Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced/Advanced Subsidiary

Monday 12 January 2004 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Mathematical Formulae (Lilac)	
Graph Paper (ASG2)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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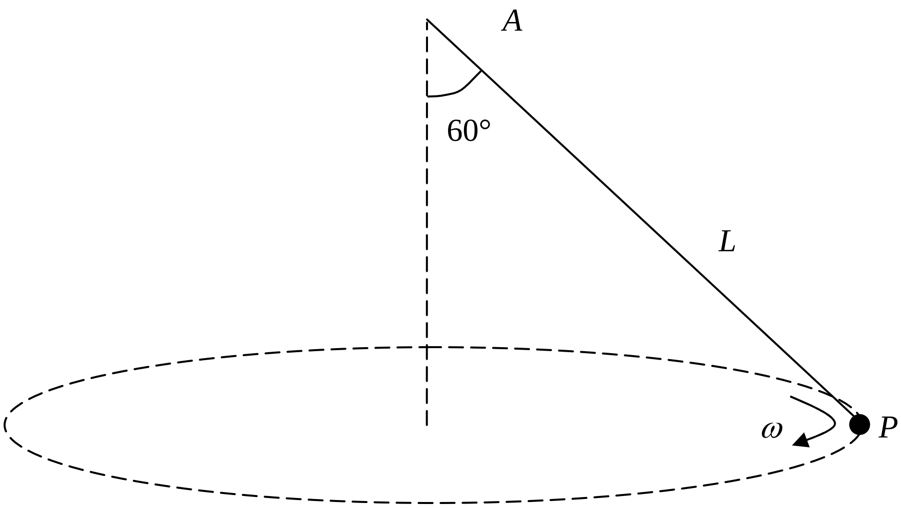
Figure 1



A particle P of mass m is attached to one end of a light string. The other end of the string is attached to a fixed point A. The particle moves in a horizontal circle with constant angular speed ω and with the string inclined at an angle of 60° to the vertical, as shown in Fig. 1. The length of the string is L.

(*a*) Show that the tension in the string is $2mg$.

(1)

(*b*) Find ω in terms of g and L.

(4)

The string is elastic and has natural length $\frac{3}{5}L$.

(*c*) Find the modulus of elasticity of the string.

(2)

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2. A particle P moves along the x-axis. At time t seconds its acceleration is $(-4e^{-2t})$ m s^{-2} in the direction of x increasing. When $t = 0$, P is at the origin O and is moving with speed 1 m s^{-1} in the direction of x increasing.

(*a*) Find an expression for the velocity of P at time t.

(3)

(*b*) Find the distance of P from O when P comes to instantaneous rest.

(6)

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3. Above the earth's surface, the magnitude of the force on a particle due to the earth's gravity is inversely proportional to the square of the distance of the particle from the centre of the earth. Assuming that the earth is a sphere of radius R, and taking g as the acceleration due to gravity at the surface of the earth,

(a) prove that the magnitude of the gravitational force on a particle of mass m when it is a distance x ($x \geq R$) from the centre of the earth is $\dfrac{mgR^2}{x^2}$.

(3)

A particle is fired vertically upwards from the surface of the earth with initial speed u, where $u^2 = \frac{3}{2}gR$. Ignoring air resistance,

(b) find, in terms of g and R, the speed of the particle when it is at a height $2R$ above the surface of the earth.

(7)

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4. A particle P of mass m is attached to one end of a light elastic string of length a and modulus of elasticity $\frac{1}{2}mg$. The other end of the string is fixed at the point A which is at a height $2a$ above a smooth horizontal table. The particle is held on the table with the string making an angle β with the horizontal, where $\tan \beta = \frac{3}{4}$.

(a) Find the elastic energy stored in the string in this position.

(3)

The particle is now released. Assuming that P remains on the table,

(b) find the speed of P when the string is vertical.

(3)

By finding the vertical component of the tension in the string when P is on the table and AP makes an angle θ with the horizontal,

(c) show that the assumption that P remains in contact with the table is justified.

(5)

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5. A piston in a machine is modelled as a particle of mass 0.2 kg attached to one end *A* of a light elastic spring, of natural length 0.6 m and modulus of elasticity 48 N. The other end *B* of the spring is fixed and the piston is free to move in a horizontal tube which is assumed to be smooth. The piston is released from rest when AB = 0.9 m.

(*a*) Prove that the motion of the piston is simple harmonic with period $\frac{\pi}{10}$ s.

(5)

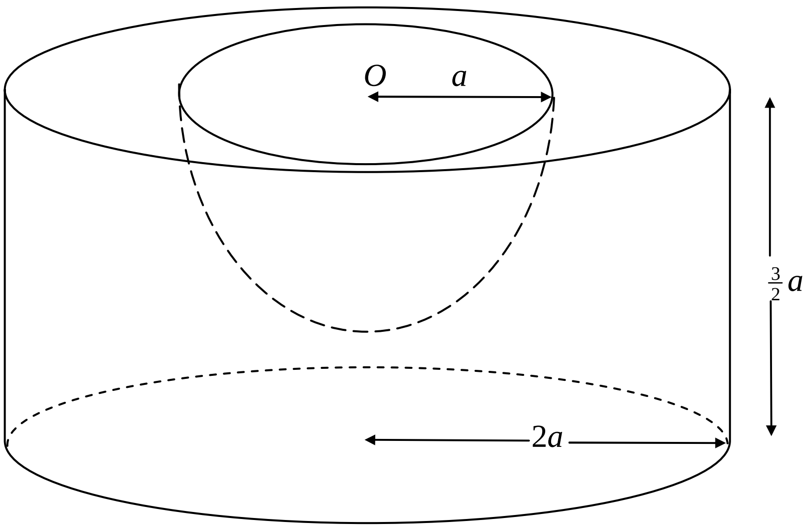
(*b*) Find the maximum speed of the piston.

(2)

(*c*) Find, in terms of π, the length of time during each oscillation for which the length of the spring is less than 0.75 m.

(5)

6. **Figure 2**



A uniform solid cylinder has radius $2a$ and height $\frac{3}{2}a$. A hemisphere of radius a is removed from the cylinder. The plane face of the hemisphere coincides with the upper plane face of the cylinder, and the centre O of the hemisphere is also the centre of this plane face, as shown in Fig. 2. The remaining solid is S.

(*a*) Find the distance of the centre of mass of S from O.

(6)

The lower plane face of S rests in equilibrium on a desk lid which is inclined at an angle θ to the horizontal. Assuming that the lid is sufficiently rough to prevent S from slipping, and that S is on the point of toppling when $\theta = \alpha$,

(*b*) find the value of α.

(3)

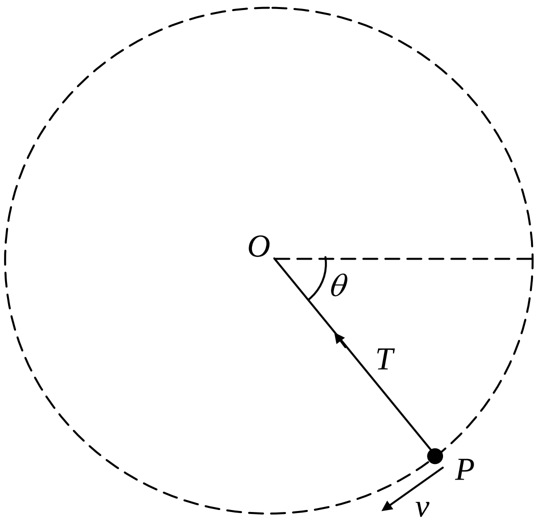
Given instead that the coefficient of friction between S and the lid is 0.8, and that S is on the point of sliding down the lid when $\theta = \beta$,

(*c*) find the value of β.

(3)

7.

Figure 3



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is fixed at a point O. The particle is held with the string taut and OP horizontal. It is then projected vertically downwards with speed u, where $u^2 = \frac{3}{2}ga$. When OP has turned through an angle θ and the string is still taut, the speed of P is v and the tension in the string is T, as shown in Fig. 3.

(*a*) Find an expression for v^2 in terms of a, g and θ.

(2)

(*b*) Find an expression for T in terms of m, g and θ.

(3)

(*c*) Prove that the string becomes slack when $\theta = 210°$.

(2)

(*d*) State, with a reason, whether P would complete a vertical circle if the string were replaced by a light rod.

(2)

After the string becomes slack, P moves freely under gravity and is at the same level as O when it is at the point A.

(*e*) Explain briefly why the speed of P at A is $\sqrt{\left(\frac{3}{2}ga\right)}$.

(1)

The direction of motion of P at A makes an angle φ with the horizontal.

(*f*) Find φ.

(4)

END