Core Mathematics 3 Paper E

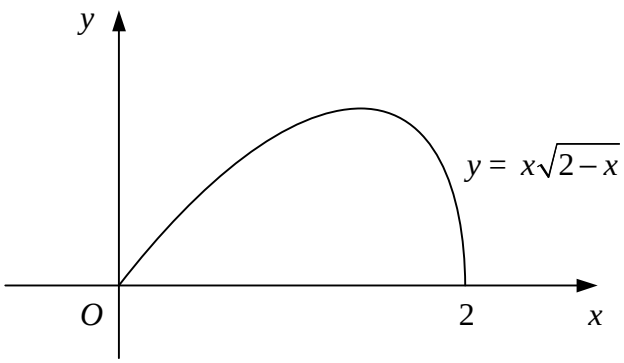
1. *(i)* Solve the inequality

$$|x - 0.2| < 0.03 \qquad [2]$$

(ii) Hence, find all integers n such that

$$|0.95^n - 0.2| < 0.03 \qquad [3]$$

2.



The diagram shows the curve with equation $y = x\sqrt{2-x}$, $0 \le x \le 2$.

Find, in terms of π, the volume of the solid formed when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. [5]

3. Solve, for $0 \le y \le 360$, the equation

$$2\cot^2 y° + 5\,\text{cosec}\, y° + \text{cosec}^2 y° = 0. \qquad [6]$$

4. A curve has the equation $x = y\sqrt{1-2y}$.

(i) Show that

$$\frac{dy}{dx} = \frac{\sqrt{1-2y}}{1-3y}. \qquad [4]$$

The point A on the curve has y-coordinate -1.

(ii) Show that the equation of tangent to the curve at A can be written in the form

$$\sqrt{3}\,x + py + q = 0$$

where p and q are integers to be found. [3]

5. The function f is defined by

$$f(x) \equiv 4 - \ln 3x, \quad x \in \mathbb{R}, \quad x > 0.$$

(i) Solve the equation $f(x) = 0$. [2]

(ii) Sketch the curve $y = f(x)$. [2]

The function g is defined by

$$g(x) \equiv e^{2-x}, \quad x \in \mathbb{R}.$$

(iii) Show that

$$fg(x) = x + a - \ln b,$$

where a and b are integers to be found. [3]

6. Find the value of each of the following integrals in exact, simplified form.

(i) $\int_{-1}^{0} e^{1-2x} \, dx$ [4]

(ii) $\int_{2}^{4} \frac{3x^2 - 2}{x} \, dx$ [4]

7. $$f(x) = 2 + \cos x + 3 \sin x.$$

(i) Express $f(x)$ in the form

$$f(x) = a + b \cos (x - c)$$

where a, b and c are constants, $b > 0$ and $0 < c < \frac{\pi}{2}$. [3]

(ii) Solve the equation $f(x) = 0$ for x in the interval $0 \leq x \leq 2\pi$. [4]

(iii) Use Simpson's rule with four strips, each of width 0.5, to find an approximate value for

$$\int_{0}^{2} f(x) \, dx.$$ [3]

Turn over

8. $\mathrm{f}(x) \equiv 2x^2 + 4x + 2, \quad x \in \mathbb{R}, \quad x \geq -1.$

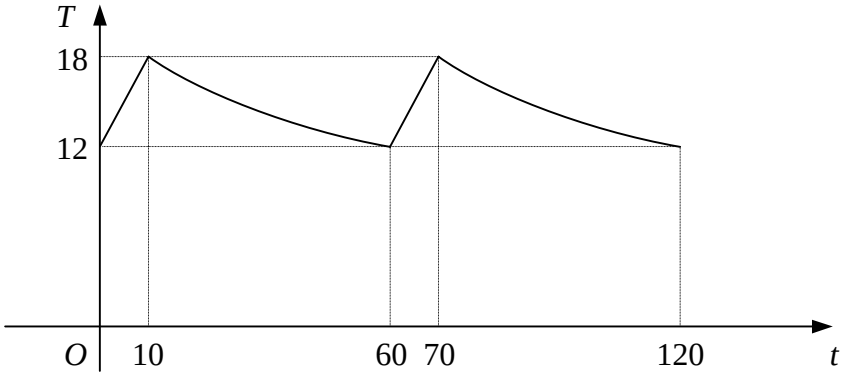
(i) Express $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$. [2]

(ii) Describe fully two transformations that would map the graph of $y = x^2, \ x \geq 0$ onto the graph of $y = \mathrm{f}(x)$. [3]

(iii) Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. [3]

(iv) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram and state the relationship between them. [3]

9.



The diagram shows a graph of the temperature of a room, T °C, at time t minutes.

The temperature is controlled by a thermostat such that when the temperature falls to 12°C, a heater is turned on until the temperature reaches 18°C. The room then cools until the temperature again falls to 12°C.

For t in the interval $10 \leq t \leq 60$, T is given by

$$T = 5 + A\mathrm{e}^{-kt},$$

where A and k are constants.

Given that $T = 18$ when $t = 10$ and that $T = 12$ when $t = 60$,

(i) show that $k = 0.0124$ to 3 significant figures and find the value of A, [6]

(ii) find the rate at which the temperature of the room is decreasing when $t = 20$. [4]

The temperature again reaches 18°C when $t = 70$ and the graph for $70 \leq t \leq 120$ is a translation of the graph for $10 \leq t \leq 60$.

(iii) Find the value of the constant B such that for $70 \leq t \leq 120$

$$T = 5 + B\mathrm{e}^{-kt}.$$ [3]