1

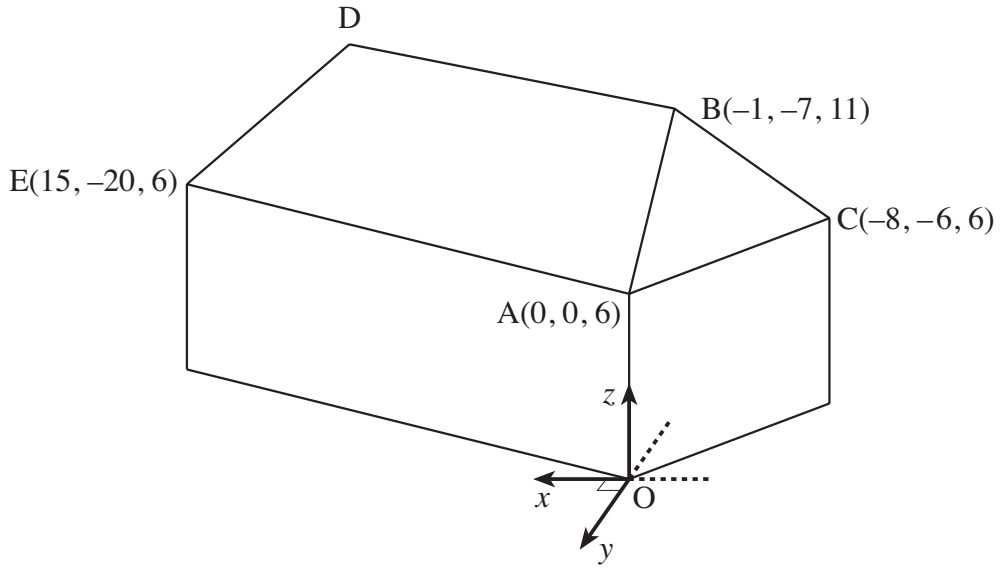

Fig. 7

Fig. 7 illustrates a house. All units are in metres. The coordinates of A, B, C and E are as shown. BD is horizontal and parallel to AE.

(i) Find the length AE. [2]

(ii) Find a vector equation of the line BD. Given that the length of BD is 15 metres, find the coordinates of D. [4]

(iii) Verify that the equation of the plane ABC is

$$-3x + 4y + 5z = 30.$$

Write down a vector normal to this plane. [4]

(iv) Show that the vector $\begin{pmatrix} 4 \\ 3 \\ 5 \end{pmatrix}$ is normal to the plane ABDE. Hence find the equation of the plane ABDE. [4]

(v) Find the angle between the planes ABC and ABDE. [4]

2 In Fig. 6, ABC, ACD and AED are right-angled triangles and BC = 1 unit. Angles CAB and CAD are θ and ϕ respectively.

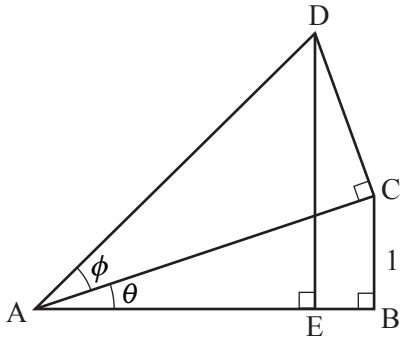

Fig. 6

(i) Find AC and AD in terms of θ and ϕ. **[2]**

(ii) Hence show that $\mathrm{DE} = 1 + \dfrac{\tan\phi}{\tan\theta}$. **[3]**

3 Verify that the vector $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is perpendicular to the plane through the points A(2, 0, 1), B(1, 2, 2) and C(0, –4, 1). Hence find the cartesian equation of the plane. **[5]**

4 Show that the straight lines with equations $\mathbf{r} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \qquad + \mu \qquad$ meet.

Find their point of intersection. **[5]**

5 The points A, B and C have coordinates (2, 0, –1), (4, 3, –6) and (9, 3, –4) respectively.

(i) Show that AB is perpendicular to BC. **[4]**

(ii) Find the area of triangle ABC. **[3]**

6 **(i)** Verify that the lines $\mathbf{r} = \begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ meet at the point (1, 3, 2). **[3]**

(ii) Find the acute angle between the lines. **[4]**