

# FPI Specimen

1)  $f(x) = x^3 - 3x^2 + 5x - 4$

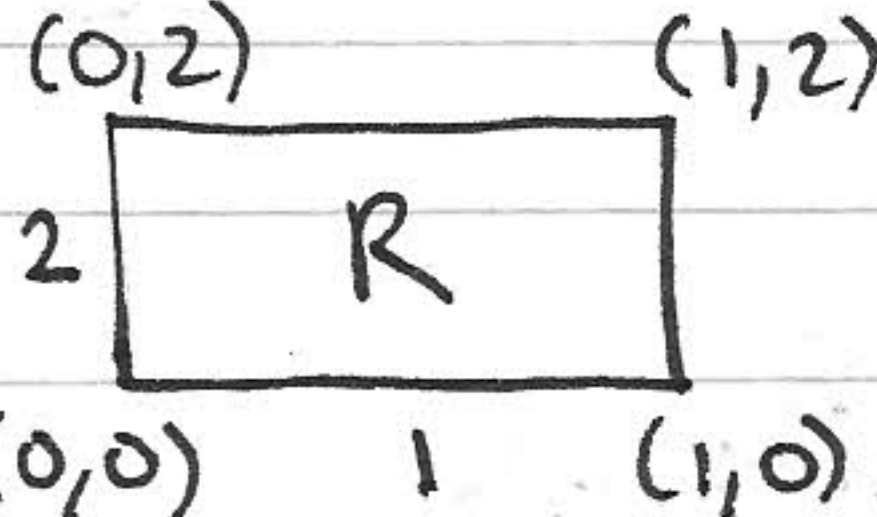
a)  $f'(x) = 3x^2 - 6x + 5$

$$x_0 = 1.4 \quad x_1 = 1.4 - \frac{f(1.4)}{f'(1.4)} = \underline{1.455} \text{ (3dp)}$$

2)  $AR = \begin{pmatrix} a & 4 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} = \begin{pmatrix} 0 & a & a+8 & 8 \\ 0 & -1 & 1 & 2 \end{pmatrix}$

$$(0,0); (a,-1); (a+8,1); (8,2)$$

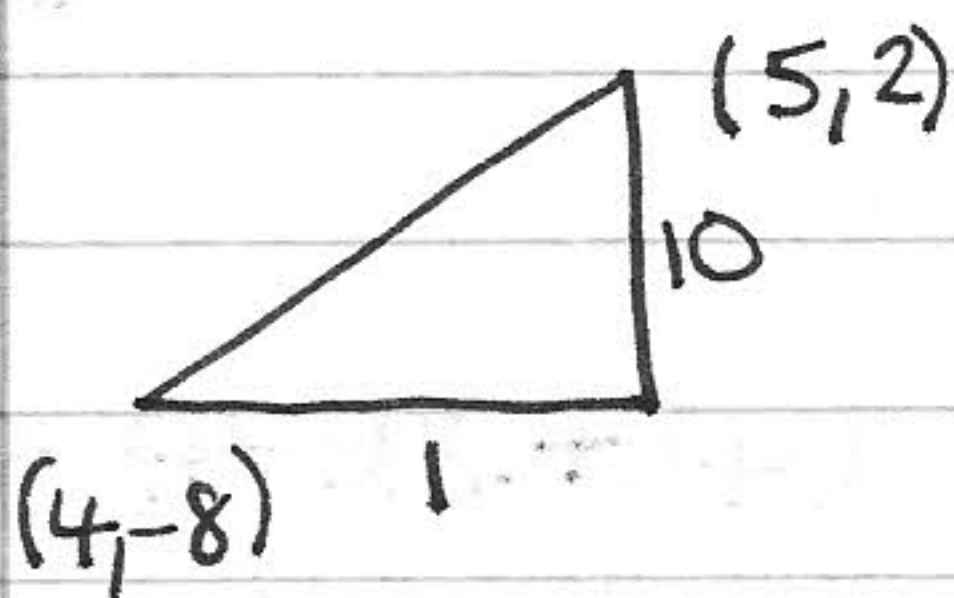
b)  $\det A = a+4$

c)   $\text{Area } R = 2$      $\text{Area Image} = 18$   
 $\Rightarrow \det A = 9 \Rightarrow 9 = a+4 \Rightarrow \underline{a=5}$

3)  $R^2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  rotation  $90^\circ$  clockwise about origin.

d)  $R =$  rotation  $45^\circ$  clockwise about origin.

4)  $f(x) = 2^x - 6x$      $f(4) = -8$      $f(5) = 2$



$$m = \frac{10}{1} \quad y - 2 = 10(x - 5)$$

$$y = 0 \Rightarrow -\frac{1}{5} = x - 5 \Rightarrow \underline{x = 4\frac{4}{5}}$$

5)  $\sum r^2 - \sum r - \sum 1 = \frac{1}{6}n(n+1)(2n+1) - \frac{1}{2}n(n+1) - n$

$$= \frac{1}{6}n[(n+1)(2n+1) - 3(n+1) - 6] = \frac{1}{6}n[2n^2 + 3n + 1 - 3n - 3 - 6]$$

$$= \frac{1}{6}n(2n^2-8) = \frac{1}{3}n(n^2-4) = \frac{1}{3}n(n+2)(n-2) \quad \#$$

$$b) \sum_{10}^{40} r^2 - r - 1 = \frac{1}{3}(40)(42)(38) - \frac{1}{3}(9)(11)(7) \\ = \underline{21049}$$

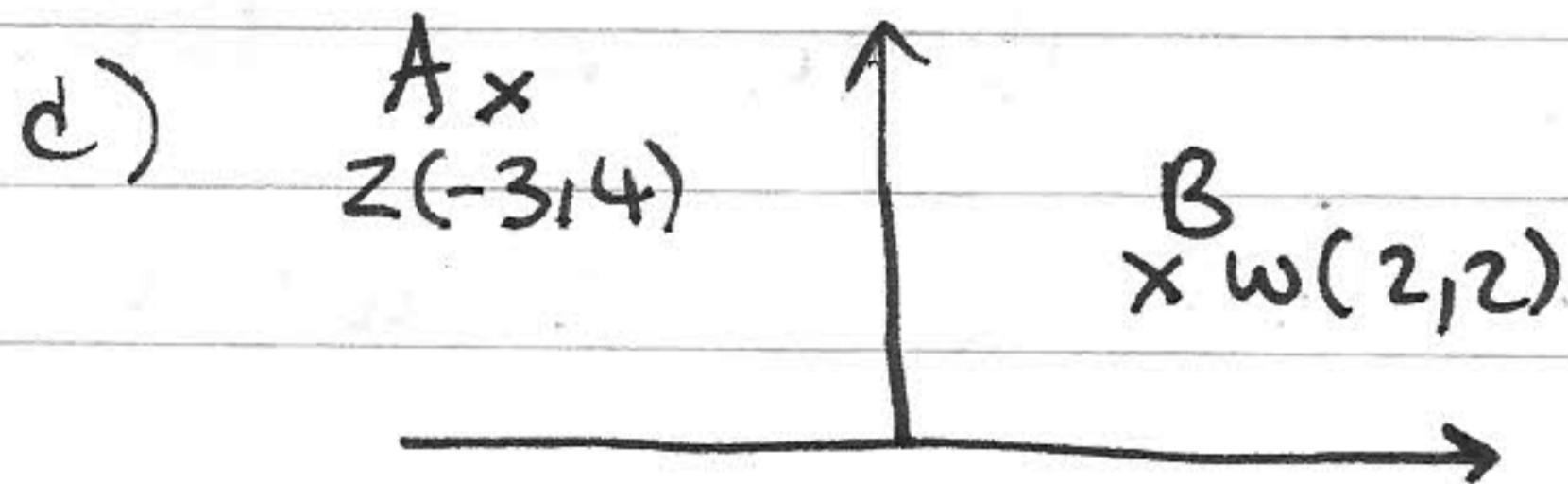
$$6) z = -3 + 4i \quad |z| = \sqrt{3^2 + 4^2} = 5$$

$$b) \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{4}{-3} \quad \theta = \tan^{-1}\left(\frac{4}{-3}\right) = -0.927 \\ + \pi$$

$$\arg(z) = 2.21$$

$$c) w = \frac{(-14+2i) \times (-3-4i)}{(-3+4i) \times (-3-4i)} = \frac{42 - 8i^2 + 56i - 6i}{9 - 16i^2} = \frac{50 + 50i}{25}$$

$$w = \underline{2+2i}$$



$$7) y^2 = 4ax \quad (4t^2, 8t) \Rightarrow 64t^2 = 4a(4t^2) \\ \Rightarrow 64t^2 = a \times 16t^2 \Rightarrow \underline{a=4}$$

$$b) \frac{d}{dx} y^2 = \frac{d}{dx} 16x \Rightarrow 2y \frac{dy}{dx} = 16 \Rightarrow \frac{dy}{dx} = \frac{8}{y} = \frac{8}{8t} \Rightarrow m_t = \frac{1}{t}$$

$$y - 8t = \frac{1}{t}(x - 4t^2) \Rightarrow yt - 8t^2 = x - 4t^2 \Rightarrow yt = x + 4t^2 \quad \#$$

$$c) \text{directrix } x+4=0 \quad (-4, 15)$$

$$15t = -4 + 4t^2 \Rightarrow 4t^2 - 15t - 4 = 0 \quad (4t+1)(t-4) = 0 \\ t = -\frac{1}{4} \quad t = 4$$

$$t=4 \quad (64, 32) \quad t = -\frac{1}{4} \quad \left(\frac{1}{4}, -2\right)$$

$$8) f(x) = 2x^3 - 5x^2 + px - 5$$

$$a) 1-2i \Rightarrow \text{other solution is } 1+2i \quad \alpha + \beta = 2$$

$$\alpha\beta = 1 - 4i^2 = 5$$

$$b) (x^2 - 2x + 5)(2x - 1) = 0 \Rightarrow x = 1-2i, 1+2i, \frac{1}{2}$$

$$c) 10x + 2x = 12x \equiv px \Rightarrow p = 12$$

$$9) \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^n = \begin{pmatrix} n+1 & n \\ -n & 1-n \end{pmatrix}$$

$$n=1 \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \quad n=1 \begin{pmatrix} 1+1 & 1 \\ -1 & 1-1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \checkmark$$

$$n=k+1 = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^{k+1} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}^k$$

$$= \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} k+1 & k \\ -k & 1-k \end{pmatrix} = \begin{pmatrix} 2k+2-k & 2k+1-k \\ -k-1 & -k \end{pmatrix} = \begin{pmatrix} k+2 & k+1 \\ -(k+1) & -k \end{pmatrix} \#$$

true for  $n=1$ ,  $n=k+1$  if true for  $n=k$   
 $\therefore$  by induction true for all  $n \in \mathbb{Z}^+$

$$b) f(n) = 4^n + 6n - 1 \quad n=1 \quad f(1) = 4 + 6 - 1 = 9 = 3 \times 3 \checkmark$$

$$n=k+1 \Rightarrow f(k+1) = 4^{k+1} + 6(k+1) - 1 = 4 \times 4^k + 6k + 5$$

$$f(k+1) - f(k) = 4 \times 4^k + 6k + 5 - 4^k - 6k + 1 = 3 \times 4^k + 6$$

$$\Rightarrow f(k+1) = 3[4^k + 2] + f(k)$$

true for  $n=1$ , true for  $n=k+1$  if true for  $n=k$   $\therefore$  by induction true for all  $n \in \mathbb{Z}^+$