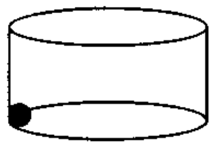
MECHANICS (C) UNIT 2 **TEST PAPER 9**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A plank of wood XY has length $5a$ m and mass 5 kg. It rests on a support at Q, where $XQ = 3a$ m. When a kitten of mass 8 kg sits on the plank at P, where $PY = a$ m, the plank just remains horizontal.
By modelling the plank as a non-uniform rod and the kitten as a particle, find
(i) the magnitude of the reaction at the support, [2]
(ii) the distance from X to the centre of mass of the plank, in terms of a. [2]

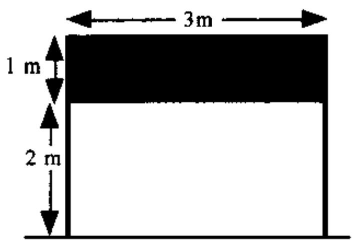
2. Charlotte, whose mass is 55 kg, is running up a straight hill inclined at 6° to the horizontal. She passes two points P and Q, 80 metres apart, with speeds $2{\cdot}5$ ms^{-1} and $1{\cdot}5$ ms^{-1} respectively.
Calculate, in J to the nearest whole number, the total work done by Charlotte as she runs from P to Q. [5]

3. A ball moves in a horizontal circle on the inside of a smooth hollow cylinder, in such a way that it remains in contact with both the side and the base of the cylinder. The mass of the ball is $0{\cdot}2$ kg and the radius of the circular cross-section of the cylinder is $0{\cdot}3$ m. The ball moves with constant angular speed 4 radians per second



(i) Find the magnitude and direction of the resultant force exerted on the ball by the cylinder. [5]
(ii) State a modelling assumption that you have made about the ball. [1]

4.



A sign-board consists of a rectangular sheet of metal, of mass M, which is 3 metres wide and 1 metre high, attached to two thin metal supports, each of mass m and length 2 metres. The board stands on horizontal ground.
(i) Calculate the height above the ground of the centre of mass of the sign-board, in terms of M and m. [3]

Given now that the centre of mass of the sign-board is $2{\cdot}2$ metres above the ground,
(ii) find the ratio $M : m$, in its simplest form. [3]

5. A ball is hit with initial speed u ms^{-1}, at an angle θ above the horizontal, from a point at a height of h m above horizontal ground. The ball, which is modelled as a particle moving freely under gravity, hits the ground at a horizontal distance d m from the point of projection.

   (i) Prove that $\frac{gd^2}{2u^2\cos^2\theta} - d\tan\theta - h = 0$. [7]

   Given further that $u = 14$, $h = 7$ and $d = 14$, and assuming the result $\frac{1}{\cos^2\theta} = 1 + \tan^2\theta$,

   (ii) find the value of θ. [5]

6. A cyclist is pedalling along a horizontal cycle track at a constant speed of 5 ms^{-1}. The air resistance opposing her motion has magnitude 42 N. The combined mass of the cyclist and her machine is 84 kg.

   (i) Find the rate, in W, at which the cyclist is working. [2]

   The cyclist now starts to ascend a hill inclined at an angle α to the horizontal, where $\sin\alpha = \frac{1}{21}$, at a constant speed.

   She continues to work at the same rate as before, against the same air resistance.

   (ii) Find the constant speed at which she ascends the hill. [4]

   In fact the air resistance is not constant, and a revised model takes account of this by assuming that the air resistance is proportional to the cyclist's speed.

   (iii) Use this model to find an improved estimate of the speed at which she ascends the hill, if her rate of working still remains constant. [7]

7. Two ships A and B, of masses m and km respectively, are moving towards each other in heavy fog along the same straight line, both with speed u. The ships collide and immediately after the collision they drift away from each other, both their directions of motion having been reversed. The speed of A after the impact is $\frac{1}{5}u$ and the speed of B after the impact is v.

   (i) Show that $v = u\left(\frac{6}{5k} - 1\right)$. [3]

   The coefficient of restitution between A and B is e.

   (ii) Show that $v = u\left(2e - \frac{1}{5}\right)$. [3]

   (iii) Use your answers to parts (i) and (ii) to find the rational numbers p and q such that $p \le k < q$. [7]

MECHANICS 2 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1.	(i) $R = 5g + 8g = 13g = 127{\cdot}4$ N	M1 A1	
	(ii) M(X) : $5g \times d + 8g \times 4a = 13g \times 3a$ $\quad 5d = 7a \quad d = 1{\cdot}4a$	M1 A1	4
2.	Loss in K.E. $= \frac{1}{2} \times 55 \times (2{\cdot}5^2 - 1{\cdot}5^2) = 110$ J	M1 A1	
	Gain in P.E. $= 55g(80 \sin 6°) = 4507$ J	M1 A1	
	Work done = total energy gained = 4397 J	A1	6
3.	(i) Vert. reaction $= 0{\cdot}2g = 1{\cdot}96$ N, hor. reaction $= mr\omega^2 = 0{\cdot}96$ N	B1 B1	
	Resultant $= \sqrt{(0{\cdot}96^2 + 1{\cdot}96^2)} = 2{\cdot}18$ N	M1 A1	
	at arctan $(1{\cdot}96/0{\cdot}96) = 63{\cdot}9°$ to horizontal	A1	
	(ii) Assumed radius is negligible compared with 0·3 m	B1	6
4.	(i) $M(2{\cdot}5) + 2m(1) = (M + 2m)\bar{x}$ $\quad \bar{x} = \frac{5M+4m}{2M+4m}$	M1 A1 A1	
	(ii) $5M + 4m = 2{\cdot}2(2M + 4m)$ $\quad 0{\cdot}6M = 4{\cdot}8m$ $\quad M : m = 8 : 1$	M1 A1 A1	6
5.	(i) $x = (u \cos \theta)t$, $\quad y = h + (u \sin \theta)t - \frac{1}{2}gt^2$	B1 M1 A1	
	$y = h + x \tan \theta - \frac{g}{2}\frac{x^2}{u^2 \cos^2 \theta}$ $\quad 0 = h + d \tan \theta - \frac{gd^2}{2u^2 \cos^2 \theta}$	M1 A1 M1	
	$\frac{gd^2}{2u^2 \cos^2 \theta} - d \tan \theta - h = 0$	A1	
	(ii) Let $\tan \theta = T$ $\quad$ Subst. given values : $4{\cdot}9(1 + T^2) - 14T - 7 = 0$	M1 A1	
	$7T^2 - 20T - 3 = 0$ $\quad (7T + 1)(T - 3) = 0$ $\quad T = 3$ $\quad \theta = 71{\cdot}6°$	M1 A1 A1	12
6.	(i) $P = Fv = 210$ W	M1 A1	
	(ii) $210 = v(42 + 84g \sin \alpha)$ $\quad v = 210 \div (42 + 4g) = 2{\cdot}59$ ms^{-1}	M1 A1 M1 A1	
	(iii) $R = kv$ $\quad 42 = 5k$, so $k = 8{\cdot}4$	M1 A1	
	$210 = v(8{\cdot}4v + 4g)$ $\quad 8{\cdot}4v^2 + 39{\cdot}2v - 210 = 0$	M1 A1	
	$3v^2 + 14v - 75 = 0$ $\quad v = (-14 + \sqrt{1096})/6 = 3{\cdot}18$ ms^{-1}	A1 M1 A1	13
7.	(i) Momentum : $mu - kmu = -m\frac{u}{5} + kmv$ $\quad \frac{6u}{5} - ku = kv$	M1 A1	
	$v = u\left(\frac{6}{5k} - 1\right)$	A1	
	(ii) Elasticity : $(v + \frac{u}{5}) / (-u - u) = -e$ $\quad v = 2eu - \frac{u}{5} = u\left(2e - \frac{1}{5}\right)$	M1 A1 A1	
	(iii) $v > 0$, so $2e > \frac{1}{5}$ $\quad$ Hence $\frac{1}{10} < e \leq 1$	M1 A1	
	$\frac{6}{5k} - 1 = 2e - \frac{1}{5}$, so $\frac{6}{5k} = 2e + \frac{4}{5}$ $\quad k = \frac{3}{5e+2}$	M1 A1 A1	
	Thus $\frac{3}{7} \leq k < \frac{6}{5}$ $\quad p = \frac{3}{7}, q = \frac{6}{5}$	M1 A1	13