$. We found the gradient of the line passing through $A(4,3)$ and $B(2,-1)$ in the previous section. I will call this m_1 where $m_1 = 2$. The gradient of the perpendicular will therefore be the negative reciprocal which gives $m_2 = \frac{-1}{2}$. All I need to do is simply substitute these values into the equation of a straight line. I can use either method outlined previously. I am going to choose $y - y_1 = m(x - x_1)$ and use the values $C(5,-3)$ and $m_2 = \frac{-1}{2}$ such that:

$$y - (-3) = \frac{-1}{2}(x - 5)$$

multiplying both sides of the equation by 2 and expanding the brackets on the RHS:

$$2y + 6 = -x + 5$$

I am going to give the final answer in the form $ax + by + c = 0$ where a, b and c are integers:

$$x + 2y + 1 = 0$$

The Midpoint of a Line Segment

This can be done geometrically or by using a formula. In Layman's terms, *add the x coordinates together and divide by 2, add the y coordinates together and divide by 2*. More formally the midpoint M is such that $M = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$. The midpoint of the line AB where $A(4,3)$ and $B(2,-1)$ is $\frac{4+2}{2}, \frac{3+(-1)}{2} = \frac{6}{2}, \frac{2}{2} = 3,1$

Expect some non integer answers in exam and be prepared for exam questions that give you the midpoint and one point where you are expected to find the other. Substituting into the formula is an easy way to tackle these types of questions.

The Distance Between Two Points or the Length of a Line Segment

Despite its quite bewildering formula this is simply Pythagoras Theorem. Plotting the points in the x, y plane should make this fairly clear. The distance formula is such that distance d is $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Using $A(4,3)$ and $B(2,-1)$ again we can find the length of the line segment AB by substituting the values in.

$$AB = \sqrt{(4-2)^2 + (3-(-1))^2}$$

$$AB = \sqrt{(2)^2 + (4)^2}$$

$$AB = \sqrt{20}$$

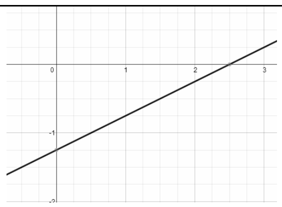
$$AB = 2\sqrt{5}$$

This is left in 'exact form' and simplified. Most questions will ask for the length in the form $p\sqrt{q}$ or similar. Here $p = 2, q = 5$

Here is a question to finish: *The line $2x - 4y = 5$ crosses the x axis at A and the y axis at B.*
Find the area of the triangle AOB where O is the origin.

When $x = 0, y = \frac{-4}{5}$ and when $y = 0, x = \frac{5}{2}$

Here is a quick sketch



We now have a right angle triangle with a base of $\frac{5}{2}$ and a height of $\frac{4}{5}$. Using the area of a triangle:

$$\therefore A = \frac{1}{2} \times \frac{5}{2} \times \frac{4}{5} = 1$$

Sequences and Series

A sequence is an ordered list that follows a given rule. A series is the summation of the terms in a sequence. The most challenging aspect of this topic for many students is the notation. There are only a few concepts that have to be understood and implemented. If you struggle with this topic, especially recurrence relations, I suggest using a table to write down your values.

Basic Sequences

You may be asked to generate terms in a sequence or solve a constant such as p or q. Here is a typical question.

A sequence is defined be the rule $u_n = 4^n - n, \ \ n > 0$. *Find the first 4 terms.*

All we have to do here is substitute values of n into the sequence, starting with $n = 1$ and write down the value. I am going to put these in the boxes below to keep on top of my work.

$n=1$ $u_1 = 4^1 - 1$ $u_1 = 3$	$n=2$ $u_2 = 4^2 - 2$ $u_2 = 16 - 2$ $u_2 = 14$	$n=3$ $u_3 = 4^3 - 3$ $u_3 = 64 - 3$ $u_3 = 61$	$n=4$ $u_4 = 4^4 - 4$ $u_4 = 256 - 4$ $u_4 = 252$

This gives us the first for terms as 3, 15, 63 and 255. You can argue the table is overkill but it can lead to fewer mistakes. We read u_n as "u sub n". a_n is often used in exams too.

Recurrence Relations

The terms in a recurrence relation are generated based on a rule linking previous terms in the sequence. An example could be $a_{n+1} = \frac{3}{a_n} - 2, \ n > 0 \ \ a_1 = 3$ where we might be asked to find the first 4 terms of the sequence.

You **could** read this sequence as *"The next terms is 3 divided by the last term then subtract 2"*
Using the 'table' below we can generate terms in the sequence. We know the first term $a_1 = 3$ and that $n > 0$, so we can start with $n = 1$.

We know $a_1 = 3$	$n=1$ $a_2 = \frac{3}{a_1} - 2$ $a_2 = \frac{3}{3} - 2$ $a_2 = -1$	$n=2$ $a_3 = \frac{3}{a_2} - 2$ $a_3 = \frac{3}{-1} - 2$ $a_3 = -5$	$n=3$ $a_4 = \frac{3}{a_3} - 2$ $a_4 = \frac{3}{-5} - 2$ $a_4 = -\frac{13}{5}$

This gives us the first 4terms as $3, -1$, -5 and $-\frac{13}{5}$. This is a fairly simple case but gives you an idea on the structure of the questions. Often the sequence will be defined in terms of a constant and question will introduce the sum of a number of terms being equal to a given value. You will have to solve accordingly.
Let's look at a typical example: (a) *Write down the first 3 terms in the sequence* $u_{n+1} = ku_n - 1$, $n \geq 1, u_1 = 1$ *where* k *is a positive constant. (b) Given* $\sum_{i=1}^{3} u_i = 8$ *find the value of the constant* k.

We know $u_1 = 1$	$n=1$ $u_2 = ku_1 - 1$ $u_2 = k(1) - 1$ $u_2 = k - 1$	$n=2$ $u_3 = ku_2 - 1$ $u_3 = k(k-1) - 1$ $u_3 = k^2 - k - 1$

This gives us the first 3 terms as 1, $k - 1$ and $k^2 - k - 1$ which is part a completed.
We are now going to part b and simply sum the terms we have and set them = 8 as we are summing from $i = 1$ to $i = 3$.

$\therefore 1+k-1+k^2-k-1=8$

$k^2=9$ *(We are told there is one value of k and that k is a positive $\therefore$ discard $k=-3$)*

$k=\pm 3$

$k>0, \therefore k=3$

Arithmetic Sequences and Series

2, 4, 6, 8… is an example of an arithmetic sequence, 5 + 8 + 11 + 14… is an example of an arithmetic series. Arithmetic sequences and series have a common difference, usually denoted as d. The difference could be 4, 3, -1 or ½ . The sequence or series will increase or decrease by a fixed amount. If it doesn't increase or decrease by a fixed amount then it's not arithmetic. The first term is given as a or a_1. Most questions simply involve finding the 4th piece of information after being given you 3. You may have to find a term in the sequence, the number of terms in a sequence or the sum of a series for example.

Is it Arithmetic?

If the sequence is arithmetic when you subtract the first term from the second, the second term from the third and so on the value will be the same.

$a_2-a_1=a_3-a_2=a_4-a_3=.....$

Remember to check more than just the first 2 terms if asked if the sequence is arithmetic!

Finding a Term in a Sequence

The n th term of an arithmetic sequence or series, a_n, is found by using $a_n=a+(n-1)d$.

Let's look at a basic example. *Find the 43rd term in the sequence 4, 7, 10, 11, 14….*

With any question like this we can simply collect the information required and substitute into the formula.

$a = 4$

$n = 43$

$d = 3$ (as we can see the common difference of the sequence is +3)

It's always a good idea to write a, n and d down the side of the page to collect information. This will really help with word based questions too.

Substituting in:

$a_{43}=4+(43-1)3$

$a_{43}=4+126$

$a_{43}=130$

A more challenging question might be: *Given the first 3 terms of an arithmetic sequence are $x-1, 2x-3$ and $4x-11$, find the value of the 12th term.*

We know an arithmetic sequence has a common difference therefore $a_2-a_1=a_3-a_2$. Initially we need to solve for x

Solving for x:

$2x-3-(x-1)=4x-11-(2x-3)$

$x-2=2x-8$

$x=6$

That gives a first term of 5 (substituting in $x=1$), a second term of 9 and a third term of 13. Applying this to the question:

$a = 5$

$n = 12$

$d = 4$

Substituting in:

$a_{12}=5+(12-1)4$

$a_{12}=5+44$

$a_{12}=49$

Finding the Sum of a Series

We can use one of 2 formulae (which you may be asked to prove in an exam) to find the sum of a series or values given a sum.

We could use $S_n = \frac{n}{2}(2a + (n-1)d)$ or $S_n = \frac{n}{2}(a + l)$ where a is the first term and l is the last term or n th term.

A straight forward example may ask us to find the sum of the first 25 terms of the series 5 + 2 + -1 + -4….
Don't be tempted to try and do this manually. Simply collect the information and substitute it into the formula.

$a = 5$

$n = 25$

$d = -3$

Using this information to find S_{25}

$S_{25} = \frac{25}{2}(2(5) + (25-1)(-3))$

$S_{25} = \frac{25}{2}(10 - 72)$

$S_{25} = 25(-31)$

$S_{25} = -775$

The second formula $S_n = \frac{n}{2}(a + l)$ could be used if you find the last term (n th term) using the formula shown previously.

Sigma Notation may also be used. A typical example could be $\sum_{r=1}^{12} 3r - 1$. This is an arithmetic series with common difference of 3. The question is asking us to sum the first 12 terms of the series from $r = 1$ to $r = 12$.
We can find the first term by substituting in $r = 1$ which gives 2 and find the last term by substituting in $r = 12$ which gives 35. There are 12 terms. Be careful with the number of terms as r may not start at 1.

$a = 2$

$n = 12$

$l = 35$

Substituting in the values:

$S_{12} = \frac{12}{2}(2 + 35)$

$S_{12} = 6(37)$

$S_{12} = 222$

If you have a word based problem simply extract the information from the question and decided whether you are find a term, a sum or another piece of information. Simply take the values and substitute into the correct formula and solve checking your answer is logical.

Differentiation

Differentiation is a branch of calculus studying "rates of change". We might ask ourselves how does one quantity change in response to another quantity changing? A nice example to look at is displacement (s), velocity (v) and acceleration (a). All 3 are functions of time (t). The rate of change of displacement 'with respect to time' is velocity. We could say $\frac{ds}{dt} = v$ which is pronounced $dee - ess, dee - tee$. We know this from basic work in maths and physics. Distance/Time = Speed.

The rate of change of velocity with respect to time is acceleration. We could say $\frac{dv}{dt} = a$. We are 'differentiating velocity with respect to time'

These are basic examples although applications generally will not be tested in C1.

The Gradient of the Tangent at a Point on a Curve.

When you find the gradient of a straight line you will use $m = \frac{y_1 - y_2}{x_1 - x_2}$ as we saw in the section on coordinate geometry. When you are finding the gradient of the tangent to a **curve** at a given point you will use the gradient function $\frac{dy}{dx}$ or $f'(x)$. These are the same thing using different notation. We say we are going to 'differentiate' the function, in this case, with respect to x. In C1 you will only be expected to differentiate functions of the form $y = x^n$ and will not be expected to prove this from 'first principles' (despite it being very interesting!)

Let's look at a standard result for differentiation:

If $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$.

If you want an algorithm, *"Multiply down by the power and drop the power by one"*.

This gives us the 'gradient function' and we can find the gradient of the tangent to the curve at any point by simply substituting the x coordinate of that point into either $\frac{dy}{dx}$ or $f'(x)$.

Some basic examples are shown in the box below. We say we are 'differentiating both sides of the equation with respect to x' in each given case. x and y will not always be the variables of choice. We might have to find $\frac{ds}{dt}$ given $s = 4t^5 - 2t + 1$ where we would be differentiating s with respect (WRT) to t. You will need to be comfortable with basic fraction work, the rules of indices and use the 'rules' for multiplying negative numbers throughout differentiation and integration. Many marks are lost in exam questions through sloppy fraction work. Be careful!

Find $\frac{dy}{dx}$ when $y = 3x^2$. $y = 3x^2$ $\frac{dy}{dx} = 2(3x^1)$ $\frac{dy}{dx} = 6x$	Find $f'(x)$ given $f(x) = 4x^3 - 6x^{\frac{1}{2}}$ $f'(x) = 12x^2 - 3x^{-\frac{1}{2}}$ *I have not shown full workings here. You must check with your teacher or exam board on the level of workings required.*	Find $\frac{dy}{dx}$ when $y = \frac{2}{x^5} - 3\sqrt{x}$ Using the rules of indices to first simplify: $y = \frac{2}{x^5} - 3\sqrt{x}$ $\therefore y = 2x^{-5} - 3x^{\frac{1}{2}}$ $\frac{dy}{dx} = -10x^{-6} - \frac{3}{2}x^{\frac{-1}{2}}$

At this stage I feel it's important to discuss 2 results that may be intuitive but often cause some confusion. Differentiating a term in x will give a constant, differentiating a constant will give 0.

For example, if $y = 3x$ then $\frac{dy}{dx} = 3$. Visually this should be fairly clear. The gradient of the line $y = 3x$ is 3.

Remember $\frac{dy}{dx}$ is the gradient function. Alternatively you can say initially the power of x is 1 such that $y = 3x^1$. When you reduce the power by 1 it will be 0 and by the rules of indices $x^0 = 1$.

Here is an example of differentiating a constant: If $y = 5$ then $\frac{dy}{dx} = 0$. The line $y = 5$ is a horizontal line which means the gradient is 0. Alternatively you could see this as $y = 5x^0$ and when you multiply down by the power it will = 0. Both of these results may seem obvious but students sometimes as why a term 'disappears'

The examples shown in the box previously have produced a 'gradient function' which allows us to find the gradient of the tangent to the curve at a given point. If we wanted to find the gradient of the tangent at a given point we could simply substitute in the given x coordinate.

Here is a typical question: *Find the gradient of the tangent to the curve* $y = 3x^2$ *at the point* $A(1,3)$.

We need to find $\frac{dy}{dx}$

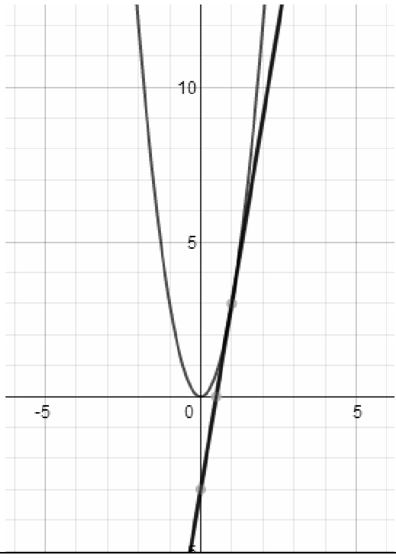
$y = 3x^2$

$\frac{dy}{dx} = 2(3x^1)$

$\frac{dy}{dx} = 6x$

Substituting in $x = 1$, $\frac{dy}{dx} = 6(1)$

This gives a gradient of 6. The gradient of the tangent (which is just a straight line) is 6. The sketch below shows a graphical representation. The tangent will touch the curve at the point $A(1,3)$



The Equation of a Tangent

A tangent is simply a straight line. As we have seen before, we need two things for the equation of a straight line. (1) A gradient and (2) A point the line passes through. We can find the gradient of a tangent using the gradient function ($\frac{dy}{dx}$ or $f'(x)$) and then simply substitute the values into the equation of a straight line.

Here is a basic example:

Find the equation of the tangent to the curve $f(x) = 2x^4 - 1$ *at the point* $P(1, y)$.

We need to find the gradient **and** the y coordinate of the point P. Let's start with the y coordinate of point P:

$f(x) = 2x^4 - 1$

$f(1) = 2(1) - 1$

$f(1) = 1$

We can now write the point P as $P(1,1)$. We need the gradient function so need to differentiate the function with respect to x.

If $f(x) = 2x^4 - 1$ then $f'(x) = 8x^3$. We say *"f dashed of x"*. You may be asked to show full workings remember!

I am now going to find $f'(1)$ which will give me the gradient of the curve at the point $P(1,1)$.

$f'(1) = 8(1)$ This gives a gradient of 8. I simply now substitute this into the equation of a straight line using one of the two methods shown below (this is covered in coordinate geometry).

$P(1,1)$ and $m = 8$ $y - y_1 = m(x - x_1)$ $y - 1 = 8(x - 1)$ $y = 8x - 7$	$P(1,1)$ and $m = 8$ $y = mx + c$ $1 = 8(1) + c$ $-7 = c$ $y = 8x - 7$

The Equation of a Normal

The normal is a straight line perpendicular to the tangent. If we find the value of the gradient of the tangent at a given point using $\frac{dy}{dx}$ or $f'(x)$ we can use the result $m_1 \times m_2 = -1$ if perpendicular to obtain the gradient of the normal at the same point. Once we have this gradient we can simply substitute the values we are given (or have to find) into the equation of a straight line to find an equation for the normal.

Let's look at a basic question:

Find the equation of the normal to the curve $y = 5\sqrt{x} - 2$ *at the point* $P(1,3)$.

We have a point the curve passes through so all we need is the gradient. Differentiating will give us the gradient function to find the gradient of the tangent:

$$y = 5\sqrt{x} - 2$$

$$y = 5x^{\frac{1}{2}} - 2$$

$$\frac{dy}{dx} = \frac{5}{2}x^{-\frac{1}{2}}$$

If we substitute $x = 1$ into $\frac{dy}{dx}$ we will find the gradient of the tangent at the point $P(1,3)$:

When $x = 1$, $\frac{dy}{dx} = \frac{5}{2}(1)$. We have a gradient of $\frac{5}{2}$. Using the result $m_1 \times m_2 = -1$ we can say the gradient of the normal is $\frac{-2}{5}$. It's the negative reciprocal. All we need to do now is substitute these values into the equation of a straight line:

$y - y_1 = m(x - x_1)$ where $P(1,3)$ *and* $m = \frac{-2}{5}$

$$y - 3 = \frac{-2}{5}(x - 1)$$

$$5y - 15 = -2x + 2$$

$$2x + 5y - 17 = 0$$

I have written the equation of the normal written in the form $ax + by + c = 0$. The question will guide you in terms of the form required. If the question states 'an equation' you can decide the form you leave it in.

Some questions will extend beyond these basics concepts and ask, for example, you may be asked "*What are coordinates of the other point on the curve where the gradient is also 2?*" or *Where does the normal intersect the curve again?* A quick sketch and basic applications of either algebra (mainly simultaneous equations) or the use of $\frac{dy}{dx}$ will allow you to find the given coordinates.

Integration

In C1 Integration is merely seen as a mechanical process and as the reverse of differentiation. Applications of integration are not considered until later units.
In a very algorithmic manner we can simply say in order to integrate we "Raise by a power, divide by the new power and add a constant of integration". The first part should make sense as it's the reverse of differentiation and follows from the previous section. The formal result is $\int x^n dx = \frac{x^{n+1}}{n+1} + c$, $n \neq -1$.
$n \neq -1$ (as division by 0 is undefined). I have used the integral sign here and it's notation you will need to be familiar with. Often questions will ask you to find an equation for y given $\frac{dy}{dx}$. Remember $\frac{dy}{dx}$, the gradient function, is found by differentiating both sides of an equation with respect to x where $y = \mathrm{f}(x)$.
We can simply write $y = \int \frac{dy}{dx} dx$. We say we are "Integrating with respect to x"
When we differentiate a function that includes a constant we end up with one less term as differentiating a constant gives 0. An example might be $y = 2x^2 - 3x + 5$ which differentiates to give $\frac{dy}{dx} = 4x - 3$.
Consider $y = 2x^2 - 3x + 6$. This would also have the derivative $\frac{dy}{dx} = 4x - 3$ as would $y = 2x^2 - 3x + k$ where k is a constant. This is why $+c$ is included when we integrate. Integrating gives us a *general solution* and a *family of curves* as we don't yet know the value of the constant we 'lost' when differentiating the original function. The *particular solution*, i.e. an equation that involves a numeric value for c can be found if we have initial conditions or have enough information in the question to find these initial conditions (this is just a value for x and y at a given point on the curve).
Let's consider basic some examples below:

Find $\int (3x^5 - 4x^2 + 2) dx$ $\int (3x^5 - 4x^2 + 2) dx = \frac{3}{6}x^6 - \frac{4}{3}x^3 + \frac{2}{1}x^1 + c$ Which simplifies to give: $\frac{1}{2}x^6 - \frac{4}{3}x^3 + 2x + c$	$y = \int \left(\frac{2}{x^4} - 5\sqrt{x} \right) dx$ $y = \int \left(2x^{-4} - 5x^{\frac{1}{2}} \right) dx$ $y = \frac{2}{-3}x^{-3} - \frac{5}{\frac{3}{2}}x^{\frac{3}{2}} + c$ Which simplifies to give: $y = -\frac{2}{3}x^{-3} - \frac{10}{3}x^{\frac{3}{2}} + c$	Given the curve C, where $y = \mathrm{f}(x)$ passes through the point $P(1,2)$ and is such that $\frac{dy}{dx} = 4x - 3$. Find an equation for C. $y = \int (4x - 3) dx$ $y = \frac{4}{2}x^2 - \frac{3}{1}x^1 + c$ $y = 2x^2 - 3x + c$ Substituting in the values for x and y: $2 = 2(1) - 3(1) + c$ $c = 3$ This gives us an equation for C which can be written $y = 2x^2 - 3x + 3$

As with questions on differentiation, you may have to use the rules of indices to simplify your equation or expression first.
Some questions will also involve both differentiation and integration.