Core 3 Numerical Methods Questions

2 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to

$$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

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6 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve $y = x^3 + 4x - 3$ intersects the x-axis at the point A where $x = \alpha$.

(a) Show that α lies between 0.5 and 1.0. *(2 marks)*

(b) Show that the equation $x^3 + 4x - 3 = 0$ can be rearranged into the form $x = \frac{3 - x^3}{4}$. *(1 mark)*

(c) (i) Use the iteration $x_{n+1} = \frac{3 - x_n{}^3}{4}$ with $x_1 = 0.5$ to find x_3, giving your answer to two decimal places. *(3 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{3 - x^3}{4}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(3 marks)*

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1 The curve $y = x^3 - x - 7$ intersects the x-axis at the point where $x = \alpha$.

(a) Show that α lies between 2.0 and 2.1. *(2 marks)*

(b) Show that the equation $x^3 - x - 7 = 0$ can be rearranged in the form $x = \sqrt[3]{x + 7}$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \sqrt[3]{x_n + 7}$ with $x_1 = 2$ to find the values of x_2, x_3 and x_4, giving your answers to three significant figures. *(3 marks)*

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6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_1^5 \ln x \, \mathrm{d}x$, giving your answer to three significant figures. *(3 marks)*

(c) The region R is bounded by the curve $y = \sec x$, the x-axis and the lines $x = 0$ and $x = 1$.

Find the volume of the solid formed when R is rotated through 2π radians about the x-axis, giving your answer to three significant figures. *(3 marks)*

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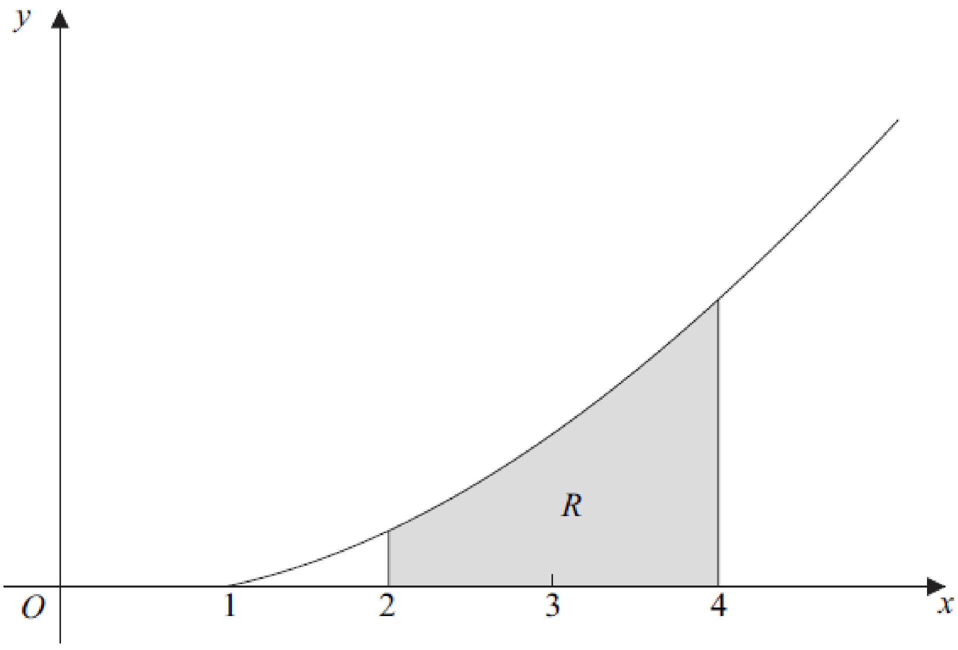
1 Use the mid-ordinate rule with four strips of equal width to find an estimate for $\int_1^5 \frac{1}{1+\ln x} \mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(b) The diagram shows the curve with equation $y = 2\sqrt{(x-1)^3}$ for $x \geqslant 1$.



The shaded region R is bounded by the curve $y = 2\sqrt{(x-1)^3}$, the lines $x = 2$ and $x = 4$, and the x-axis.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

(c) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \sqrt{x^3}$ onto the graph of $y = 2\sqrt{(x-1)^3}$. *(4 marks)*

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4 [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^2 3^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(b) The curve $y = 3^x$ intersects the line $y = x + 3$ at the point where $x = \alpha$.

(i) Show that α lies between 0.5 and 1.5. *(2 marks)*

(ii) Show that the equation $3^x = x + 3$ can be rearranged into the form

$$x = \frac{\ln(x+3)}{\ln 3}$$ *(2 marks)*

(iii) Use the iteration $x_{n+1} = \frac{\ln(x_n + 3)}{\ln 3}$ with $x_1 = 0.5$ to find x_3 to two significant figures. *(2 marks)*

(iv) The sketch on **Figure 1** shows part of the graphs of $y = \frac{\ln(x+3)}{\ln 3}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

Figure 1 (for Question 6)

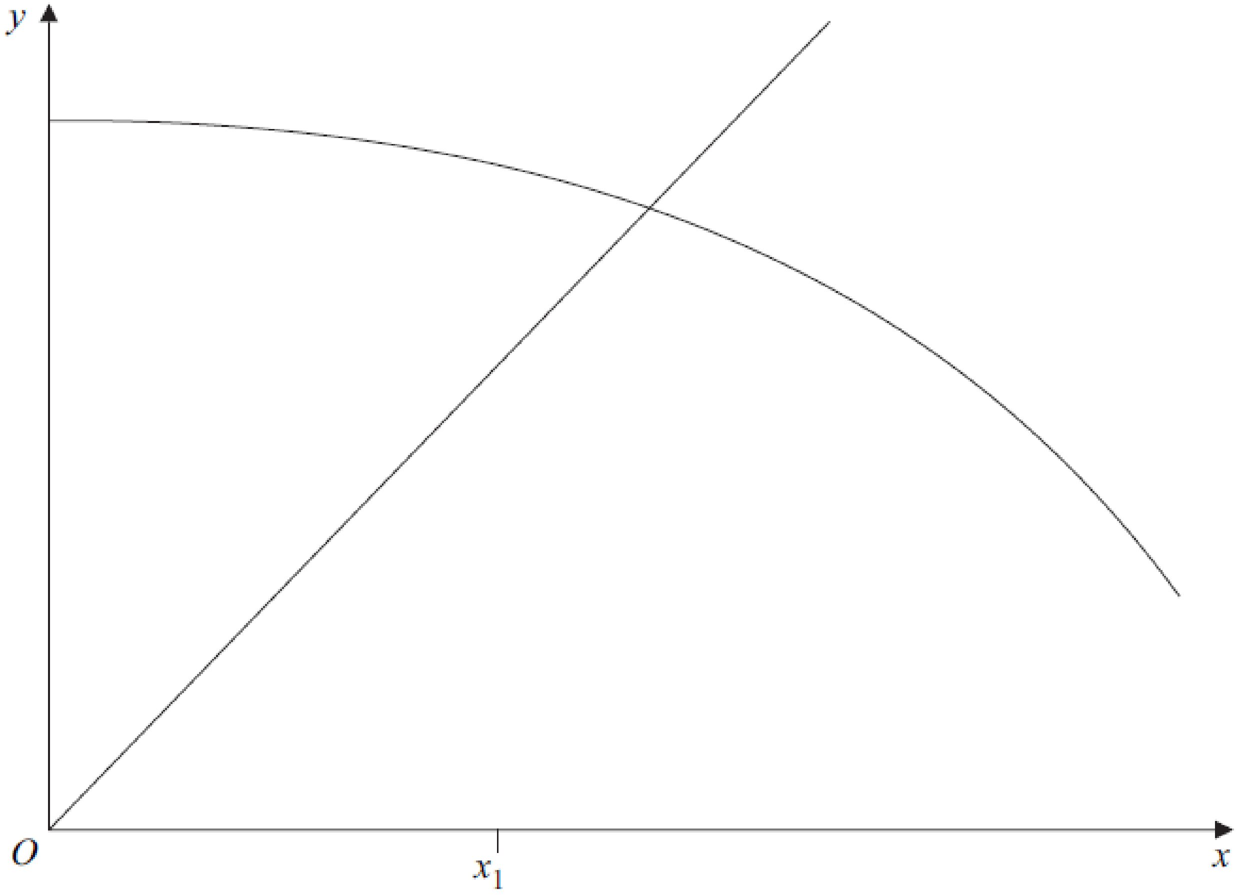
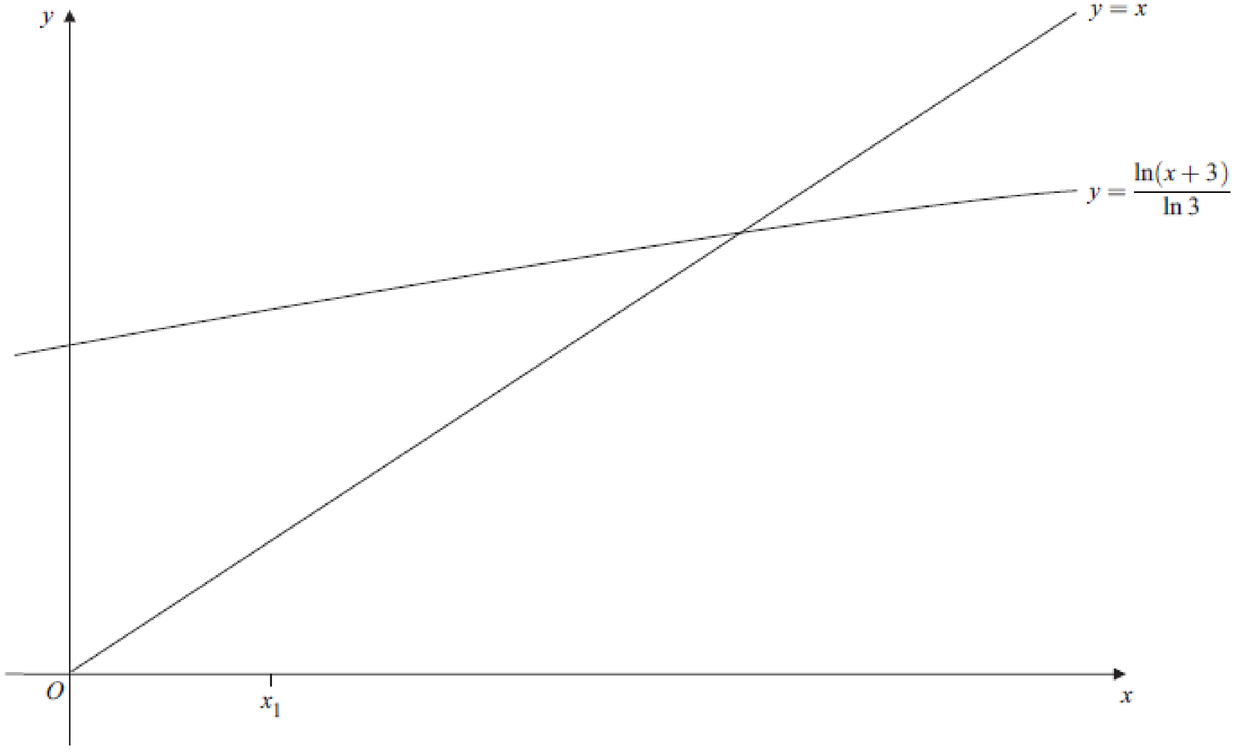
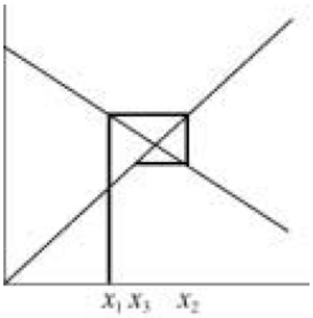


Figure 1 (for use in Question 4)



Core 3 Numerical Methods Answers

2	$\int_1^3 \frac{1}{\sqrt{1+x^3}}\,\mathrm{d}x$			
	x y 1 0.707(1) 1.5 0.478(1) 2 0.333(3) 2.5 0.245(3) 3 0.189(0)	B1 B1		3 correct all correct **SC** B1 for all correct expressions but wrongly evaluated
	$A=\frac{1}{3}\times 0.5\left[\begin{array}{l} y(1)+y(3)+ \\ 4\left(y(1.5)+y(2.5)\right)+2(y(2))\end{array}\right]$	M1		use of Simpson's rule
	$=0.743$	A1	4	
	Total		**4**	

6(a)	$f(0.5)=-0.875$ $f(1)=2$ Change of sign $\therefore$ root	M1 A1	2	
(b)	$x^3+4x-3=0$ $4x=3-x^3$ $x=\frac{3-x^3}{4}$	B1	1	AG
(c)(i)	$x_1=0.5$ $x_2=0.71875$ 0.72 AWRT $x_3=0.66$	M1 A1 A1	3	
(ii)		M1 A1 A1	3	For cobweb, x_1 to curve For x_2 All correct
	Total		**9**	

1(a)	$\left.\begin{array}{l} \mathrm{f}(2) = -1 \\ \mathrm{f}(2.1) = +0.161 \end{array}\right\}$	M1		both attempted
	change of sign $\therefore 2 < \alpha < 2.1$	A1	2	
(b)	$x^3 - x - 7 = 0$ $x^3 = x + 7$ $x = \sqrt[3]{x+7}$	B1	1	AG
(c)	$x_1 = 2$	M1		
	$x_2 = 2.0801\ldots$	A1		AWRT 2.08
	$x_3 = 2.0862\ldots$			AWRT 2.09
	$x_4 = 2.09$	A1	3	
	Total		**6**	

6(a)	$\therefore \int \ln x = 1(\ln 1.5 + \ln 2.5 + \ln 3.5 + \ln 4.5)$	M1 A1		use of 1.5, 2.5,… ; 3 or 4 correct x values AWFW 4 to 4.2
	$= 4.08$	A1	3	CAO

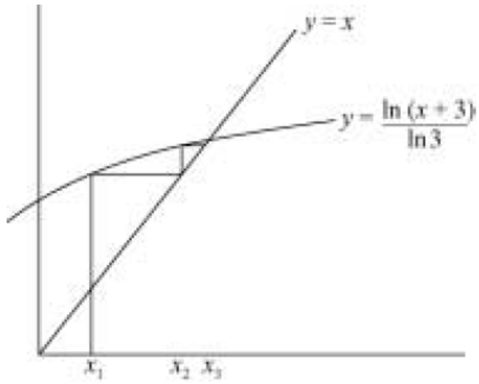
(c)	$V = (k)\int \sec^2 x \, \mathrm{d}x$ $= (k)[\tan x]_0^1$	M1 A1		
	$= 4.89$	A1	3	CAO

1	$x = 1.5, 2.5, 3.5, 4.5$	M1 A1		Method x values
	$\left.\begin{array}{ll} y_1 = 0.7115 & 0.712 \\ y_2 = 0.5218 & 0.522 \\ y_3 = 0.4439 & 0.444 \\ y_4 = 0.3993 & 0.399 \end{array}\right\}$ AWRT	A1		3 correct y's
	$A = 1 \times (y_1 + y_2 + y_3 + y_4)$ $= 2.08$	A1	4	
	Total		**4**	

8(a)	$A(-1,\pi)$	B1		
	$B\left(0,\frac{\pi}{2}\right)$	B1	2	
(b)	$\cos^{-1}x-3x-1=0$			
	$\mathrm{f}(0.1)=0.17$ allow 0.2, 0.1	M1		Or comparing 'sides'
	$\mathrm{f}(0.2)=-0.23$ allow -0.2			
	Change of sign ∴ root	A1	2	
(c)	$x_1=0.1$	M1		
	$x_2=0.1569=0.157$	A1		
	$x_3=0.1378=0.138$			
	$x_4=0.144$	A1	3	
	Total		**7**	

(b)	$V=4\,(\pi)\int_2^4(x-1)^3\,\mathrm{d}x$	M1		$(\pi)\int y^2\,\mathrm{d}x$
	$=\cancel{4}\pi\left[\frac{(x-1)^4}{\cancel{4}}\right]_2^4$	M1 m1		$k(x-1)^4\ (\pi)$ or in expanded form correct substitution of limits into $k(x-1)^4$
	$=\pi(81-1)=80\pi$	A1	4	CAO
(c)	Translate	E1		
	$\begin{pmatrix}1\\0\end{pmatrix}$	B1		OE
	Stretch (I) SF 2 (II)	M1		for I and (II or III)
	// y axis (III)	A1	4	for I and II and III

4(a)	y			
	x_0 1 3	B1		x values PI
	x_1 1.25 3.948(2)			
	x_2 1.5 5.196(2)	B1		(4 +) y values correct
	x_3 1.75 6.838(5)			
	x_4 2 9			
	$A=\frac{1}{3}\times\frac{1}{4}(3+4\times3.9482+2\times5.1962$ $+4\times6.8385+9)$	M1		Simpson's rule
	$=5.46$	A1	4	CAO
(b)(i)	$\mathrm{f}(x)=3^x-x-3$			
	$\left.\begin{array}{l}\mathrm{f}(0.5)=-1.77\\ \mathrm{f}(1.5)=0.696\end{array}\right\}$ change of sign ∴ root	M1A1	2	

(ii)	$3^x = x + 3$			
	$\ln 3^x = \ln(x+3)$	M1		correct use of logs
	$x \ln 3 = \ln(x+3)$			
	$x = \frac{\ln(x+3)}{\ln 3}$	A1	2	correct with no mistakes; AG
(iii)	$x_1 = 0.5$			
	$(x_2 = 1.14)$	M1		
	$x_3 = 1.29 = 1.3$	A1	2	CAO
(iv)		M1 A1	2	staircase x_2, x_3 correct and labelled on x-axis
	Total		**12**	