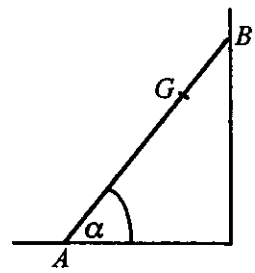
MECHANICS (A) UNIT 2 — TEST PAPER 3

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A ball, of mass m kg, is moving with velocity $(5\mathbf{i} - 3\mathbf{j})$ ms^{-1} when it receives an impulse of $(-2\mathbf{i} - 4\mathbf{j})$ Ns. Immediately after the impulse is applied, the ball has velocity $(3\mathbf{i} + k\mathbf{j})$ ms^{-1}. Find the values of the constants k and m. **(6 marks)**

2. A particle P, initially at rest at the point O, moves in a straight line such that at time t seconds after leaving O its acceleration is $(12t - 15)$ ms^{-2}. Find
   (a) the velocity of P at time t seconds after it leaves O, **(3 marks)**
   (b) the value of t when the speed of P is 36 ms^{-1}. **(3 marks)**

3. A non-uniform ladder AB, of length $3a$, has its centre of mass at G, where $AG = 2a$. The ladder rests in limiting equilibrium with the end B against a smooth vertical wall and the end A resting on rough horizontal ground. The angle between AB and the horizontal in this position is α, where $\tan\alpha = \frac{14}{9}$.
   Calculate the coefficient of friction between the ladder and the ground. **(7 marks)**



4. A particle P starts from the point O and moves such that its position vector $\mathbf{r}$ m relative to O after t seconds is given by $\mathbf{r} = at^2\mathbf{i} + bt\mathbf{j}$.
   60 seconds after P leaves O it is at the point Q with position vector $(90\mathbf{i} + 30\mathbf{j})$ m.
   (a) Find the values of the constants a and b. **(3 marks)**
   (b) Find the speed of P when it is at Q. **(4 marks)**
   (c) Sketch the path followed by P for $0 \leq t \leq 60$. **(2 marks)**

5. A lorry of mass 4200 kg can develop a maximum power of 84 kW. On any road the lorry experiences a non-gravitational resisting force which is directly proportional to its speed. When the lorry is travelling at 20 ms^{-1} the resisting force has magnitude 2400 N.
   Find the maximum speed of the lorry when it is
   (a) travelling on a horizontal road, **(4 marks)**
   (b) climbing a hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{7}$. **(6 marks)**

6. Two railway trucks, P and Q, of equal mass, are moving towards each other with speeds $4u$ and $5u$ respectively along a straight stretch of rail which may be modelled as being smooth. They collide and move apart. The coefficient of restitution between P and Q is e.

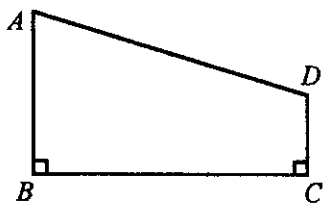
(a) Find, in terms of u and e, the speed of Q after the collision. **(6 marks)**

(b) Show that $e > \frac{1}{9}$. **(2 marks)**

Q now hits a fixed buffer and rebounds along the track. P continues to move with the speed that it had immediately after it collided with Q.

(c) Prove that it is impossible for a further collision between P and Q to occur. **(3 marks)**

7. A uniform lamina is in the form of a trapezium $ABCD$, as shown. AB and DC are perpendicular to BC. $AB = 17$ cm, $BC = 21$ cm and $CD = 8$ cm.



(a) Find the distances of the centre of mass of the lamina from

(i) AB, (ii) BC. **(8 marks)**

The lamina is freely suspended from C and rests in equilibrium.

(b) Find the angle between CD and the vertical. **(3 marks)**

8. A stone, of mass 1·5 kg, is projected **horizontally** with speed 4 ms^{-1} from a height of 7 m above horizontal ground.

(a) Show that the stone travels about 4·78 m horizontally before it hits the ground. **(4 marks)**

(b) Find the height of the stone above the ground when it has travelled half of this horizontal distance. **(4 marks)**

(c) Calculate the potential energy lost by the stone as it moves from its point of projection to the ground. **(2 marks)**

(d) Showing your method clearly, use your answer to part (c) to find the speed with which the stone hits the ground. **(3 marks)**

(e) State **two** modelling assumptions that you have made in answering this question. **(2 marks)**

