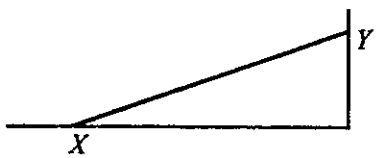
MECHANICS (A) UNIT 2 — TEST PAPER 4

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A small ball A is moving with velocity $(7\mathbf{i} + 12\mathbf{j})$ ms^{-1}. It collides in mid-air with another ball B, of mass 0·4 kg, moving with velocity $(-\mathbf{i} + 7\mathbf{j})$ ms^{-1}. Immediately after the collision, A has velocity $(-3\mathbf{i} + 4\mathbf{j})$ ms^{-1} and B has velocity $(6{\cdot}5\mathbf{i} + 13\mathbf{j})$ ms^{-1}.
Calculate the mass of A. **(4 marks)**

2. A stick of mass 0·75 kg is at rest with one end X on a rough horizontal floor and the other end Y leaning against a smooth vertical wall. The coefficient of friction between the stick and the floor is 0·6. Modelling the stick as a uniform rod, find the smallest angle that the stick can make with the floor before it starts to slip. **(6 marks)**

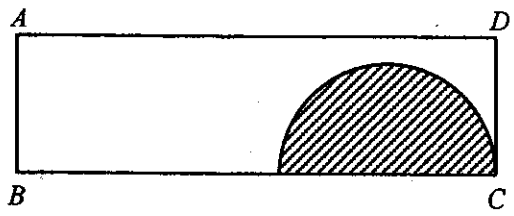


3. An engine of mass 20 000 kg climbs a hill inclined at 10° to the horizontal. The total non-gravitational resistance to its motion has magnitude 35 000 N and the maximum speed of the engine on the hill is 15 ms^{-1}.
   (a) Find, in kW, the maximum rate at which the engine can work. **(4 marks)**
   (b) Find the maximum speed of the engine when it is travelling on a horizontal track against the same non-gravitational resistance as before. **(3 marks)**

4. Relative to a fixed origin O, the points X and Y have position vectors $(4\mathbf{i} - 5\mathbf{j})$ m and $(12\mathbf{i} + \mathbf{j})$ m respectively, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in the directions due east and due north respectively. A particle P starts from X, and t seconds later its position vector relative to O is $(2t + 4)\mathbf{i} + (kt^2 - 5)\mathbf{j}$.
   (a) Find the value of k if P takes 4 seconds to reach Y. **(3 marks)**
   (b) Show that Q has constant acceleration and find the magnitude and direction of this acceleration. **(4 marks)**

5. Three particles A, B and C, of equal size and each of mass m, are at rest on the same straight line on a smooth horizontal surface. The coefficient of restitution between A and B, and between B and C, is e.
   A is projected with speed 7 ms^{-1} and strikes B directly. B then collides with C, which starts to move with speed 4 ms^{-1}.
   Calculate the value of e. **(10 marks)**

6. A rectangular piece of cardboard $ABCD$, measuring 30 cm by 12 cm, has a semicircle of radius 5 cm removed from it as shown.



(a) Calculate the distances of the centre of mass of the remaining piece of cardboard from AB and from BC. **(7 marks)**

The remaining cardboard is suspended from A and hangs in equilibrium.

(b) Find the angle made by AB with the vertical. **(4 marks)**

7. A rocket is fired from a fixed point O. During the first phase of its motion its velocity, v ms^{-1}, is given at time t seconds after firing by the formula

$$v = pt^2 + qt.$$

5 seconds after firing, the rocket is travelling at 500 ms^{-1}.

30 seconds after firing, the rocket is travelling at 12 000 ms^{-1}.

(a) Find the constants p and q. **(4 marks)**

(b) *Sketch* a velocity-time graph for the rocket for $0 \leq t \leq 30$. **(2 marks)**

(c) Find the initial acceleration of the rocket. **(2 marks)**

(d) Find the distance of the rocket from O 30 seconds after firing. **(4 marks)**

From time $t = 30$ onwards, the rocket maintains a constant speed of 12 000 ms^{-1}.

(e) Find the average speed of the rocket during its first 50 seconds of motion. **(3 marks)**

8. A golf ball is hit with initial velocity u ms^{-1} at an angle of 45° above the horizontal. The ball passes over a building which is 15 m tall at a distance of 30 m horizontally from the point where the ball was hit.

(a) Find the smallest possible value of u. **(7 marks)**

When u has this minimum value,

(b) show that the ball does not rise higher than the top of the building. **(4 marks)**

(c) Deduce the total horizontal distance travelled by the ball before it hits the ground. **(2 marks)**

(d) Briefly describe two modelling assumptions that you have made. **(2 marks)**

