OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4752**: Concepts for Advanced Mathematics

Advanced Subsidiary GCE

Mark Scheme for June 2015

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$kx^{\frac{1}{3}-1}$ oe $4x^{\frac{-2}{3}}$ isw cao	**M1** **A1** **[2]**	k is any non-zero constant ignore + c	 allow any equivalent exact simplified form
1	(ii)		kx^{-3+1} oe $-3x^{-2}$ isw $+ c$	**M1** **A1** **A1** **[3]**	k is any non-zero constant	 allow any equivalent exact simplified form
2			$u_2 = \frac{10}{2^2}, u_3 = \frac{10}{\text{their } 2.5^2}, u_4 = \frac{10}{\text{their } 1.6^2}$ isw $2 + u_2 + u_3 + u_4$ soi 10.00625 or $\frac{1601}{160}$ or $10\frac{1}{80}$ cao isw	**M1*** **M1dep*** **A1** **[3]**	 must be the sum of 4 terms only **B3** if unsupported	NB 2.5, 1.6, 3.90625 or $\frac{10}{4}, \frac{8}{5}, \frac{125}{32}$ may be implied by eg sight of 3.9 and answer of 10.0 NB 2.5, 1.1, 0.625 scores **M0M0**

3			$a + (10 - 1)d = 11.1$ and $a + (50 - 1)d = 7.1$	**M1**	may be implied by $40d = \pm 4$ or embedded in attempt to solve	condone one slip in coefficient of d
			$d = -0.1$	**A1**	if unsupported, **B2** for one of these and **B3** for both	
			$a = 12$	**A1**		
			$\frac{1}{2} \times 50(\textit{their } a + 7.1)$ with $a > 11.1$	**M1**	or $\frac{50}{2}(2a + (50-1)d)$ with $a > 11.1$ and $d < 0$	
			477.5 or 477½ or $\frac{955}{2}$ cao	**A1**		if **M0**, **B2** for any form of correct answer www
				[5]		
4			$27 = ½ r^2 \times 1.5$ oe	**M1**	or $27 = \frac{85.943669...}{360} \times \pi r^2$	angle in degrees rounded to 2 sf or more
			$r = 6$ soi	**A1**	may be embedded in formula for arc length	may be implied by later work eg 9 or 21
			their $r \times 1.5$	**M1**	or their $\frac{85.943639}{360} \times 2\pi \times$ their r	if r is incorrect, we must **see** their $r \times 1.5$ [$+ 2r$] for **M1** if r is correct, **M1** may be implied by 9 or 21
			21 [cm] cao	**A1**	allow full marks for recovery from working with rounded value of θ in degree form	**B4** for 21 unsupported www
				[4]		

5			$3x^2 - 6$ seen	**B1**		
			their $y' = 0$ or $y' > 0$ or $y' \geq 0$	**M1**	must be quadratic with at least one of only two terms correct	
			$\sqrt{2}$ and $-\sqrt{2}$ identified	**A1**	may be implied by use with inequalities or by ±1.41[4213562] to 3 sf or more	$\lvert x\rvert = \sqrt{2}$ implies **A1**
			$x < -\sqrt{2}$ or $x \leq -\sqrt{2}$ isw	**A1**	if **A1A0A0**, allow **SC1** for fully correct answer in decimal form to 3 sf or more	NB **just** $-\sqrt{2} > x > \sqrt{2}$ or $\sqrt{2} < x < -\sqrt{2}$ or $x > \pm\sqrt{2}$ implies the first **A1** then **A0A0**
			$x > \sqrt{2}$ or $x \geq \sqrt{2}$	**A1**	or **A2** for $\lvert x\rvert > \sqrt{2}$ or $\lvert x\rvert \geq \sqrt{2}$	
				[5]		
6	**(i)**		both curves with positive gradients in 1st and 2nd quadrants; ignore labels for this mark	**M1**	do not award if clearly not exponential shape; condone touching negative x-axis but not crossing it	consider each curve independently; ignore scales and points apart from $(0, 1)$
			both through $(0, 1)$	**A1**		allow if indicated in table of values or commentary if not marked on graph
			$y = 3^{2x}$ above $y = 3^x$ in first quadrant and below it in second	**A1**	must be clearly labelled, **A0** if wrongly attributed or if coincide for negative x from $(0, 1)$	if **M0** allow **SC1** for one graph fully correct
				[3]		
6	**(ii)**		$x = 3$	**B1**	**B0** if wrongly attributed	
			$3^x = 27$	**B1**	**B0** if wrongly attributed	allow $3^3 = 27$ with $x = 3$ stated
				[2]		

7			$1 - \cos^2 x = 3\cos x - 2$ oe	**M1***		
			$\cos^2 x + 3\cos x - 3$ [= 0]	**M1*dep**	or $-\cos^2 x - 3\cos x + 3 = 0$	condone one sign error *or* constant term of -1 (in LH version) or $+1$ (in RH version)
			$\cos x =$ their $\frac{-3+\sqrt{21}}{2}$ or $\cos x =$ their 0.79 to 0.7913 soi	**M1**	dependent on award of previous method mark, must be correct for their quadratic	ignore other values (eg –3.79...); condone recovery from $x = 0.791287847\ldots$ but **M0** if no recovery
			[x =] 0.6578 to 0.66 isw cao	**A1**	**A0** for eg 0.66π if 0.66 not seen separately	NB $x = 0.65788395\ldots$
			[x =] 5.625 to 5.63 isw cao	**A1**	if **A1A1** extra values in range incur a penalty of 1; ignore extra values outside range	NB $x = 5.625301357\ldots$
					if **A0A0** allow **SC1** for 37.69 to 37.7° *and* 322 to 322.31° *or* for (0.209 to 0.21)π *and* (1.79 to 1.791)π	no **SC** mark available if extra values in range
				[5]		
8			$m = 3$ seen	**B1**		
			$\log y = m\log x + 2$ or $\log y = m\log x + \log 100$	**M1**	or $\log y - 8 = m(\log x - 2)$	condone lack of base; "$c = 2$" is insufficient
			$\log y = \log x^3 + 2$ or $\log y = \log x^3 + \log 100$ or better	**M1**	or $10^{\log y} = 10^{3\log x + 2}$ or $10^{3\log x + \log 100}$ or better	condone lack of base, but not bases other than 10 unless fully recovered
			$y = 100x^3$ or $y = 10^{3\log x + 2}$ or $y = 10^{\log x^3 + 2}$ www isw	**A1**	$y = 10^{3\log x + \log 100}$ or $y = 10^{\log x^3 + \log 100}$	
				[4]		

9	**(i)**		$[\cos A =]\dfrac{20^2+13^2-8^2}{2\times13\times20}$	**M1***	or $8^2 = 20^2+13^2-2\times13\times20\times\cos A$	
			$[\cos A =]\dfrac{505}{520}$ oe soi	**A1**	or 0.971 to 0.9712	
			$A =$ 13.79 to 13.8° or 14°	**A1**	or 0.24077 to 0.241 or 0.24 (radians); allow **B3** if given to 3sf or more unsupported	or 15.32 (grad)
			[Area =] ½ × 20 × 13 × sin *their A*	**M1dep***	or **M1** for eg ½ × 20 × 8 × sin 22.8, as long as angle calculated correctly from their A (other angles are 22.79824…° and 143.40645..° or 36.59355..°)	or $\sqrt{\frac{41}{2}(\frac{41}{2}-8)(\frac{41}{2}-13)(\frac{41}{2}-20)}$ NB $13\sin A = 3.099899192$ if ½×b×h used
			30.99 to 31.01 isw	**A1**	allow **B2** for unsupported answer within range	
			or $\dfrac{5\sqrt{615}}{4}$ oe isw	**[5]**		

9	**(ii)**		$h = 4$ soi	**B1**		
			$\frac{\text{their } 4}{2} \times (0 + 0 + 2(1.45+1.56+1.27+1.04))$ *or* $\frac{\text{their } 4}{2} \times (0 + 0 + 2(\pm 0.85 \pm 0.76 \pm 0.55 \pm 0.30))$	**M1***	shape of formula correct with 2, 3 or 4 y-values in inner bracket with their h; allow recovery from bracket errors **M0** if any non-zero x-values used or if y-values used twice	eg $\frac{\text{their } 4}{2} \times \{1.45 + 1.04 + 2(1.56 + 1.27)\}$; signs must be consistent in 2nd alternative
				B1	all y-values correctly placed with their h, condone omission of zeros and/or omission of outer brackets	
			either 21.28 or $\pm$ 9.84	**A1**		or **B1** + **B3*** if area of 2 triangles and 3 trapezia calculated to give correct answer www The final **M1dep* A1** may then be earned. NB 2.9 + 6.02 + 5.66 + 4.62 + 2.08 or $\pm$ 1.7 $\pm$ 3.22 $\pm$ 2.62 $\pm$ 1.7 $\pm$ 0.60 with consistent signs throughout
			their 21.28 + their 9.84	**M1dep***		
			31.12	**A1**	ignore subsequent rounding, but **A0** if answer spoiled by eg multiplication by 20	

9	**(ii)**		*alternatively* $h = 4$ soi	 **B1**		
			attempt to find all y-values	**M1**	$y_{upper} - y_{lower}$	**M0** if values are added to obtain 0.60, 0.80 etc
			2.3, 2.32, 1.82, 1.34	**A1**	all y-values correct	
			$\frac{\text{their } 4}{2} \times (0 + 0 + 2(2.3+2.32+1.82+1.34))$	**M1**	shape of formula correct with 2, 3 or 4 of their y-values in inner bracket with their h; allow recovery from bracket errors **M0** if any non-zero x-values used or if y-values used twice	eg ½ × 4 × {2.3 + 1.34 + 2(2.32+1.82)}
				B1FT	all their y-values correctly placed, condone omission of zeros and/or omission of outer brackets	
			31.12	**A1**	ignore subsequent rounding, but **A0** if answer spoiled by eg multiplication by 20	or **B1M1A1** + **B3** if area of 2 triangles and 3 trapezia calculated to give correct answer www NB 4.6 + 9.24 + 8.28 + 6.32 + 2.68
				[6]		

10	**(i)**		$\left[\frac{dy}{dx}=\right] 4 \times 2 + 3$ or 11 isw	**M1***		
			9 = their $(4 \times 2 + 3) \times 2 + c$	**M1dep***	or $y - 9$ = their $(4 \times 2 + 3) \times (x - 2)$	
			$y = 11x - 13$ or $y = 11x + c$ and $c = -13$ stated isw	**A1** **[3]**	or $y - 9 = 11(x - 2)$ isw	
10	**(ii)**		$\frac{4x^2}{2} + 3x$	**M1***		
			$[y =] 2x^2 + 3x + c$	**A1**	must see "2" and " + c"; may be earned later eg after attempt to find c	
			$9 = 2\times2^2 + 3\times2 + c$	**M1dep***	must include constant, which may be implied by answer	
			$y = 2x^2 + 3x - 5$ cao	**A1**	allow first 4 marks for $y = 2x^2 + 3x + c$ and $c = -5$ stated	
			(1, 0) *and* (–2.5, 0) oe cao	**B1**	or for $x = 1$, $y = 0$ and $x = -2.5$, $y = 0$	**B0** for just stating $x = 1$ and $x = -2.5$
			$x = -\frac{3}{4}$	**B1**		
			$y = -\frac{49}{8}$	**B1**	–6.125 or − 6⅛	
				[7]		

10	**(iii)**		substitution to obtain [y =] f($2x$) in polynomial form	**M1**	f(x) must be the quadratic in x with linear and constant term obtained in part (ii), may be in factorised form	or their $x = 1 \rightarrow$ their 0.5 and their $x = -2.5 \rightarrow$ their $x = -1.25$
			$y = (2x - 1)(4x + 5)$ or $y = 8x^2 + 6x - 5$ or $y = 2\left(2x + \frac{3}{4}\right)^2 - \frac{49}{8}$	**A1FT**	must be simplified to one of these forms, **FT** their quadratic in x with linear and constant term obtained in part (ii)	hence $y = (2x - 1)(4x + 5)$ FT their x-intercepts from their quadratic in x with linear and constant term obtained in part (ii)
			$\left(-\frac{3}{8}, -\frac{49}{8}\right)$ oe	**B1** **[3]**	or **FT** their (both non-zero) co-ordinates for minimum point or their quadratic in x with linear and constant term obtained in part (ii)	
11	**(i)**		3×3^7 oe 6561	**M1** **A1** **[2]**	condone 1×3^7 or **B2** if unsupported	do not award if only seen in sum of terms of GP if **0, SC1** for 2187 unsupported
11	**(ii)**		valid attempt to sum a GP with $r = 3$ and $n = 15$ $\frac{3(3^{15} - 1)}{3 - 1}$ oe 21 523 359	**M1** **M1** **A1** **[3]**	eg $3 + 3^2 + \ldots\ldots + 3^{15}$ or **B2** if **M1M0** or **B3** if unsupported	must see at least first two terms and last term NB 7 174 453 implies **M1** from $1 + 3 + \ldots + 3^{14}$

11	**(iii)**		$\frac{3(3^n-1)}{3-1} > 1000000$ oe eg $3^{n+1} > 2000003$ or $3^n > \frac{2000000}{3} + 1$ www correctly taking logs of both sides eg $(n+1)\log 3 > \log 2000003$ or $n\log 3 > \log 2000003 - \log 3$ eg $n+1 > \frac{\log 2000003}{\log 3}$ and completion to $n > \frac{\log 2000003}{\log 3} - 1$ $n = 13$ seen	**M1*** **M1dep*** **A1** **B1** **[4]**	eg $\log 3^{n+1} > \log 2000003$ www or $\log 3^n + \log 3 > \log 2000003$ www; may be implied by next stage of working without any wrong working **B0** for $n \geq 13$ or $n > 13$	**M0** for working backwards **M0** if = or < used at least one previous progressive interim step needed with no wrong working; **M0dep*** for $\log(3^n - 1) > \ldots$ do not allow recovery from bracket errors at any stage
11	**(iv)**		valid attempt to sum a GP with $r = 2$ and $n = 15$ their 21 523 359 – their 65 534 21 457 825 isw	**M1*** **M1dep*** **A1** **[3]**	if correct eg $2 + 2^2 + \ldots\ldots + 2^{15} = 65\,534$ with their 65 534 < their 21 523 359 allow **B3** for 21 457 825 unsupported	NB 32767 implies **M1** from $1 + 2 + \ldots + 2^{14}$

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk

For staff training purposes and as part of our quality assurance programme your call may be recorded or monitored

Oxford Cambridge and RSA Examinations
is a Company Limited by Guarantee
Registered in England
Registered Office; 1 Hills Road, Cambridge, CB1 2EU
Registered Company Number: 3484466
OCR is an exempt Charity

OCR (Oxford Cambridge and RSA Examinations)
Head office
Telephone: 01223 552552
Facsimile: 01223 552553