4763 Mechanics 3

1(a)(i)	[Velocity] $= \mathrm{L\,T^{-1}}$ [Acceleration] $= \mathrm{L\,T^{-2}}$ [Force] $= \mathrm{M\,L\,T^{-2}}$	B1 B1 B1 **3**	*(Deduct 1 mark if kg, m, s are consistently used instead of M, L, T)*
(ii)	$[\,\lambda\,] = \dfrac{[\text{ Force }]}{[\,v^2\,]} = \dfrac{\mathrm{M\,L\,T^{-2}}}{(\mathrm{L\,T^{-1}})^2}$ $= \mathrm{M\,L^{-1}}$	M1 A1 cao **2**	
(iii)	$\left[\dfrac{U^2}{2g}\right] = \dfrac{(\mathrm{L\,T^{-1}})^2}{\mathrm{L\,T^{-2}}} = \mathrm{L}$ $\left[\dfrac{\lambda U^4}{4mg^2}\right] = \dfrac{(\mathrm{M\,L^{-1}})(\mathrm{L\,T^{-1}})^4}{\mathrm{M}(\mathrm{L\,T^{-2}})^2}$ $= \dfrac{\mathrm{M\,L^3\,T^{-4}}}{\mathrm{M\,L^2\,T^{-4}}} = \mathrm{L}$ [H] = L ; all 3 terms have the same dimensions	B1 cao M1 A1 cao E1 **4**	*(Condone constants left in)* *Dependent on B1M1A1*
(iv)	$(\mathrm{M\,L^{-1}})^2\,(\mathrm{L\,T^{-1}})^{\alpha}\,\mathrm{M}^{\beta}\,(\mathrm{L\,T^{-2}})^{\gamma} = \mathrm{L}$ $\beta = -2$ $-2 + \alpha + \gamma = 1$ $-\alpha - 2\gamma = 0$ $\alpha = 6$ $\gamma = -3$	B1 cao M1 A1 A1 cao A1 cao **5**	 At least one equation in α, γ One equation correct

(b)	EE is $\frac{1}{2}\times\frac{2060}{24}\times 6^2 \quad (=1545)$ (PE gained) = (EE lost) + (KE lost)	B1 M1		Equation involving PE, EE and KE Can be awarded from start to point where string becomes slack *or* any complete method (e.g. SHM) for finding v^2 at natural length If B0, give A1 for $v^2 = 88.2$ correctly obtained
	$50\times 9.8\times h = 1545 + \frac{1}{2}\times 50\times 12^2$ $490h = 1545 + 3600$ $h = 10.5$ OA $= 30 - h = 19.5$ m	F1 A1	**4**	*or* $0 = 88.2 - 2\times 9.8\times s \quad (s = 4.5)$ *Notes* $\frac{1}{2}\times\frac{2060}{24}\times 6$ *used as EE can earn B0M1F1A0* $\frac{2060}{24}\times 6$ *used as EE gets B0M0*

2 (i)	$T\cos\alpha = mg$ $3.92\cos\alpha = 0.3\times 9.8$ $\cos\alpha = 0.75$ Angle is $41.4°$ (0.723 rad)	M1 A1 **2**	Resolving vertically *(Condone sin / cos mix for M marks throughout this question)*
(ii)	$T\sin\alpha = m\frac{v^2}{r}$ $3.92\sin\alpha = 0.3\times\frac{v^2}{4.2\sin\alpha}$ Speed is $4.9\ \text{m s}^{-1}$	M1 B1 A1 A1 **4**	Force and acceleration towards centre (condone $v^2/4.2$ or $4.2\omega^2$) For radius is $4.2\sin\alpha$ $(=2.778)$ Not awarded for equation in ω unless $v = (4.2\sin\alpha)\omega$ also appears
(iii)	$T - mg\cos\theta = m\frac{v^2}{a}$ $T - 0.3\times 9.8\times\cos 60° = 0.3\times\frac{8.4^2}{4.2}$ Tension is 6.51 N	M1 A1 A1 **3**	Forces and acceleration towards O
(iv)	$\frac{1}{2}mv^2 - mg\times 4.2\cos\theta = \frac{1}{2}m\times 8.4^2 - mg\times 4.2\cos 60°$ $v^2 - 82.32\cos\theta = 70.56 - 41.16$ $v^2 = 29.4 + 82.32\cos\theta$	M1 M1 A1 E1 **4**	For $(-)mg\times 4.2\cos\theta$ in PE Equation involving $\frac{1}{2}mv^2$ and PE
(v)	$(T) - mg\cos\theta = m\frac{v^2}{a}$ $(T) - m\times 9.8\cos\theta = m\times\frac{29.4 + 82.32\cos\theta}{4.2}$ String becomes slack when $T = 0$ $-9.8\cos\theta = 7 + 19.6\cos\theta$ $\cos\theta = -\frac{7}{29.4}$ $\theta = 104°$ (1.81 rad)	M1 M1 A1 M1 A1 **5**	Force and acceleration towards O Substituting for v^2 *Dependent on first M1* *No marks for* $v = 0 \Rightarrow \theta = 111°$

3 (i)	$T_{PB} = 35(x - 3.2)$ $[= 35x - 112]$ $T_{BQ} = 5(6.5 - x - 1.8)$ $= 5(4.7 - x)$ $[= 23.5 - 5x]$	B1 M1 A1 **3**	Finding extension of BQ
(ii)	$T_{BQ} + mg - T_{PB} = m\frac{d^2x}{dt^2}$ $5(4.7 - x) + 2.5 \times 9.8 - 35(x - 3.2) = 2.5\frac{d^2x}{dt^2}$ $160 - 40x = 2.5\frac{d^2x}{dt^2}$ $\frac{d^2x}{dt^2} = 64 - 16x$	M1 A2 E1 **4**	Equation of motion (condone one missing force) Give A1 for three terms correct
(iii)	At the centre, $\frac{d^2x}{dt^2} = 0$ $x = 4$	M1 A1 **2**	
(iv)	$\omega^2 = 16$ Period is $\frac{2\pi}{\sqrt{16}} = \frac{1}{2}\pi = 1.57$ s	M1 A1 **2**	Seen or implied *(Allow M1 for $\omega = 16$)* Accept $\frac{1}{2}\pi$
(v)	Amplitude $A = 4.4 - 4 = 0.4$ m Maximum speed is $A\omega$ $= 0.4 \times 4 = 1.6$ m s^{-1}	B1 ft M1 A1 cao **3**	ft is $\lvert 4.4 - \text{(iii)} \rvert$
(vi)	$x = 4 + 0.4\cos 4t$ $v = (-)1.6\sin 4t$ When $v = 0.9$, $\sin 4t = -\frac{0.9}{1.6}$ $4t = \pi + 0.5974$ Time is 0.935 s	M1 A1 M1 A1 cao **4**	For $v = C\sin\omega t$ or $C\cos\omega t$ *This M1A1 can be earned in (v)* Fully correct method for finding the required time e.g. $\frac{1}{4}\arcsin\frac{0.9}{1.6} + \frac{1}{2}$period
	OR $0.9^2 = 16(0.4^2 - y^2)$ $y = -0.3307$ M1 $y = 0.4\cos 4t$ A1 $\cos 4t = -\frac{0.3307}{0.4}$ $4t = \pi + 0.5974$ M1 Time is 0.935 s A1 cao		Using $v^2 = \omega^2(A^2 - y^2)$ *and* $y = A\cos\omega t$ or $A\sin\omega t$ For $y = (\pm)0.331$ *and* $y = 0.4\cos 4t$

4 (a)(i)	$V = \int \pi x^2 \, dy = \int_0^8 \pi (4 - \frac{1}{2} y) \, dy$ $= \pi \left[4y - \frac{1}{4} y^2 \right]_0^8 = 16\pi$ $V \bar{y} = \int \pi y x^2 \, dy$ $= \int_0^8 \pi y (4 - \frac{1}{2} y) \, dy$ $= \pi \left[2y^2 - \frac{1}{6} y^3 \right]_0^8 = \frac{128}{3} \pi$ $\bar{y} = \frac{\frac{128}{3} \pi}{16\pi}$ $= \frac{8}{3} \quad (\approx 2.67)$	M1 A1 M1 A1 A1 M1 A1 **7**	π *may be omitted throughout* Limits not required for M marks throughout this question *Dependent on M1M1*
(ii)	CM is vertically above lower corner $\tan\theta = \frac{2}{\bar{y}} = \frac{2}{\frac{8}{3}} \quad (= \frac{3}{4})$ $\theta = 36.9° \quad (= 0.6435 \text{ rad})$	M1 M1 A1 A1 **4**	Trig in a triangle including θ *Dependent on previous M1* Correct expression for $\tan\theta$ or $\tan(90 - \theta)$ *Notes* $\tan\theta = \frac{2}{\text{cand's } \bar{y}}$ *implies M1M1A1* $\tan\theta = \frac{\text{cand's } \bar{y}}{2}$ *implies M1M1* $\tan\theta = \frac{1}{\text{cand's } \bar{y}}$ *without further evidence is M0M0*

(b)			*May use* $0 \le x \le 2$ *throughout*
	$A = \int_{-2}^{2} (8 - 2x^2)\,\mathrm{d}x$	M1	or $(2)\int_{0}^{8} \sqrt{4 - \frac{1}{2}y}\,\mathrm{d}y$
	$= \left[8x - \frac{2}{3}x^3 \right]_{-2}^{2} = \frac{64}{3}$	A1	
	$A\bar{y} = \int_{-2}^{2} \frac{1}{2}(8 - 2x^2)^2\,\mathrm{d}x$	M1	or $(2)\int_{0}^{8} y\sqrt{4 - \frac{1}{2}y}\,\mathrm{d}y$
			(M0 if ½ is omitted)
	$= \left[32x - \frac{16}{3}x^3 + \frac{2}{5}x^5 \right]_{-2}^{2}$	M1	For $32x - \frac{16}{3}x^3 + \frac{2}{5}x^5$ *Allow one error*
			or $-\frac{8}{3}y(4 - \frac{1}{2}y)^{\frac{3}{2}} - \frac{32}{15}(4 - \frac{1}{2}y)^{\frac{5}{2}}$
	$= \frac{1024}{15}$		or $-\frac{64}{3}(4 - \frac{1}{2}y)^{\frac{3}{2}} + \frac{16}{5}(4 - \frac{1}{2}y)^{\frac{5}{2}}$
		A1	
	$\bar{y} = \frac{1024/15}{64/3}$		
	$= \frac{16}{5} = 3.2$	M1	*Dependent on first two M1's*
		A1	
		7	