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June 2005 **Final Version**

6679 Mechanics M3
Mark Scheme

The following abbreviations are used in this scheme.

M	A method mark. These are awarded for 'knowing a method and attempting to apply it'.
A	An accuracy mark. Can only be awarded if the relevant method mark(s) have been earned.
B	These marks are independent of method marks.
cso	correct solution only. There must be no errors in this part of the question to obtain this mark.
cao	correct answer only.
ft	follow through. The scheme or marking guidance will specify what is to be followed through.
oe	or equivalent.
awrt	answers which round to

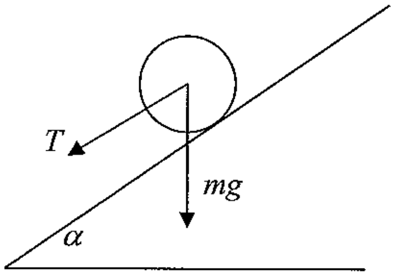
The second mark is dependent on gaining the first mark.

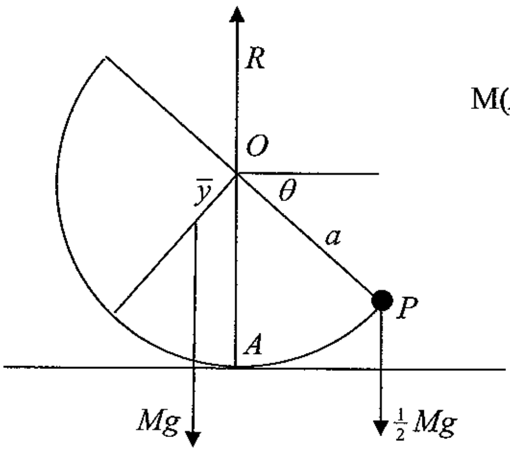
N2L	Newton's second law	LHS	Left hand side of an equation
LM	Linear momentum	RHS	Right hand side of an equation
HL	Hooke's Law.	EPE	Elastic potential energy

$\rightarrow$, $\downarrow$ etc. Resolving in the appropriate direction

M(A) Taking moments about A.

* The answer is printed on the paper.

Question Number	Scheme		Marks
1.		HL $T = \frac{20 \times 0.4}{2}$ $(=4)$ accept -4	M1 A1
		[$mg\sin\alpha + T = ma$	M1 A1
		$0.8g \times 0.6 + 4 = 0.8a$	M1
		$a = 10.88 \approx 10.9 \left(\text{m s}^{-2}\right)$ accept 11	A1
			[6]

Question Number	Scheme	Marks
2.	(a) Bowl, Lid, C	
	Mass ratio: Bowl 2, Lid 1, C 3 — anything in ratio 2 : 1 : 3	B1
	$\bar{y}$: Bowl $\frac{1}{2}a$, Lid 0, C $\bar{y}$	B1
	M(O) $\quad 2\times\frac{1}{2}a = 3\bar{y}$	M1
	$\bar{y} = \frac{1}{3}a$ * — cso	A1
		(4)
	(b) 	
	M(A) $\quad Mg\times\frac{1}{3}a\sin\theta = \frac{1}{2}Mg\times a\cos\theta$	M1 A1=A1
	$\tan\theta = \frac{3}{2}$	M1
	$\theta \approx 56°$ — cao	A1 **(5)**
		[9]
	Methods involving the location of the combined centre of mass of C and P are considered on the next page.	

Question Number	Scheme	Marks

2.

(b) *Methods involving the location of the combined centre of mass of C and P.*

G is the centre of mass of C; G' is the combined centre of mass of C and P.

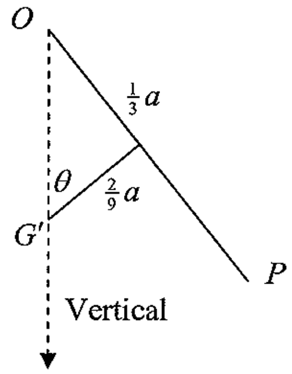
First Alternative

	C	P	C and P
Mass ratios	2	1	3
$\bar{y}$	$\frac{1}{3}a$	0	$\bar{y}$
$\bar{x}$	0	a	$\bar{x}$

Finding both coordinates of G' — M1

$\frac{2}{3}a = 3\bar{y} \quad \Rightarrow \quad \bar{y} = \frac{2}{9}a$ — A1

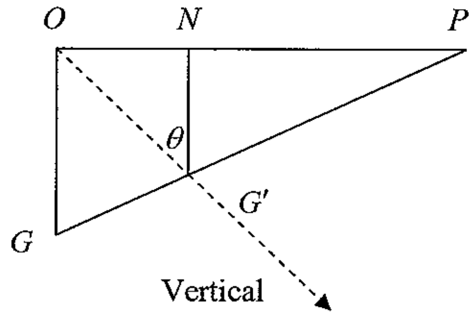
$a = 3\bar{x} \quad \Rightarrow \quad \bar{x} = \frac{1}{3}a$ — A1



$$\tan\theta = \frac{\frac{1}{3}a}{\frac{2}{9}a} = \frac{3}{2}$$ M1

$\theta \approx 56°$ cao — A1 **(5)**

Second Alternative



$GG' : G'P = \frac{1}{2}M : M = 1:2$

$OG = \frac{1}{3}a, \quad OP = a$

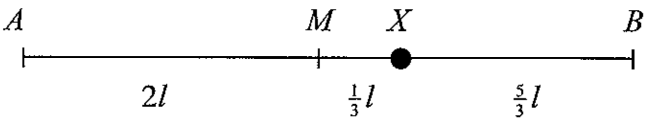
By similar triangles

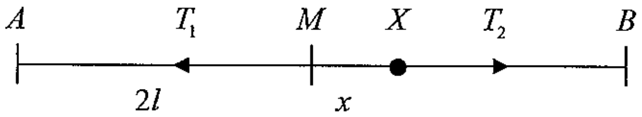
$ON = \frac{1}{3}OP = \frac{1}{3}a$ — M1 A1

$NG' = \frac{2}{3}OG = \frac{2}{9}a$ — A1

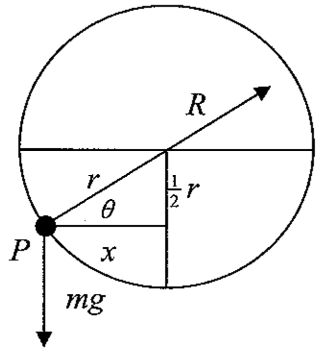
$$\tan\theta = \frac{ON}{NG'} = \frac{\frac{1}{3}a}{\frac{2}{9}a} = \frac{3}{2}$$ M1

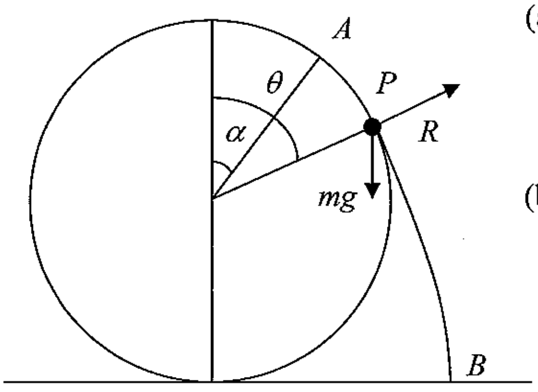
$\theta \approx 56°$ cao — A1 **(5)**

Question Number	Scheme	Marks	
3.	(a) 		
	Elastic energy when P is at X: $E = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l}$ $\left(= \frac{40mgl}{9}\right)$	M1 A1	
	$\frac{1}{2}mV^2 + 2\times\frac{4mgl^2}{2l} = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l}$	M1A1=A1ft	
	$\frac{1}{2}V^2 + 4gl = \frac{8}{9}gl + \frac{32}{9}gl$		
	$V^2 = \frac{8gl}{9}$ solving for V^2	M1	
	$V = \left(\frac{8gl}{9}\right)^{\frac{1}{2}}$ or exact equivalents	A1	(7)
	(b) The maximum speed occurs when $a = 0$	B1	
	At M the particle is in equilibrium (the sum of the forces is zero) $\Rightarrow a = 0$	B1	(2)
			[9]
	The alternative method using Newton's Second Law is considered on the next page.		

Question Number	Scheme	Marks
3.	*Alternative using Newton's second law.*	
	(a)	
		
	HL $\quad T_1 = \dfrac{4mg(l+x)}{l}, \quad T_2 = \dfrac{4mg(l-x)}{l}$	
	N2L $\quad m\ddot{x} = T_2 - T_1 = -\dfrac{8mg}{l}x$	M1 A1
	This is SHM, centre M	
	$a = \dfrac{l}{3}, \quad \omega^2 = \dfrac{8g}{l}$	A1, A1ft
	$v^2 = \omega^2\left(a^2 - x^2\right) \Rightarrow v^2 = \dfrac{8g}{l}\left(\dfrac{l^2}{9} - x^2\right)$ Depends on showing SHM	M1
	At M, $x = 0$, $\quad V^2 = \dfrac{8gl}{9}, \quad V = \left(\dfrac{8gl}{9}\right)^{\frac{1}{2}}$ or exact equivalents	M1, A1 **(7)**
	(b) The particle is performing SHM about the mid-point of AB.	B1
	The maximum speed occurs at the centre of the oscillation (when $x = 0$)	B1 **(2)**
		[9]

Question Number	Scheme	Marks
4.	(a) $\sin\theta = \frac{\frac{1}{2}r}{r} = \frac{1}{2}$ $(\Rightarrow \theta = 30°)$	B1
	$\uparrow$ $R\sin\theta = mg$	M1 A1
	$R = 2mg$	A1 **(4)**
	(b) $\rightarrow$ $R\cos\theta = mx\omega^2$	M1 A1
	$= m(r\cos\theta)\omega^2$	A1
	$\omega = \left(\frac{2g}{r}\right)^{\frac{1}{2}}$	A1
	$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{r}{2g}\right)^{\frac{1}{2}}$ or exact equivalent	M1 A1 **(6)**
	Note: $x = \frac{\sqrt{3}}{2}r$	**[10]**



Question Number	Scheme		Marks
5.		(a) $\frac{1}{2}mv^2 = mg\left(\underline{a\cos\alpha - a\cos\theta}\right)$	M1 A1 A1
		$v^2 = 2ga\left(\cos\alpha - \cos\theta\right)$ * cso	A1 (4)
		(b) [$mg\cos\theta\,(-R) = \frac{mv^2}{a}$ $(R = 0)$	M1 A1=A1
		$g\cos\theta = 2g\left(\frac{3}{4} - \cos\theta\right)$	M1
		$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ (accept 60°)	A1 (5)
	(c) From A to B $\frac{1}{2}mw^2 = mg\left(\underline{a + a\cos\alpha}\right)$		M1 A1 A1
	$w^2 = 2ga\left(1 + \frac{3}{4}\right) \Rightarrow w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$		A1 (4)
			[13]
	Alternative solutions to 5(c) *are considered on the next page.*		

Question Number	Scheme	Marks
5.	*Alternatives to* 5(c)	
	From P to C	
	$v_P^2 = 2ga\left(\frac{3}{4}-\frac{1}{2}\right) = \frac{ga}{2}$	
	$\frac{1}{2}mw^2 - \frac{1}{2}m\left(\frac{ga}{2}\right) = mg\left(\underline{a + a\cos\theta}\right)$	M1 A1 <u>A1</u>
	$w^2 - \frac{ga}{2} = 2mga\left(1+\frac{1}{2}\right) \quad \Rightarrow \quad w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$	A1 **(4)**
	Alternatives using projectile motion from P	
	$v_P = \left(\frac{ga}{2}\right)^{\frac{1}{2}}$, as above	
	$\downarrow \quad u_y = \left(\frac{ga}{2}\right)^{\frac{1}{2}}\sin 60° = \left(\frac{3ga}{8}\right)^{\frac{1}{2}}$	
	$\downarrow \quad v_y^2 = u_y^2 + 2g\times\frac{3a}{2}, = \frac{27ga}{8}$	M1, A1
	$\rightarrow \quad u_x = \left(\frac{ga}{2}\right)^{\frac{1}{2}}\cos 60° = \left(\frac{ga}{8}\right)^{\frac{1}{2}}$	A1
	$w^2 = u_x^2 + v_y^2 = \frac{ga}{8} + \frac{27ga}{8} = \frac{7ga}{2} \quad \Rightarrow \quad w = \left(\frac{7ga}{2}\right)^{\frac{1}{2}}$	A1 **(4)**
	There are also longer projectile methods using time of flight	
	In outline, solving $\frac{3a}{2} = \left(\frac{3ga}{8}\right)^{\frac{1}{2}}t + \frac{1}{2}gt^2$ gives $t = \left(\frac{3a}{2g}\right)^{\frac{1}{2}}$,	
	then, using $v = u + at$ gives $v_y = \left(\frac{3ga}{8}\right)^{\frac{1}{2}} + g\left(\frac{3a}{2g}\right)^{\frac{1}{2}} = \left(\frac{27ga}{8}\right)^{\frac{1}{2}}$, then as before.	M1 A1

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Question Number	Scheme	Marks
6.	(a) $a=3,\quad T=12\quad \left(\text{or } \tfrac{1}{2}T=6\right)$	B1, B1
	$T=\dfrac{2\pi}{\omega}=12 \Rightarrow \omega=\dfrac{\pi}{6}\quad (\approx 0.52)$	M1 A1
	In the scheme below, when a and/or ω appear in a line, accept the symbols or the candidates' values of a and/or ω for the marks in that line.	
	(Taking $x=a$ when $t=0$) $\quad x=a\cos\omega t$	M1
	$\dot{x}=-a\omega\sin\omega t$	M1 A1
	When $t=5$ $\quad \dot{x}=-3\times\dfrac{\pi}{6}\sin\dfrac{5\pi}{6}$	M1
	$\lvert\dot{x}\rvert=\dfrac{\pi}{4}\quad \left(\mathrm{m\,h^{-1}}\right)$ awrt 0.79	A1 **(9)**
	(b) Depth of 5.5 m $\Rightarrow x=-1.5$	
	$-1.5=a\cos\omega t$	M1
	$\cos\omega t=-\dfrac{1}{2}$	A1ft
	$\dfrac{\pi}{6}t=\dfrac{2\pi}{3},\ \left(\dfrac{4\pi}{3}\right)$	M1
	$t=4, 8$	A1
	Required time is $\quad t_2-t_1=8-4=4\quad$ (h)	A1 **(5)**
		[14]
	In 6(b), the following should be accepted	
	$1.5=a\cos\omega t$	M1
	$\cos\omega t=\dfrac{1}{2}$	A1ft
	$\dfrac{\pi}{6}t=\dfrac{\pi}{3}$	M1
	$t=2$	A1
	Required time is $\quad 2t=4\quad$ (h)	A1 **(5)**
	Further alternatives are given over the page.	

Question Number	Scheme	Marks
6.	*Alternative to* 6(a) The last 5 marks of 6(a) can be gained as follows. The first 4 marks are as above.	
	When $t=5$ $\quad x = 3\cos\frac{5\pi}{6} = -\frac{3\sqrt{3}}{2} \quad (\square -2.60)$	M1
	$v^2 = \omega^2\left(a^2 - x^2\right)$	M1
	$= \frac{\pi^2}{6^2}\left(9 - \frac{9\times 3}{4}\right) \quad \left(= \frac{\pi^2}{16}\right)$	M1 A1
	$\lvert v\rvert = \frac{\pi}{4} \quad \left(\text{m h}^{-1}\right)$ awrt 0.79	A1
	Alternatives measuring x from the centre of oscillation	
	(a) (Using 1400 as $t = 0$)	
	The first 4 marks are as above	B1 B1 M1 A1
	$x = a\sin\omega t$	M1
	$\dot{x} = a\omega\cos\omega t$	M1 A1
	When $t = 2$ $\quad \dot{x} = 3\times\frac{\pi}{6}\cos\frac{2\pi}{6}$ $\quad t=2$ oe is essential for this M	M1
	$= \frac{\pi}{4} \quad \left(\text{m h}^{-1}\right)$	A1 **(9)**
	(b) $\quad 1.5 = 3\sin\omega t$	M1
	$\sin\omega t = \frac{1}{2}$	A1ft
	$\frac{\pi}{6}t = \frac{\pi}{6}, \left(\frac{5\pi}{6}\right)$	M1
	$t = 1, 5$	A1
	Required time is $\quad t_2 - t_1 = 5 - 1 = 4 \quad$ (h)	A1 **(5)** **[14]**

Question Number	Scheme	Marks
7.	(a) $\frac{1}{3}\ddot{x} = -\frac{k}{(x+1)^2}$	M1
	$\frac{1}{3}v\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{k}{(x+1)^2}$	M1
	$\int v\,\mathrm{d}v = \int -\frac{3k}{(x+1)^2}\,\mathrm{d}x$ Separating variables &	
	$\frac{1}{2}v^2 = \frac{3k}{x+1}$ $(+C)$ attempting integration of both sides	M1 A1=A1
	$v^2 = \frac{6k}{x+1} + A$	
	Using boundary values to obtain two simultaneous equations.	M1
	$(1, 4)$ $16 = 3k + A$	A1
	$(8, \sqrt{2})$ $2 = \frac{2k}{3} + A$	A1
	$14 = \frac{7}{3}k \Rightarrow k = 6$	M1 A1 **(10)**
	(b) $A = -2$	B1
	$v^2 = \frac{36}{x+1} - 2 = 0$	M1
	$x = 17$ (m)	M1 A1 **(4)**
		[14]