Edexcel GCE Further Pure Mathematics (FP1)

Required Knowledge Information Sheet

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FP1 Formulae Given in Mathematical Formulae and Statistical Tables Booklet

- Summations
  - $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$
  - $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

- Area of a sector
  - $A = \frac{1}{2}\int r^2 \, d\theta$ (polar coordinates)

- Numerical Solution Of Equations
  - The Newton-Raphson iterations for solving $f(x) = 0 \Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Complex Numbers

- $\sqrt{-1} = i$ and $i^2 = -1$
- An imaginary number is a number of the form *bi*, where b is a real number ($b \in \mathbb{R}$)
- A complex number is a number of the form *a* + *bi*, where ($a \in \mathbb{R}$) and ($b \in \mathbb{R}$)
- For the complex number *a*+ *bi*, a is called the real part and b is called the imaginary part
- The complex number $z^* = a - bi$ is called the complex conjugate of the complex number $z = a + bi$
- LǂűKŜ ꟿƻƻűă ʰ ŀȳŘ ʲ ƻǂ ŀ ĳǳŀŘꟿŀűƛŎ ŜĳǳŀűƛƻƳ ŀꟿŜ ŎƻϒǇƒŜȅΣ ʰ ŀȳŘ ʲ øƛff ŀƒøŀȅă ōŜ ŀ complex conjugate pair
- The complex number $z = x + iy$ is represented on an Argand diagram by the point (x,y) where x and y are Cartesian coordinates
- The complex number $z = x + iy$ can also be represented by the vector $\overrightarrow{OP}$, where O is the origin and P is the point (x,y) on the Argand diagram
- Addition of complex numbers can be represented on the Argand diagram by the addition of their respective vectors on the diagram
- The modulus of the complex number $z = x + iy$ is given by $\sqrt{x^2 + y^2}$
- The modulus of the complex number $z = x + iy$ is written as *r* or $|z|$ or $|x + iy|$
  - So
    - $r = \sqrt{x^2 + y^2}$
    - $|z| = \sqrt{x^2 + y^2}$
    - $|x + iy| = \sqrt{x^2 + y^2}$
- The modulus of any non-zero complex number is positive
- The argument (arg z) of the complex number $z = x + iy$ ƛă űKŜ ŀȳƎƒŜ ɸ ōŜűøŜŜȳűKŜ positive real axis and the vector representing z on the Argand diagram
- CƻꟿűKŜ ŀꟿƎǳϒŜȳű ɸ ƻǂ ŀȳȅ ŎƻϒǇƒŜȅ ȳǳϒōŜꟿ $z = x + iy$, $\tan\theta = \frac{y}{x}$
- ¢KŜ ŀꟿƎǳϒŜȳű ɸ ƻǂ ŀȳȅ ŎƻϒǇƒŜȅ ȳǳϒōŜꟿ ƛă ăǳŎK űKŀű $-\pi < \theta \le \pi$ This is referred to as the principle argument and is sometimes seen as $-180° < \theta \le 180°$
- The modulus-argument form of the complex number $z = x + iy$ is,
  - $z = r(\cos\theta + i\sin\theta)$
  - [r ƛă ŀ LJƻăƛűƛØŜ ꟿŜŀƒ ȳǳϒōŜꟿ ŀȳŘ ɸ is an angle within the principle argument]
- For complex numbers z_1 and z_2, $|z_1 z_2| = |z_1||z_2|$
- If $x_1 + iy_1 = x_2 + iy_2$, then $x_1 = x_2$ and $y_1 = y_2$
- An equation of the form $ax^3 + bx^2 + cx + d = 0$ is called a cubic equation, and has three roots.
- For a cubic equation, either
  - All three roots are real, or
  - One root is real and the other two roots form a complex conjugate pair

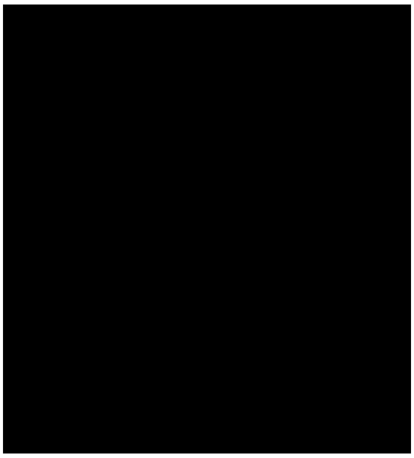
- An equation of the form $ax^4 + bx^3 + cx^2 + dx + e = 0$ is called a quartic equation, and has four roots
- For a quartic equation, either
    - All four roots are real, or
    - Two roots are real and the other two roots form a complex conjugate pair, or
    - Two roots form a complex conjugate pair and the other two roots also form a complex conjugate pair

Numerical Solutions of Equations

- You can solve equations of the form *f(x) = 0* using interval bisection
    - If you find an interval in which *f(x)* changes sign, then the interval must contain a root of the equation *f(x) =0*. You then take the midpoint as the first approximation and repeat this process until you get your required accuracy
- You can solve equations of the form *f(x) = 0* using linear interpolation
- You can solve equations of the form *f(x) = 0* using the Newton-Raphson process
- The Newton-Raphson formula is,
    - $f(x) = 0 \Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
- The Newton-Raphson process may not always give you a better approximation and may take you further away from the root

<u>Coordinate Systems</u>

- To find the Cartesian equation of a curve given parametrically you eliminate the parameter *t* between the parametric equations



- A parabola is a set of points which are equidistant from the focus *S* and a line called the directrix. So, for the parabola opposite,
  - SP = PX
  - The focus, S, has coordinates (a, 0)
  - The directrix has equation $x + a = 0$

- The curve opposite is a sketch of a parabola with a Cartesian equation of $y^2 = 4ax$, where a, is a positive constant.
  - The curve has parametric equations:
    - $x = at^2$ (ò ᴺ я)
    - $y = 2at$ (ò ᴺ я)

- The curve opposite is a sketch of a rectangular hyperbola with a Cartesian equation of $xy = c^2$, where c is a positive constant.
  - The curve has parametric equations:
    - $x = ct$ ò ᴺ я , t 0
    - ώ = ώ/ò ò ᴺ я , t 0

Matrix Algebra

- An *n x m* matrix has *n* rows and *m* columns
- The transformation represented by the matrix ABC means,
  - First do the transformation represented by C
  - Second do the transformation represented by B
  - Third do the transformation represented by A
- The matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity matrix or transformation. It does not change a matrix or an object
- The determinant of matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is det(A) = ad − bc
- The inverse of a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $A^{-1} = \frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Series

- The notation $\sum_{r=1}^{n} U_r$ defines the series $U_1 + U_2 + U_3 + \ldots + U_n$ where U_r is the general ůŜŝǇ ƚǇƉůKŜ ăŜŴƚŜăƚ ƚů ŀƚăƨ ƚă ĚǌăŜŘ ůƨ ǇŜŀǇΨăĚǌǇ ůKŜ ăŜŴƚŜăΩ
- If the series is summed from *r* = *k* to *r* = *n* then, $\sum_{r=k}^{n} U_r = \sum_{r=1}^{n} U_r - \sum_{r=1}^{k-1} U_r$
- The sums of powers of the first *n* natural numbers, for the cases k = 0, 1, 2 and 3 are
  - $\sum_{r=1}^{n} r^0 = \sum_{r=1}^{n} 1 = 1 + 1 + 1 + \ldots + 1 = n$
  - $\sum_{r=1}^{n} r = 1 + 2 + 3 + \ldots + n = \frac{n}{2}(n + 1)$
  - $\sum_{r=1}^{n} r^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n}{6}(n + 1)(2n + 1)$
  - $\sum_{r=1}^{n} r^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2}{4}(n + 1)^2$
- $\sum_{r=1}^{n} r^3 = (\sum_{r=1}^{n} r)^2$
- ǈƚ ůKŜ ∃ŜǇŜŴŀƭ ůŜŴǇ ƚă ŀ ǇƨŴŜ ŎƨǇǈƭƚŎŀůŜŘ ƚĚǌǇŎůƚƨǇƭƚ ŴΣ ůKŜ ăĚǌǇ ŎŀǇōŜ ΨăǈƭƚůĚǌǈΩ ăƨ that some, or all, of the above basic results can be used. For example:
  - $\sum_{r=1}^{n} (r^3 + 3r^2 - 2r + 5) = \sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r + 5\sum_{r=1}^{n} 1$
  - $\sum_{r=1}^{n} (2r + 1)(r + 2) = \sum_{r=1}^{n} (2r^2 + 5r + 2) = 2\sum_{r=1}^{n} r^2 + 5\sum_{r=1}^{n} r + 2\sum_{r=1}^{n} 1$

Proof by Mathematical Induction

- Mathematical induction is used to prove whether or not general statements are true, usually for positive integers, *n*.
- When performing a proof by mathematical induction you need to apply the following four steps:
  - Basis: show the general statement if true for $n=1$
  - Assumption: assume that the general statement is true for $n=k$
  - Induction: show the general statement is true for $n=k+1$
  - Conclusion: Then state that the general statement is then true for all positive integers, *n*.
- Proof by induction is of no use for deriving formulae from first principles. Proof by induction is used, however, to check whether or not a general statement is true.