Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	9	/	0	1	Signature	

Paper Reference(s)

6669/01

Edexcel GCE

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 25 June 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

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1. The hyperbola H has equation

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Find

(a) the coordinates of the foci of H,

(3)

(b) the equations of the directrices of H.

(2)

Leave blank

2.

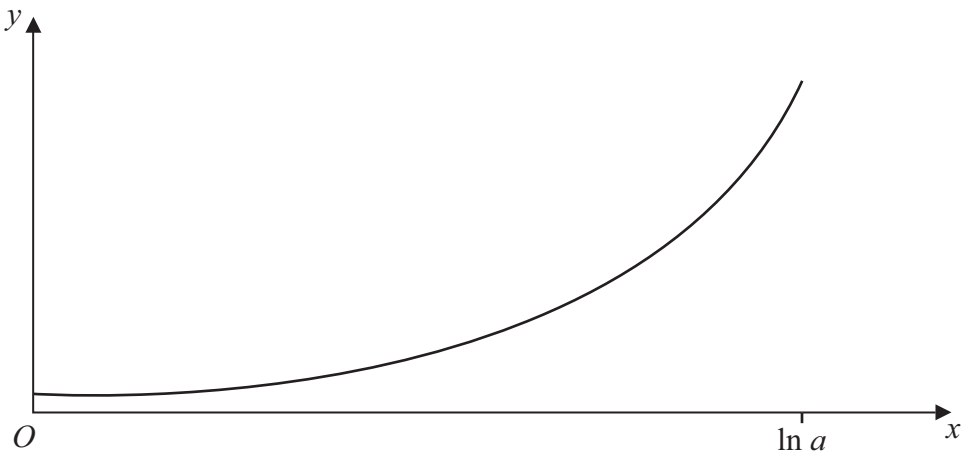

Figure 1

The curve C, shown in Figure 1, has equation

$$y = \frac{1}{3}\cosh 3x, \qquad 0 \leqslant x \leqslant \ln a$$

where a is a constant and $a > 1$

Using calculus, show that the length of curve C is

$$k\left(a^3 - \frac{1}{a^3}\right)$$

and state the value of the constant k.

(6)

Leave blank

3. The position vectors of the points A, B and C relative to an origin O are $\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, $7\mathbf{i} - 3\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j}$ respectively.

Find

(a) $\overrightarrow{AC} \times \overrightarrow{BC}$,

(4)

(b) the area of triangle ABC,

(2)

(c) an equation of the plane ABC in the form $\mathbf{r}.\mathbf{n} = p$

(2)

Leave blank

Question 3 continued

Leave blank

4. $$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \, \mathrm{d}x, \qquad n \geqslant 0$$

(a) Prove that, for $n \geqslant 2$,

$$I_n = \frac{1}{4} n \left(\frac{\pi}{4} \right)^{n-1} - \frac{1}{4} n(n-1) I_{n-2}$$

(5)

(b) Find the exact value of I_2

(4)

(c) Show that $I_4 = \frac{1}{64}(\pi^3 - 24\pi + 48)$

(2)

Leave blank

Question 4 continued

Leave blank

5. (a) Differentiate $x \operatorname{arsinh} 2x$ with respect to x.

(3)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \, \mathrm{d}x$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

Leave blank

Question 5 continued

Leave blank

6. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The line l_1 is a tangent to E at the point $P\ (a\cos\theta,\ b\sin\theta)$.

(a) Using calculus, show that an equation for l_1 is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

(4)

The circle C has equation

$$x^2 + y^2 = a^2$$

The line l_2 is a tangent to C at the point $Q\ (a\cos\theta,\ a\sin\theta)$.

(b) Find an equation for the line l_2.

(2)

Given that l_1 and l_2 meet at the point R,

(c) find, in terms of a, b and θ, the coordinates of R.

(3)

(d) Find the locus of R, as θ varies.

(2)

Leave blank

Question 6 continued

Leave blank

7. $f(x) = 5\cosh x - 4\sinh x, \qquad x \in \mathbb{R}$

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$

(2)

Hence

(b) solve $f(x) = 5$

(4)

(c) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\,dx = \frac{\pi}{18}$

(5)

Leave blank

Question 7 continued

Leave blank

8. The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}$$

(a) Show that 4 is an eigenvalue of **M**, and find the other two eigenvalues.

(5)

(b) For the eigenvalue 4, find a corresponding eigenvector.

(3)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix **M**.

The equation of l_1 is $\mathbf{(r - a)} \times \mathbf{b} = 0$, where $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

(c) Find a vector equation for the line l_2.

(5)

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END