Further Pure 1 Past Paper Questions Pack B
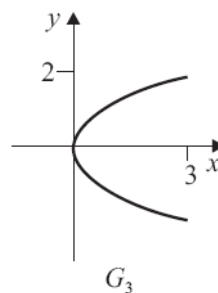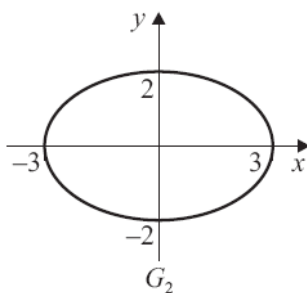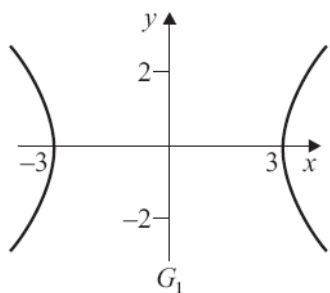
Taken from MBP1, MBP3, MBP4, MBP5

Parabolas, Ellipses and Hyperbolas

Pure 3 January 2002

1 (a) The graph of $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ is one of those sketched below.

Identify whether it is graph G_1, G_2 or G_3.

(1 mark)

(b) On separate sets of axes, sketch the graphs of:

  (i) $\dfrac{(x+3)^2}{9} - \dfrac{y^2}{4} = 1$;

(2 marks)

  (ii) $\dfrac{y^2}{9} - \dfrac{x^2}{4} = 1$.

(2 marks)

Pure 3 January 2003

3 (a) Sketch the graph of $\dfrac{x^2}{16} + \dfrac{y^2}{49} = 1$, marking the values of the intercepts with the coordinate axes.

(3 marks)

(b) Describe a sequence of geometrical transformations that maps the graph of $\dfrac{x^2}{16} + \dfrac{y^2}{49} = 1$ onto the graph of $\dfrac{x^2}{4} + \dfrac{(y-3)^2}{49} = 1$.
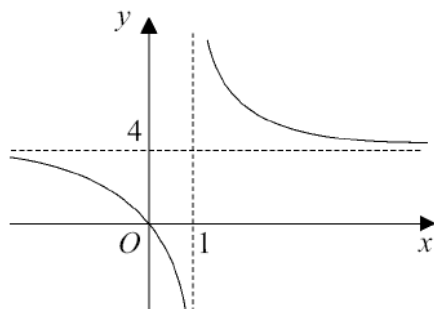
(4 marks)

Pure 3 January 2004

2 (a) Sketch the curve with equation $y^2 = 8x$.

(2 marks)

(b) Write down the equation of the curve obtained when the curve $y^2 = 8x$ is reflected in the line $y = x$.

(2 marks)

(c) Describe a geometrical transformation that maps the curve $y^2 = 8x$ onto the curve with equation $y^2 = 8x - 16$.

(2 marks)

Rational Functions and Asymptotes

Pure 3 January 2004

3 The graph of $y = f(x)$ is sketched below.
The asymptotes have equations $x = 1$ and $y = 4$.

(a) Given that $f(x) = \dfrac{ax}{x - b}$, use the sketch to find the values of a and b. *(2 marks)*

(b) Sketch the graph of $y^2 = f(x)$ and state the equations of its asymptotes. *(5 marks)*

Pure 3 January 2002

3 (a) Sketch the graph of $y = \dfrac{5x - 7}{x - 3}$.

State the coordinates of the points where the curve crosses the coordinate axes and write down the equations of its asymptotes. *(5 marks)*

(b) Using the graph from part (a), or otherwise, solve the inequality

$$\frac{5x - 7}{x - 3} > 0.$$ *(2 marks)*

Pure 3 June 2002

2 (a) Sketch the graph of $y = \dfrac{3x + 4}{x - 2}$.

State the coordinates of the points where the curve crosses the coordinate axes and write down the equations of its asymptotes. *(6 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{3x + 4}{x - 2} > 1.$$ *(3 marks)*

Pure 3 January 2003

1 (a) Sketch the graph of $y = \dfrac{3 - 4x}{2x - 5}$.

State the coordinates of the points where the graph crosses the coordinate axes and write down the equations of its asymptotes. *(6 marks)*

(b) Hence, or otherwise, solve the inequality

$$\frac{3 - 4x}{2x - 5} < 0$$

(3 marks)

Pure 5 June 2002

7 A curve has equation $y = \dfrac{3x^2 - 9x + 7}{(2x - 3)(x - 2)}$.

(a) Write down the equations of the three asymptotes to the curve. *(3 marks)*

(b) (i) Prove that there are no real values of x for which $-3 < y < 1$. *(7 marks)*

(ii) Hence find the coordinates of the turning points on the curve. *(4 marks)*

Pure 5 January 2003

2 A curve has equation $y = \dfrac{2x^2 + 1}{x^2}$.

(a) Find the equations of the asymptotes to the curve. *(2 marks)*

(b) Sketch the curve. *(2 marks)*

Pure 5 June 2003

3 A curve has equation $y = \dfrac{x^2 + 2}{2x + 1}$.

Prove that there are no real values of x for which $-2 < y < 1$. *(6 marks)*

Pure 3 June 2004

2 (a) A curve has equation $y = \dfrac{3x + 4}{1 - 2x}$.

(i) Find the coordinates of the points where the curve crosses the coordinate axes.
(2 marks)

(ii) State the equations of its asymptotes.
(2 marks)

(iii) Sketch the curve.
(2 marks)

(b) Calculate the x-coordinate of the point where the curve $y = \dfrac{3x + 4}{1 - 2x}$ intersects the line $y = 1$.
(2 marks)

(c) Hence, or otherwise, solve the inequality $\dfrac{3x + 4}{1 - 2x} \leqslant 1$.
(3 marks)

Pure 5 January 2004

5 A curve has equation $y = \dfrac{x^2}{x^2 + 3x + 3}$.

(a) Write down the equation of the horizontal asymptote to the curve.
(1 mark)

(b) (i) Prove that, for all real values of x, y satisfies the inequality $0 \leqslant y \leqslant 4$.
(6 marks)

(ii) Hence find the coordinates of the turning points on the curve.
(3 marks)

(c) Given that there are no vertical asymptotes, sketch the curve.
(3 marks)

Pure 5 June 2004

5 A curve has equation $y = \dfrac{x^2}{x + 1}$.

(a) Find the equations of the two asymptotes to the curve.
(3 marks)

(b) Given that $y \leqslant -4$ or $y \geqslant 0$ for all real values of x, and that there are no values of y for which $-4 < y < 0$, find the coordinates of the two turning points of the curve. *(3 marks)*

(c) Sketch the curve.
(3 marks)

Complex Numbers

Pure 3 June 2002

4 (a) Given that $z = -2 + 2\sqrt{3}\,i$, show that $z^2 + 4z$ is real. *(3 marks)*

Pure 3 January 2004

6 (a) Find the value of the following, giving each answer in the form $a + bi$, where a and b are integers.

    (i) $(2 + 3i)^2$ *(2 marks)*

    (ii) $(2 + 3i)^4$ *(2 marks)*

Roots of Quadratic Equations

Pure 3 January 2002

9   (a)   The roots of the quadratic equation $x^2 + 4x + 13 = 0$ are α and β.

Without solving the equation, find the value of:

(i) $\alpha^3 + \beta^3$;   *(4 marks)*

(ii) $\dfrac{\alpha}{\beta^2} + \dfrac{\beta}{\alpha^2}$.   *(2 marks)*

(b)   Determine a quadratic equation with integer coefficients which has roots $\dfrac{\alpha}{\beta^2}$ and $\dfrac{\beta}{\alpha^2}$.   *(3 marks)*

(c)   Find the complex roots of the equation $x^2 + 4x + 13 = 0$.   *(3 marks)*

Pure 3 June 2002

1   (a)   The roots of the quadratic equation $x^2 + 4x - 3 = 0$ are α and β.

Without solving the equation, find the value of:

(i) $\alpha^2 + \beta^2$;

(ii) $\left(\alpha^2 + \dfrac{2}{\beta} \right) \left(\beta^2 + \dfrac{2}{\alpha} \right)$.   *(6 marks)*

(b)   Determine a quadratic equation with integer coefficients which has roots

$$\left(\alpha^2 + \frac{2}{\beta} \right) \text{ and } \left(\beta^2 + \frac{2}{\alpha} \right).$$ *(4 marks)*

Pure 3 January 2003

7   The roots of the quadratic equation $x^2 + 3x - 2 = 0$ are α and β.

(a)   Write down the values of $\alpha + \beta$ and $\alpha\beta$.   *(1 mark)*

(b)   Without solving the equation, find the value of:

(i) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ *(3 marks)*

(ii) $\left(\alpha - \dfrac{3}{\beta^2} \right) \left(\beta - \dfrac{3}{\alpha^2} \right)$ *(3 marks)*

(c)   Determine a quadratic equation with integer coefficients which has roots

$$\alpha - \frac{3}{\beta^2} \quad \text{and} \quad \beta - \frac{3}{\alpha^2}$$ *(4 marks)*

Pure 3 June 2003

9   The roots of the quadratic equation $x^2 - 3x + 1 = 0$ are α and β.

    (a)   Without solving the equation:

        (i)   show that $\alpha^2 + \beta^2 = 7$;   *(3 marks)*

        (ii)   find the value of $\alpha^3 + \beta^3$.   *(3 marks)*

    (b)   (i)   Show that $\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$.   *(1 mark)*

        (ii)   Hence find the value of $\alpha^4 + \beta^4$.   *(2 marks)*

    (c)   Determine a quadratic equation with integer coefficients which has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$.   *(5 marks)*

Pure 3 January 2004

1   The roots of the quadratic equation $x^2 + 2x + 3 = 0$ are α and β.

    (a)   Without solving the equation:

        (i)   write down the value of $\alpha + \beta$ and the value of $\alpha\beta$;   *(2 marks)*

        (ii)   show that $\alpha^3 + \beta^3 = 10$;   *(3 marks)*

        (iii)   find the value of $\dfrac{1}{\alpha^3} + \dfrac{1}{\beta^3}$.   *(2 marks)*

    (b)   Determine a quadratic equation with integer coefficients which has roots

$$\frac{1}{\alpha^3} \quad \text{and} \quad \frac{1}{\beta^3}$$
                                                                        (3 marks)

Pure 3 June 2004

3   The roots of the quadratic equation $x^2 + (7 + p)x + p = 0$ are α and β.

    (a)   Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$, in terms of p.   *(2 marks)*

    (b)   Find the value of $\alpha^2 + \beta^2$ in terms of p.   *(2 marks)*

    (c)   (i)   Show that $(\alpha - \beta)^2 = p^2 + 10p + 49$.   *(2 marks)*

        (ii)   Given that α and β differ by 5, find the possible values of p.   *(3 marks)*

Series

Pure 1 June 2003

3 (a) Find the value of:

(i) $\displaystyle\sum_{r=1}^{100} r^3$ *(1 mark)*

(ii) $\displaystyle\sum_{r=51}^{100} r^3$ *(2 marks)*

(b) Find the sum of the fifty integers from 51 to 100 inclusive.   *(3 marks)*

(c) Hence find the value of $\displaystyle\sum_{r=51}^{100} (r^3 - 6325r)$.   *(2 marks)*

Pure 1 June 2004

6 (a) Find the value of $\displaystyle\sum_{r=1}^{29} r^2$.   *(2 marks)*

(b) (i) The first two terms of an arithmetic series are 3 and 7 respectively.

Write down the rth term of the series, giving your answer in its simplest form.
(3 marks)

(ii) Express the sum of the following arithmetic series in sigma notation

$$3 + 7 + 11 + \ldots + 799$$

(*You are not required to evaluate this sum.*)   *(2 marks)*

Calculus

Pure 1 June 2004

(d) The points P and Q lie on the curve with equation $y = x^2 - 6x + 10$.
The x-coordinate of P is 1 and the x-coordinate of Q is $1 + h$.

(i) Show that the gradient of the chord PQ is $h - 4$.   *(3 marks)*

(ii) Deduce the value of the gradient of the curve at the point P.   *(1 mark)*

Linear Laws

Pure 3 January 2002

6 [A sheet of graph paper is supplied for use in this question.]

The energy, E, lost in a cycle of magnetization of a transformer core is thought to relate to the flux density, B, by a law of the form $E = kB^{\alpha}$ where k and α are constants.

(a)   Express $\ln E$ in terms of $\ln k$, α and $\ln B$.   *(1 mark)*

For a given material, the values of B and E in appropriate units are:

B	3.16	9.56	18.3	29.0	41.4
E	1	2	3	4	5

(b)   Plot $\ln E$ against $\ln B$ on graph paper.   *(3 marks)*

(c)   Draw a suitable straight line to illustrate the relationship between the data.   *(1 mark)*

(d)   Use your line to estimate:

   (i)   the value of E when $B = 25.5$ giving your answer to 2 significant figures;
                                                                                                              (3 marks)

   (ii)   the values of k and α, giving your answers to 2 significant figures.   *(4 marks)*

Pure 3 January 2003

5   A mathematical model is used by an astronomer to investigate features of the moons of a particular planet. The mean distance of a moon from the planet, measured in millions of kilometres, is denoted by x, and the corresponding period of its orbit is P days.

The model assumes that the graph of $\ln P$ against $\ln x$ is the straight line drawn below.
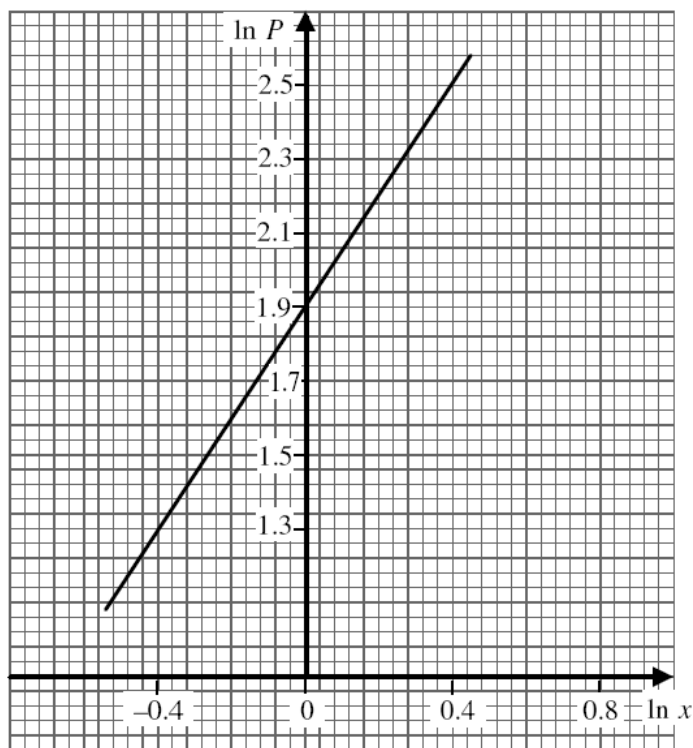
(a)   Use the graph to estimate the period of the orbit of a moon for which $x = 1.43$.   *(3 marks)*

(b)   The graph would suggest that P and x are related by an equation of the form

$$P = kx^{\alpha}$$

where k and α are constants.

  (i)   Express $\ln P$ in terms of $\ln k$, $\ln x$ and α.   *(1 mark)*

  (ii)   Use the graph to determine the values of k and α, giving your answers to 2 significant figures.   *(4 marks)*

Pure 3 June 2003

7 A mathematical model is required to estimate the number, N, of a certain strain of bacteria in a test tube at time t hours after a certain instant.

   After values of $\log_{10} N$ are plotted against t, a straight line graph can be drawn through the points as shown below.
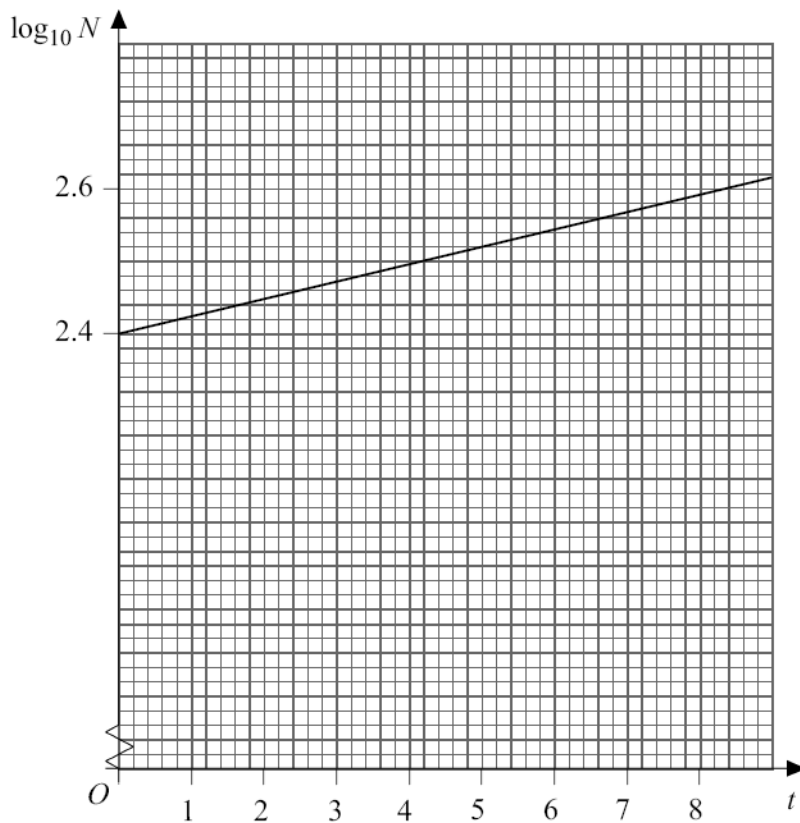
(a) Use the graph to estimate the number of bacteria when $t = 5$. *(3 marks)*

(b) The graph would suggest that N and t are related by an equation of the form

$$N = a \times b^t$$

   where a and b are constants.

   (i) Express $\log_{10} N$ in terms of $\log_{10} a$, $\log_{10} b$ and t. *(2 marks)*

   (ii) Use the graph to determine the values of a and b, giving your answers to 3 significant figures. *(4 marks)*

(c) Suggest why the model $N = a \times b^t$ is likely to give an overestimate of the number of bacteria in the test tube for large values of t. *(1 mark)*

Pure 3 January 2004

5   [An insert is provided for use in answering this question.]

The variables Q and x satisfy a relationship of the form $Q = ax^b$, where a and b are constants.

Measurements of Q for given values of x gave the following results.

| x	0.4	0.5	0.6	0.7	0.8
Q	1.72	3.02	4.74	6.98	9.73

(a)   Express $\ln Q$ in terms of $\ln a$, b and $\ln x$.   *(1 mark)*

(b)   (i)   Complete the table on the insert and plot $\ln Q$ against $\ln x$ on the axes provided.
                                                                                                                    (3 marks)

        (ii)   Draw a suitable straight line to illustrate the relationship between the data.
                                                                                                                    (1 mark)

(c)   Use your line to estimate:

        (i)   the value of Q when $x = 0.54$, giving your answer to two significant figures;
                                                                                                                    (2 marks)

        (ii)   the values of a and b, giving your answers to two significant figures.   *(4 marks)*

Pure 3 June 2004

6   A student performs an experiment and records data for two variables x and y.

Values of $\ln x$ and $\ln y$ are calculated and a line of best fit is drawn as shown below.
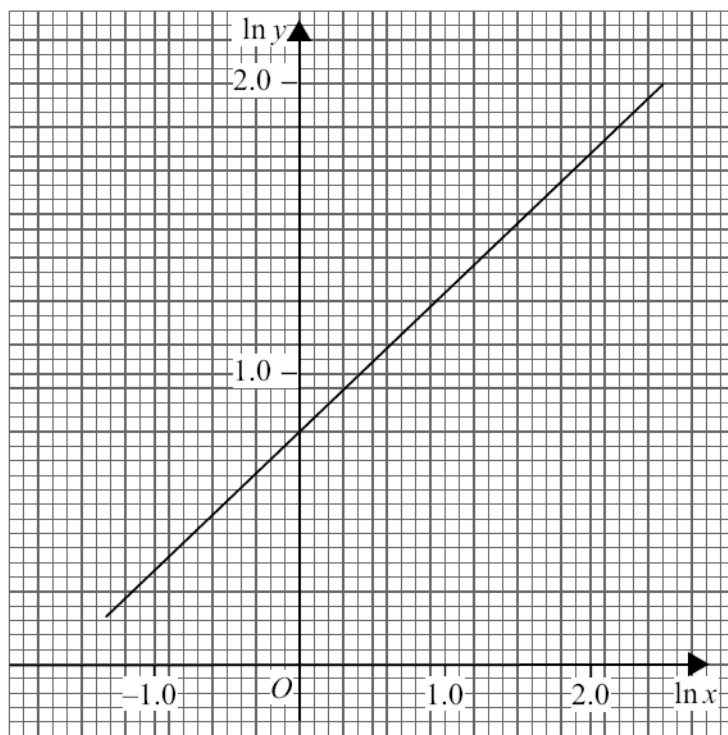
(a)   Use this graph to find the value of y when $x = 3.0$, giving your answer to two significant figures. *(3 marks)*

(b)   The student believes there is a relationship between x and y of the form $y = Ax^n$, where A and n are constants.

　(i)   Express $\ln y$ in terms of $\ln A$, $\ln x$ and n. *(1 mark)*

　(ii) Use the graph to estimate the values of A and n, giving your answers to two significant figures. *(4 marks)*

Numerical Methods

Pure 4 January 2002

2   A polynomial is defined by $p(x) = 4x^3 - 5x^2 + 2$.

    (a)   Find the remainder when $p(x)$ is divided by $(2x + 1)$.   *(2 marks)*

    (b)   The equation $p(x) = 0$ has a single real root, α.

        Use the Newton-Raphson method once with first approximation -0.5 to find a second approximation to α, giving your answer to 3 decimal places.   *(3 marks)*

Pure 4 January 2003

2   (a)   Use logarithms to solve the equation $2^x = 7$, giving your answer to three significant figures.
                   (2 marks)

    (b)   The equation

$$2^x = 7 - x$$

       has a single root, α.

       (i)   Show that α lies between 2.0 and 2.4.   *(1 mark)*

       (ii)   Use the bisection method to find an interval of width 0.1 in which α lies.   *(3 marks)*

Pure 4 June 2003

5   A curve is defined for $0 \leqslant x \leqslant \pi$ by the equation

$$y = 2x - 1 + \sin 2x$$

    (a)   (i)   Find $\dfrac{dy}{dx}$.   *(2 marks)*

       (ii)   The curve crosses the x-axis when $x = \alpha$. Use the Newton-Raphson iterative formula with first approximation 0.2 to find a second approximation for α, giving your answer to three significant figures.   *(2 marks)*

Pure 4 June 2004

4   The polynomial $p(x)$ is given by

$$p(x) = x^3 - 6x^2 + 12x - 11$$

    (a)   Find the remainder when $p(x)$ is divided by $(x - 3)$.   *(2 marks)*

    (b)   The equation $p(x) = 0$ has a single real root α.

       (i)   Show that α lies between 3 and 4.   *(1 mark)*

       (ii)   Use the bisection method to find an interval of width 0.25 in which α lies.
                   (3 marks)

Matrix Transformations

Pure 3 January 2002

2 The matrix A is $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(a) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}.$$

Describe fully the geometrical transformation represented by **T**. *(2 marks)*

(b) Find the matrix A^3. *(2 marks)*

Pure 3 January 2003

2 The matrix M is $\begin{bmatrix} \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} \end{bmatrix}$.

(a) Find:

(i) M^2; *(2 marks)*

(ii) M^3. *(1 mark)*

(b) The transformation **T** is given by

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = M \begin{bmatrix} x \\ y \end{bmatrix}$$

Describe fully the geometrical transformation represented by **T**. *(2 marks)*