Worked Solutions

Edexcel C3 Paper D

1. (*a*) $\dfrac{(x+3)(x+4)}{x(x+3)} = \dfrac{x+4}{x}$ **(3)**

(*b*) $\log_3\left(\dfrac{x^2+7x+12}{x^2+3x}\right) = 2$

$$\frac{x+4}{x} = 3^2$$

$$x + 4 = 9x$$

$$x = \frac{1}{2}$$ **(4)**

---

2. (*a*) $\mathrm{f}(x) > 0$ **(2)**

(*b*) $\mathrm{f}\left(1\frac{1}{2}\right) = 2$

$\mathrm{g}(2) = 9 + k$

$9 + k = 14 \Rightarrow k = 5$ **(4)**

---

3. (*a*) $\cos 4x = \cos 2(2x)$

$= 2\cos^2 2x - 1$

$= 2\left(2\cos^2 x - 1\right)^2 - 1$

$= 8\cos^4 x - 8\cos^2 x + 1$ **(4)**

(*b*) $8\cos^2 x - 8\cos^4 x - 1 = 0 \Rightarrow \cos 4x = 0$

$4x = 90,\ 270,\ 450,\ 630$

$\therefore x = 22\frac{1}{2},\ 67\frac{1}{2},\ 112\frac{1}{2},\ 157\frac{1}{2}$ **(3)**

---

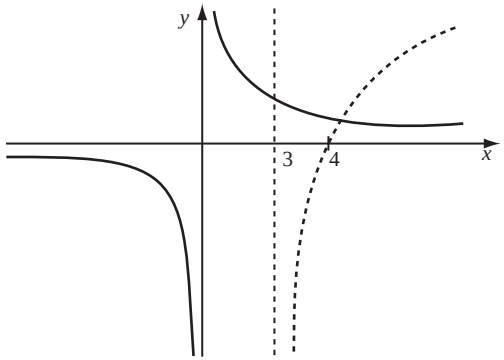
4. (*a*) (i) $x^3 \cdot 3\mathrm{e}^{3x} + 3x^2 \cdot \mathrm{e}^{3x}$ **(3)**

(ii) $\dfrac{\cos x \cdot 2 + 2x \cdot \sin x}{\cos^2 x}$ **(3)**

(*b*) $\dfrac{\mathrm{d}x}{\mathrm{d}y} = -\sin y^2 \cdot 2y \qquad \dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{2y\sin y^2}$ **(4)**

---

5. (*a*)



(3)

(*b*) curves cross once **(1)**

(*c*) $\mathrm{f}(4) = \ln 1 - \dfrac{2}{4} = -\dfrac{1}{2}$

$\mathrm{f}(5) = \ln 2 - \dfrac{2}{5} = 0.293$

} change in sign $\Rightarrow$ root in interval

(2)

(*d*) $x_1 = 4.4918$

$x_2 = 4.5609$

$x_3 = 4.5504$

$x_4 = 4.5520$

$x = 4.55$ (2 d.p.) **(3)**

---

6. (*a*) $\cot x = \frac{4}{3}$:

$\text{cosec}^2 x = 1 + \cot^2 x$

$= 1 + \frac{16}{9} = \frac{25}{9}$

$\text{cosec}\, x = \pm\frac{5}{3}$

x is reflex, $\therefore \text{cosec}\, x = \frac{-5}{3}$ **(4)**

$\left[\text{or } \tan x = \frac{3}{4}, \quad \sin x = -\frac{3}{5}, \ x \text{ is reflex}\right]$

(*b*) $\sin x = \frac{-3}{5}$

so $\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow \cos 2x = 1 - 2 \times \frac{9}{25}$

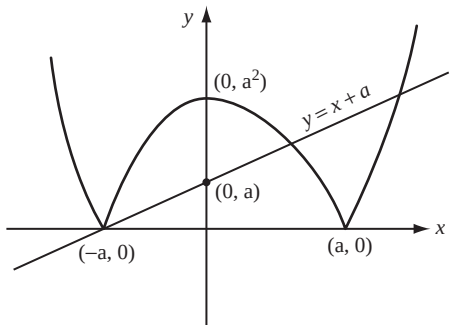
$= \frac{7}{25}$ (**4**)

(*c*) $\tan x = \frac{3}{4}$

$\left.\begin{aligned} \tan 2x &= \frac{2\tan x}{1-\tan^2 x} \\ &= \frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}} \end{aligned}\right\} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{24}{7}$ **(3)**

---

7. (*a*)



(4)

(*b*) see graph, $y = 0$, $x^2 = a^2$ $\therefore x = \pm a$ **(1)**

(*c*) $x^2 - a^2 = x + a$

$x^2 - x - a^2 - a = 0$ **(2)**

(*d*) $-\left(x^2 - a^2\right) = x + a$

$x^2 + x + a - a^2 = 0$ **(2)**

(*e*) $x^2 - x - 12 = 0$ $(a = 3)$

$(x-4)(x+3) = 0$ 2^{nd} pt. (4, 7)

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$ 3^{rd} pt. (2, 5) (**2**)

---

8. $\dfrac{dx}{dt} = e^{-t}(-\sin t + \cos t) + \left(-e^{-t}\right)(\cos t + \sin t) = -2e^{-t}\sin t$

$\dfrac{d^2x}{dt^2} = 2e^{-t}\sin t - 2e^{-t}\cos t$

$2e^{-t}\sin t - 2e^{-t}\cos t - 4e^{-t}\sin t + 2e^{-t}(\cos t + \sin t) = 0$ **(6)**

(*a*) $x = e^0(\cos 0 + \sin 0) = 1$

$\dfrac{dx}{dt} = -2e^{-0} \cdot \sin 0 = 0$ **(2)**

(*b*) $\sin t = 0 \Rightarrow t = 0, \pi, 2\pi$ etc. $t = \pi$ **(2)**

(*c*) $x = 1, x = e^{-\pi}(-1) = -e^{-\pi}$ **(2)**

(*d*) $\dfrac{d^2x}{dt^2} = -2x - 2\dfrac{dx}{dt} = -2x$

$x = 1, \dfrac{d^2x}{dt^2} < 0 \quad \therefore$ max.

$x = -e^{-\pi}, \dfrac{d^2x}{dt^2} > 0 \quad \therefore$ min. (**2**)