Core Mathematics C3   For Edexcel
Advanced Level

Paper B
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

The booklet 'Mathematical Formulae and Statistical Tables', available from Edexcel, may be used.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner.
Answers without working may gain no credit.

1. Solve each equation, giving your answers in exact form.

(a) $\ln(3x+7) = 1$

(3)

(b) $2e^y + 5 = 3e^{-y}$

(5)

---

2. (a) Prove by counter-example, that the statement

$$\text{"cosec}(A+B) \equiv \operatorname{cosec} A + \operatorname{cosec} B \text{ for all } A \text{ and } B\text{" is false.}$$

(2)

(b) Prove that

$$2\operatorname{cosec} 2A \equiv \sec A \operatorname{cosec} A, \quad A \neq \frac{n\pi}{2}, n \in \mathbb{Z}.$$

(3)

---

3. (a) Express as a fraction in its simplest form

$$\frac{2}{x-5} + \frac{11}{(x-5)(x+3)}$$

(3)

(b) Hence solve

$$\frac{2}{x-5} + \frac{11}{(x-5)(x+3)} = 1$$

(4)

---

4. The function f is given by

$$\text{f}: x \mapsto k \ln x \qquad x \in \mathbb{R}, \quad x > 0 \text{ where } k \text{ is a positive constant.}$$

(a) Find $\text{f}^{-1}(x)$.

(3)

The function g is given by

$$\text{g}: x \mapsto e^x$$

(b) Find $\text{gf}(x)$.

(3)

(c) If $\text{gf}(2) = 16$, find the value of k.
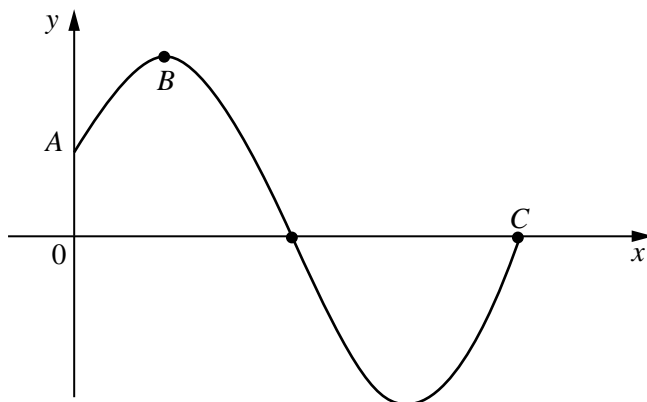
(2)

---

5. Given that $\sin x = \dfrac{3}{5}$ and that x is an obtuse angle, use an appropriate double angle formula to find the exact value of $\cot 2x$.

(7)

6. (*a*) Find the coordinates of the turning point on the graph of $y = xe^{2x}$ and determine its nature.

(8)

(*b*) Show that the equation of the tangent to this curve at the origin is the line $y = x$.

(2)

7. The function f is defined by

$$f(x) = \sin(x + \alpha), \qquad x \in \mathbb{R}, \quad x \geq 0, \quad 0 < \alpha < \frac{\pi}{6}$$

The diagram shows part of the graph of $y = f(x)$ which meets the *x*-axis at *C*, the *y*-axis at *A* and has a turning point at *B*.

(*a*) State the coordinates of the points *A*, *B* and *C*.

(3)

(*b*) On separate diagrams sketch the graphs of

(i) $y = 3f(x) + 1$

(2)

(ii) $y = f(x - \alpha)$

(2)

Show the coordinates of the new positions of points *A*, *B* and *C*.

(6)

8. On the same pair of axes sketch the graphs of

$$y = x^2 - 4x \quad \text{and} \quad y = \left|4x - x^2\right|.$$

(4)

Label each clearly.

(*a*) Find the area bounded by the 2 curves.

(5)

(*b*) Find the angle between the tangents to the curves at the point (4, 0). Give your answer correct to one decimal place.

(8)

END   **TOTAL 75 MARKS**