AQA

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname __________

Forename(s) __________

Candidate signature ANSWERS

A-level
MATHEMATICS
Unit Pure Core 3

A

Wednesday 15 June 2016 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Given that $y = (4x+1)^3 \sin 2x$, find $\frac{dy}{dx}$.

[2 marks]

(b) Given that $y = \frac{2x^2+3}{3x^2+4}$, show that $\frac{dy}{dx} = \frac{px}{(3x^2+4)^2}$, where p is a constant.

[2 marks]

(c) Given that $y = \ln\left(\frac{2x^2+3}{3x^2+4}\right)$, find $\frac{dy}{dx}$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

1a) $y = (4x+1)^3 \sin 2x$

$u = (4x+1)^3$

$\frac{du}{dx} = 12(4x+1)^2$

$v = \sin 2x$

$\frac{dv}{dx} = 2\cos 2x$

$\frac{dy}{dx} = 12(4x+1)^2 \sin 2x + 2(4x+1)^3 \cos 2x$

b) $y = \frac{2x^2+3}{3x^2+4}$

$u = 2x^2+3$

$\frac{du}{dx} = 4x$

$v = 3x^2+4$

$\frac{dv}{dx} = 6x$

$\frac{dy}{dx} = \frac{4x(3x^2+4) - 6x(2x^2+3)}{(3x^2+4)^2}$

$= \frac{12x^3 + 16x - 12x^3 - 18x}{(3x^2+4)^2} = \frac{-2x}{(3x^2+4)^2}$

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1

c) $y = \ln\left(\frac{2x^2+3}{3x^2+4}\right)$ $\quad u = \frac{2x^2+3}{3x^2+4}$ $\quad y = \ln u$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{(3x^2+4)^2}} \qquad \frac{dy}{du} = \frac{1}{u}$$

$$\boxed{\frac{dy}{du} = \frac{3x^2+4}{2x^2+3}}$$

$$\frac{dy}{dx} = \frac{-2x\,\cancel{(3x^2+4)}}{(\cancel{3x^2+4})^2(2x^2+3)}$$

$$= \frac{-2x}{(3x^2+4)(2x^2+3)}$$

Turn over ▶

2 The curve with equation $y = x^x$, where $x > 0$, intersects the line $y = 5$ at a single point, where $x = \alpha$.

(a) Show that α lies between 2 and 3.

[2 marks]

(b) Show that the equation $x^x = 5$ can be rearranged into the form

$$x = e^{\left(\frac{\ln 5}{x}\right)}$$

[3 marks]

(c) Use the iterative formula

$$x_{n+1} = e^{\left(\frac{\ln 5}{x_n}\right)}$$

with $x_1 = 2$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(d) (i) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to

$$\int_{0.5}^{1.7} (5 - x^x)\, dx$$

giving your answer to three significant figures.

[4 marks]

(ii) Hence find an approximation to $\int_{0.5}^{1.7} x^x \, dx$.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 2
2a)	$y = x^x \qquad y = 5$
	$x^x = 5$
	$x^x - 5 = 0 \qquad f(x) = x^x - 5$
	$f(2) = 2^2 - 5 = -1$
	$f(3) = 3^3 - 5 = 22$
	change of sign therefore α lies between 2 and 3

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QUESTION PART REFERENCE	Answer space for question 2

b) $x^x = 5$

$x \ln x = \ln 5$

$\ln x = \frac{\ln 5}{x}$

$e^{\ln x} = e^{\frac{\ln 5}{x}}$

$x = e^{\left(\frac{\ln 5}{x}\right)}$ (as required)

c) $x_1 = 2$

$x_2 = 2.236067977\ldots = \underline{2.236}$

$x_3 = 2.053965. = \underline{2.054}$

di) $\int_{0.5}^{1.7} (5 - x^x)\,dx$ $\quad h = \frac{1.7 - 0.5}{6} = 0.2$

x	0.5	0.7	0.9	1.1	1.3	1.5	1.7
y	4.29289	4.22074	4.09046	3.88946	3.59354	3.1628	2.535305
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

$\approx \frac{0.2}{3}\left(4.29289. + 2.535305.. + 4(4.220\ldots + 3.889\ldots + 3.1628\ldots) + 2(4.09\ldots + 3.59\ldots)\right)$

$= 4.485954.\ldots$

$= \underline{4.49 \text{ (3sf)}}$

ii) $\int_{0.5}^{1.7} (5)\,dx - \int_{0.5}^{1.7} (x^x)\,dx = 4.485\ldots$

$[5x]_{0.5}^{1.7} - \int_{0.5}^{1.7} (x^x)\,dx = 4.485\ldots$

$6 - \int_{0.5}^{1.7} (x^x)\,dx = 4.485\ldots \rightarrow \int_{0.5}^{1.7} (x^x) = \underline{\underline{1.5140}}$

Turn over ▶

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QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	Answer space for question 2

Do not write outside the box

3 Solve

$$x^2 \geqslant |5x - 6|$$

[5 marks]

QUESTION PART REFERENCE

Answer space for question 3

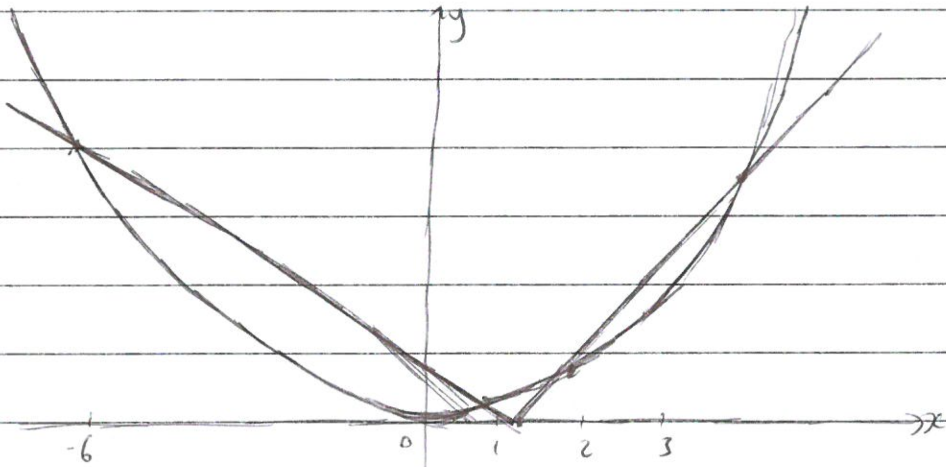
3) $x^2 \geqslant |5x-6|$

$x^2 \geqslant 5x - 6$ or $x^2 \geqslant -5x + 6$

$x^2 - 5x + 6 \geqslant 0$ $x^2 + 5x - 6 \geqslant 0$

$(x-3)(x-2)$ $(x+6)(x-1)$

$x = 3$ or $x = 2$ $x = -6$ or $x = 1$



$x \leqslant -6$, $1 \leqslant x \leqslant 2$, $x \geqslant 3$

QUESTION PART REFERENCE

Answer space for question 3

Turn over ▶

4 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = e^x$ onto the graph of $y = e^{2x-5}$.

[4 marks]

(b) The **normal** to the curve $y = e^{2x-5}$ at the point $P(2, e^{-1})$ intersects the x-axis at the point A and the y-axis at the point B.

Show that the area of the triangle OAB is $\dfrac{(e^2+1)^m}{e^n}$, where m and n are integers.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 4

4a) $y = e^x \rightarrow y = e^{2x-5}$

Translation $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and stretch scale factor $\frac{1}{2}$ in x direction

or

stretch scale factor $\frac{1}{2}$ in x direction and translation $\begin{pmatrix} 2.5 \\ 0 \end{pmatrix}$

b) $y = e^{2x-5}$

$\dfrac{dy}{dx} = 2e^{2x-5}$

when $x = 2$, gradient of tangent :-

$2 \times e^{2(2)-5} = 2e^{-1}$

$= \dfrac{2}{e}$

gradient of normal $\rightarrow \dfrac{-e}{2}$

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QUESTION PART REFERENCE

Answer space for question 4

$P(2, e^{-1})$ $\quad m = \frac{-e}{2}$

$$y - e^{-1} = \frac{-e}{2}(x-2)$$

when $x = 0$,

$y - e^{-1} = e$

$y = e + \frac{1}{e}$ (point B)

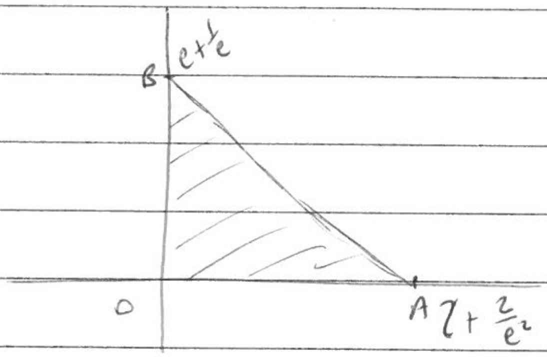
when $y = 0$,

$-\frac{1}{e} = \frac{-e}{2}(x-2)$

$-\frac{2}{e} = -e(x-2)$

$\frac{2}{e^2} = x - 2$

$x = 2 + \frac{2}{e^2}$ (point A)



$$\text{Area} = \frac{1}{2}\left(2 + \frac{2}{e^2}\right)\left(e + \frac{1}{e}\right)$$

$$= \left(1 + \frac{1}{e^2}\right)\left(e + \frac{1}{e}\right)$$

$$= e + \frac{1}{e} + \frac{1}{e} + \frac{1}{e^3}$$

$$= \frac{e^4 + e^2 + e^2 + 1}{e^3} = \frac{e^4 + 2e^2 + 1}{e^3}$$

$$= \frac{(e^2+1)^2}{e^3}$$

Turn over ▶

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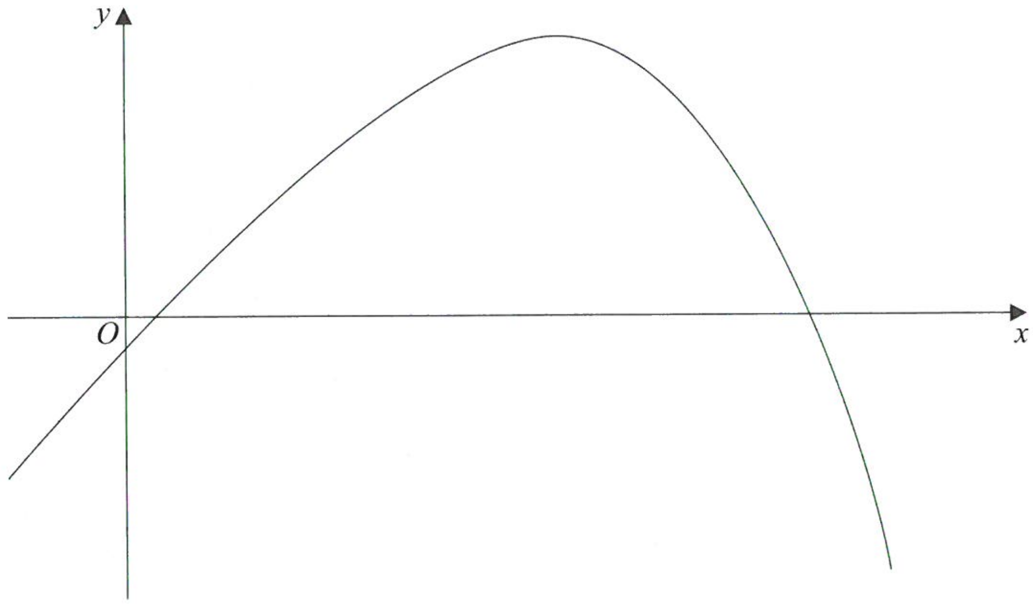
5 The function f is defined by

$$f(x) = 16x - e^{2x}, \quad \text{for all real } x$$

The graph of $y = f(x)$ is sketched below.



(a) Find the range of f.

[5 marks]

(b) The composite function fg is defined by

$$fg(x) = \frac{16}{x} - e^{\frac{2}{x}}, \quad \text{for real } x, \ x \neq 0$$

Find an expression for $gg(x)$.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 5**
5a)	$f(x) = 16x - e^{2x}$
	$\frac{dy}{dx} = 16 - 2e^{2x}$
	when $\frac{dy}{dx} = 0$. $16 - 2e^{2x} = 0$
	$2e^{2x} = 16$
	$e^{2x} = 8$
	$2x = \ln 8$
	$x = \frac{1}{2}\ln 8$

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QUESTION PART REFERENCE

Answer space for question 5

when $x = \frac{1}{2}\ln 8$,

$f(x) = 16\left(\frac{1}{2}\ln 8\right) - e^{2(\frac{1}{2}\ln 8)}$

$= 8\ln 8 - e^{\ln 8}$

$= 8\ln 8 - 8$

$f(x) \leq 8\ln 8 - 8$ or $f(x) \leq 8(\ln 8 - 1)$

b) $f(x) = 16x - e^{2x}$

$fg(x) = \frac{16}{x} - e^{\frac{2}{x}}$ 'x' replaced with '$\frac{1}{x}$'

so $g(x) = \frac{1}{x}$

$gg(x) = \frac{1}{\frac{1}{x}}$

$= \underline{\underline{x}}$

Turn over ▶

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6 (a) Use integration by parts to find $\int \frac{\ln(3x)}{x^2}\,dx$.

[4 marks]

(b) The region bounded by the curve $y = \frac{\ln(3x)}{x}$, the x-axis from $\frac{1}{3}$ to 1, and the line $x = 1$ is rotated through 2π radians about the x-axis to form a solid.

Find the exact value of the volume of the solid generated.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 6

6a) $\int \ln(3x)\, x^{-2}\, dx$

$$\boxed{\begin{array}{ll} u = \ln(3x) & \frac{dv}{dx} = x^{-2} \\ \frac{du}{dx} = \frac{1}{x} & \\ & v = -x^{-1} \\ & v = -\frac{1}{x} \end{array}}$$

$-\frac{1}{x}\ln 3x - \int -\frac{1}{x} \times \frac{1}{x}\, dx$

$-\frac{1}{x}\ln 3x - \int -x^{-2}\, dx$

$-\frac{1}{x}\ln 3x + \int x^{-2}\, dx$

$-\frac{1}{x}\ln 3x - \frac{1}{x} + c$

b) $\pi \int_{1/3}^{1} y^2\, dx$

$y = \frac{\ln(3x)}{x}$

$y^2 = \left(\frac{\ln(3x)}{x}\right)^2 = \frac{(\ln(3x))^2}{x^2}$

$\pi \int_{1/3}^{1} \frac{\ln 3x \times \ln 3x}{x^2}\, dx$

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QUESTION PART REFERENCE	Answer space for question 6

$$\pi \int_{\frac{1}{3}}^{1} \frac{\ln 3x}{x^2} \cdot \ln 3x \, dx$$

$u = \ln 3x$ $\quad \frac{dv}{dx} = \frac{\ln 3x}{x^2}$

$\frac{du}{dx} = \frac{1}{x}$

$v = -\frac{1}{x}\ln 3x - \frac{1}{x}$

$$\pi \left[\ln 3x\left(-\frac{1}{x}\ln 3x - \frac{1}{x}\right) - \int \frac{1}{x}\left(-\frac{1}{x}\ln 3x - \frac{1}{x}\right) dx \right]_{\frac{1}{3}}^{1}$$

$$\pi \left[\ln 3x\left(-\frac{1}{x}\ln 3x - \frac{1}{x}\right) + \int \left(\frac{\ln 3x}{x^2} + \frac{1}{x^2}\right) dx \right]_{\frac{1}{3}}^{1}$$

$$\pi \left[\ln 3x\left(-\frac{1}{x}\ln 3x - \frac{1}{x}\right) + \left(-\frac{1}{x}\ln 3x - \frac{1}{x} - \frac{1}{x}\right) \right]_{\frac{1}{3}}^{1}$$

$$\pi \left[\ln 3x\left(-\frac{1}{x}\ln 3x - \frac{1}{x}\right) - \frac{1}{x}\ln 3x - \frac{2}{x} \right]_{\frac{1}{3}}^{1}$$

$$\pi \left[\left(\ln 3(-\ln 3 - 1) - \ln 3 - 2\right) - \left(\ln 1(-3\ln 1 - 3) - 3\ln 1 - 6\right) \right]$$

$$\pi \left[\left(-(\ln 3)^2 - \ln 3 - \ln 3 - 2\right) - \left(0 - 6\right) \right]$$

$$\pi \left(-(\ln 3)^2 - 2\ln 3 - 2 + 6\right)$$

$$\pi \left(4 - 2\ln 3 - (\ln 3)^2\right)$$

7 (a) By writing $\sec x = (\cos x)^{-1}$, use the chain rule to show that, if $y = \sec x$, then

$$\frac{dy}{dx} = \sec x \tan x$$

[2 marks]

(b) The function f is defined by

$$f(x) = 2\tan x - 3\sec x, \quad \text{for} \quad 0 < x < \frac{\pi}{2}$$

Find the value of the y-coordinate of the stationary point of the graph of $y = f(x)$, giving your answer in the form $p\sqrt{q}$, where p and q are integers.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 7

7a) $\sec x = (\cos x)^{-1}$

$$\boxed{\begin{array}{ll} u = \cos x & y = u^{-1} \\ \frac{du}{dx} = -\sin x & \frac{dy}{du} = -u^{-2} \\ & \frac{dy}{du} = \frac{-1}{\cos^2 x} \end{array}}$$

$y = \sec x$

$y = (\cos x)^{-1}$

$\frac{dy}{dx} = \frac{-\sin x}{-\cos^2 x}$

$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$

$= \tan x \sec x \rightarrow \underline{\sec x \tan x}$ (as req)

b) $f(x) = 2\tan x - 3\sec x$

$\frac{dy}{dx} = 2\sec^2 x - 3\sec x \tan x$

stationary when $\frac{dy}{dx} = 0$

$2\sec^2 x - 3\sec x \tan x = 0$

$\sec x(2\sec x - 3\tan x) = 0$

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QUESTION PART REFERENCE	Answer space for question 7

$\sec x = 0$

$\cos x = \frac{1}{0}$ (undefined)

OR

$2\sec x - 3\tan x = 0$

$\frac{2}{\cos x} - \frac{3\sin x}{\cos x} = 0$

$\frac{2 - 3\sin x}{\cos x} = 0$

$2 - 3\sin x = 0$

$\boxed{\sin x = \frac{2}{3}}$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{2}{3}\right)^2 + \cos^2 x = 1$

$\frac{4}{9} + \cos^2 x = 1$

$\cos^2 x = \frac{5}{9} \rightarrow \boxed{\cos x = \frac{\sqrt{5}}{3}}$

$f(x) = 2\tan x - 3\sec x$

$= \frac{2\sin x}{\cos x} - \frac{3}{\cos x}$

$= 2\left(\frac{2/3}{\sqrt{5}/3}\right) - \frac{3}{\sqrt{5}/3}$

$= 2\left(\frac{2\sqrt{5}}{5}\right) - \frac{9\sqrt{5}}{5}$

$= \frac{4\sqrt{5}}{5} - \frac{9\sqrt{5}}{5}$

$= \frac{-5\sqrt{5}}{5} = -\sqrt{5}$

Do not write outside the box

8 Use the substitution $u = 4x - 1$ to find the exact value of

$$\int_{\frac{1}{4}}^{\frac{1}{2}} (5 - 2x)(4x - 1)^{\frac{1}{3}}\,dx$$

[7 marks]

QUESTION PART REFERENCE

Answer space for question 8

8) $\int_{\frac{1}{4}}^{\frac{1}{2}} (5-2x)(4x-1)^{\frac{1}{3}}dx$ 　　 $u = 4x-1$

$\frac{du}{dx} = 4$

$\int_0^1 (5-2x)\, u^{\frac{1}{3}} \frac{du}{4}$ 　　 $\boxed{dx = \frac{du}{4}}$

$\int_0^1 5 - 2\left(\frac{u+1}{4}\right) u^{1/3} \frac{du}{4}$ 　　 $\boxed{x = \frac{u+1}{4}}$

$\frac{1}{4}\int_0^1 \left(5 - \left(\frac{u+1}{2}\right)\right) u^{1/3} du$ 　　 $\boxed{\begin{array}{l}\text{when } x = 1/2 \to u = 4\left(\frac{1}{2}\right) - 1 = 1 \\ x = 1/4 \to u = 4(1/4) - 1 = 0\end{array}}$

$\frac{1}{4}\int_0^1 \left(\frac{10 - (u+1)}{2}\right) u^{1/3} du$

$\frac{1}{4}\int_0^1 \left(\frac{9 - u}{2}\right) u^{1/3} du$

$\frac{1}{4}\int_0^1 \left(\frac{9u^{1/3}}{2} - \frac{u^{4/3}}{2}\right) du$

$\frac{1}{4}\left[\frac{9u^{4/3}}{2(4/3)} - \frac{u^{7/3}}{2(7/3)}\right]_0^1$

$\frac{1}{4}\left[\frac{27u^{4/3}}{8} - \frac{3u^{7/3}}{14}\right]_0^1$

$\frac{1}{4}\left(\frac{27}{8} - \frac{3}{4}\right) = \frac{177}{224}$

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QUESTION PART REFERENCE	Answer space for question 8

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9 (a) It is given that $\sec x - \tan x = -5$.

(i) Show that $\sec x + \tan x = -0.2$. **[2 marks]**

(ii) Hence find the exact value of $\cos x$. **[3 marks]**

(b) Hence solve the equation

$$\sec(2x - 70°) - \tan(2x - 70°) = -5$$

giving all values of x, to one decimal place, in the interval $-90° \leqslant x \leqslant 90°$. **[3 marks]**

QUESTION PART REFERENCE	Answer space for question 9
9ai)	$\tan^2 x + 1 = \sec^2 x$
	$\sec^2 x - \tan^2 x = 1$
	$(\sec x + \tan x)(\sec x - \tan x) = 1$
	$(\sec x + \tan x)(-5) = 1$
	$\sec x + \tan x = \frac{-1}{5}$ (as req)
ii)	$\sec x - \tan x = -5$
	$+\ \sec x + \tan x = -0.2$
	$2\sec x = -5.2$
	$\sec x = -2.6$
	$\cos x = -\frac{1}{2.6} = -\frac{5}{13}$

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QUESTION PART REFERENCE | **Answer space for question 9**

b) when $\sec\theta - \tan\theta = -5$

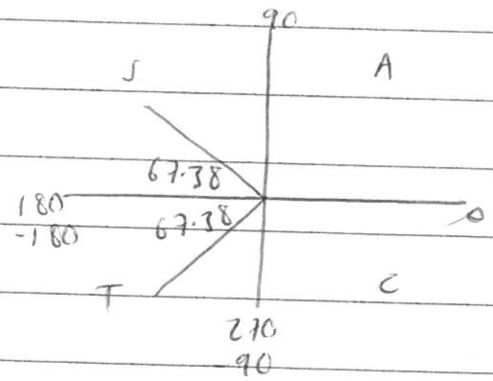
$\cos\theta = \frac{-5}{13}$ $\qquad -90^\circ \leq x \leq 90$

$-250 \leq 2x - 70 \leq 110$

$\cos(2x - 70) = \frac{-5}{13}$

$2x - 70 = \cos^{-1}\left(\frac{-5}{13}\right)$

$2x - 70 = 112.6198649 \rightarrow$ not in range



$2x - 70 = -112.6198649, \ -247.3801351...$

$x = -21.309932..., \ -88.69006...$

$x = \underline{-21.3^\circ}, \ \underline{-88.7^\circ}$

<u>However</u>, considering $\sec x - \tan x = -5$

$\underline{\sec x + \tan x = -0.2}$

$2\tan x = 4.8$

$\tan x = 2.4$

$x = \tan^{-1}(2.4) = 67.3801, \ 247.3801, \ -112.61986, \ -292.619$

only -112.61986 is in required range and both lists

so, $\underline{x = -21.3^\circ}$ is <u>only</u> solution for $-90^\circ \leq x \leq 90$

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QUESTION PART REFERENCE

Answer space for question 9

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QUESTION PART REFERENCE	Answer space for question 9

END OF QUESTIONS

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