Write your name here

Surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Further Pure Mathematics F3

Advanced/Advanced Subsidiary

Monday 22 June 2015 – Morning
Time: 1 hour 30 minutes

Paper Reference
WFM03/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1. Find the exact values of x for which

$$\cosh 2x - 7\sinh x = 5$$

giving your answers as natural logarithms.

(7)

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2. The hyperbola H has equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where a and b are positive constants.

The hyperbola H has eccentricity $\frac{\sqrt{21}}{4}$ and passes through the point (12, 5).

Find

(a) the value of a and the value of b,

(4)

(b) the coordinates of the foci of H.

(1)

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Question 2 continued

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3. $\mathbf{M} = \begin{pmatrix} 0 & 1 & 9 \\ 1 & 4 & k \\ 1 & 0 & -3 \end{pmatrix}$, where k is a constant.

Given that $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$ is an eigenvector of the matrix **M**,

(a) find the eigenvalue of **M** corresponding to $\begin{pmatrix} 7 \\ 19 \\ 1 \end{pmatrix}$,

(2)

(b) show that $k = -7$

(2)

(c) find the other two eigenvalues of the matrix **M**.

(4)

The image of the vector $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ under the transformation represented by **M** is $\begin{pmatrix} -6 \\ 21 \\ 5 \end{pmatrix}$.

(d) Find the values of the constants p, q and r.

(4)

Leave blank

Question 3 continued

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4. $$I_n = \int \cosh^n x \,\mathrm{d}x, \quad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$

$$nI_n = \sinh x \cosh^{n-1} x + (n-1)I_{n-2}$$

(6)

(b) Hence find the exact value of

$$\int_0^{\ln 2} \cosh^5 x \,\mathrm{d}x$$

(4)

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Question 4 continued

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5. The ellipse E has equation $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

The line L has equation $y = mx + c$, where m and c are constants.

Given that L is a tangent to E,

(a) show that

$$c^2 - 25m^2 = 9$$

(4)

(b) find the equations of the tangents to E which pass through the point (3, 4).

(5)

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Question 5 continued

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6.

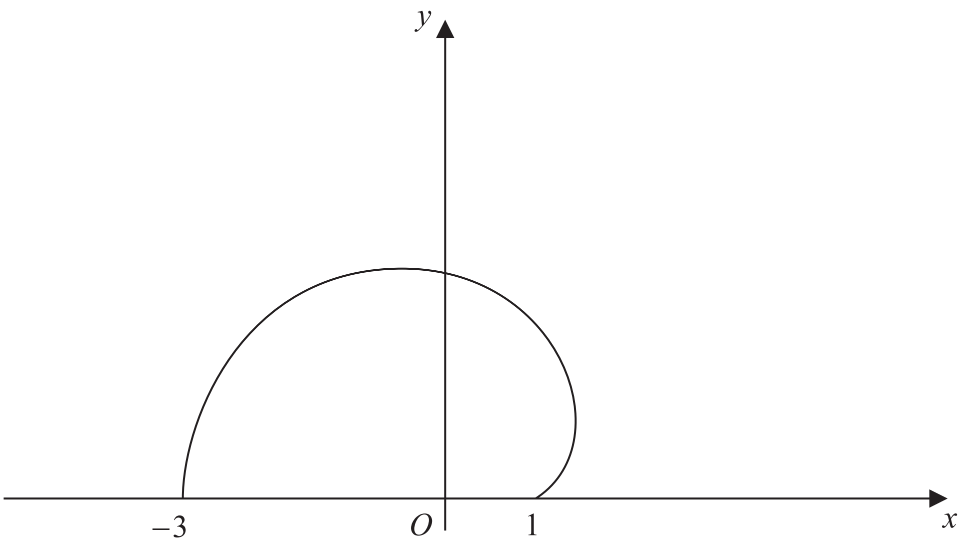

Figure 1

Figure 1 shows the curve C with parametric equations

$$x = 2\cos\theta - \cos 2\theta,\ y = 2\sin\theta - \sin 2\theta, \qquad 0 \leqslant \theta \leqslant \pi$$

(a) Show that

$$\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2 = 8(1 - \cos\theta)$$

(5)

The curve C is rotated through 2π radians about the x-axis.

(b) Find the area of the surface generated, giving your answer in the form $k\pi$, where k is a rational number.

(5)

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Question 6 continued

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7. The plane Π_1 contains the point $(3, 3, -2)$ and the line $\dfrac{x-1}{2} = \dfrac{y-2}{-1} = \dfrac{z+1}{4}$

(a) Show that a cartesian equation of the plane Π_1 is

$$3x - 10y - 4z = -13$$

(5)

The plane Π_2 is parallel to the plane Π_1

The point $(\alpha, 1, 1)$, where α is a constant, lies in Π_2

Given that the shortest distance between the planes Π_1 and Π_2 is $\dfrac{1}{\sqrt{5}}$

(b) find the possible values of α.

(6)

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Question 7 continued

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8. (a) Show that, under the substitution $x = \frac{3}{4}\sinh u$,

$$\int \frac{x^2}{\sqrt{16x^2 + 9}}\,dx = k\int (\cosh 2u - 1)\,du$$

where k is a constant to be determined.

(6)

(b) Hence show that

$$\int_0^1 \frac{64x^2}{\sqrt{16x^2 + 9}}\,dx = p + q\ln 3$$

where p and q are rational numbers to be found.

(5)

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Question 8 continued