Worked Solutions

Edexcel C3 Paper I

1. (*a*) $y = \dfrac{1-2x}{2-x} \Rightarrow$

$$2y - xy = 1 - 2x$$

$$x(2-y) = 1 - 2y$$

$$x = \frac{1-2y}{2-y} \qquad \textbf{(3)}$$

So $\mathrm{f}'(x) = \dfrac{1-2x}{2-x}$

(*b*) $\mathrm{ff}\left(k^2\right) = \mathrm{ff}^{-1}\left(k^2\right)$

$= k^2$ **(2)**

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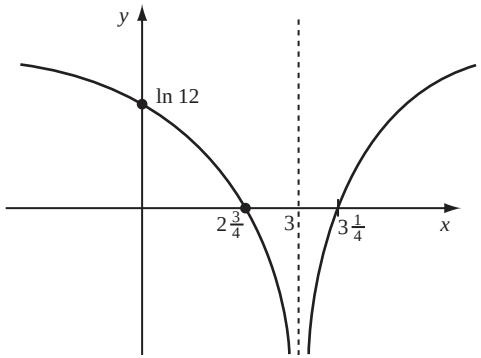
2. (*a*)



(3)

(*b*) $(0, \ln 12)$; $(2\frac{3}{4}, 0)$; $(3\frac{1}{4}, 0)$ **(3)**

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3. (*a*) $\dfrac{x(x-2)(x+3) - (x+3) + 5}{(x-2)(x+3)}$

$$\frac{x^3 + x^2 - 7x + 2}{(x-2)(x+3)}$$

$$\frac{\cancel{(x-2)}(x^2+3x-1)}{\cancel{(x-2)}(x+3)} \qquad \textbf{(5)}$$

(*b*) $\mathrm{f}'(x) = \dfrac{(x+3)(2x+3) - \left(x^2+3x-1\right)}{(x+3)^2}$

$$= \frac{x^2+6x+10}{x^2+6x+9}$$

$$= 1 + \frac{1}{(x+3)^2}$$

$$\frac{1}{(x+3)^2} = \frac{1}{25} \Rightarrow x + 3 = \pm 5$$

$$x = 2 \text{ or } -8$$

but $x > 2$, so there are no solutions. **(5)**

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4. (*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x \cdot 5\mathrm{e}^{5x} - \mathrm{e}^{5x}}{x^2} = 0$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \quad \frac{(5x-1)\mathrm{e}^{5x}}{x^2} = 0 \Rightarrow x = \frac{1}{5} \qquad \textbf{(4)}$$

(*b*) (i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 \cdot \sin 3y \cdot \cos 3y \cdot 3$ **(3)**

(ii) $y = \dfrac{\pi}{12}, 3y = \dfrac{\pi}{4} : \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2 \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 3} = \dfrac{1}{3}$ **(3)**

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5. (*a*) $f'(x) = \frac{1}{2x} - \frac{2}{x^3}, \quad x > 0$

$\frac{1}{2x} = \frac{2}{x^3} \Rightarrow x^2 = 4, \quad x = 2$ **(4)**

(*b*) $f(x) = \frac{1}{2}\ln 2 + \frac{1}{4}$

$= 2^{-1}\ln 2 + 2^{-2} \quad k = 2$ **(2)**

(*c*) $f(x) = \frac{1}{2}\ln 1 + \frac{1}{1^2} = 1 \quad Q(1, 1)$

$x = 1 \qquad f'(x) = \frac{1}{2} - 2 = -1\frac{1}{2}$

gradient of normal is $\frac{2}{3}$

$y - 1 = \frac{2}{3}(x - 1)$

$3y - 3 = 2x - 2$

$3y - 2x - 1 = 0$ **(3)**

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6. (*a*) $R^2 = 2.5^2 + 6^2$

$R = 6.5$

$\tan\alpha = \frac{6}{2.5}$

$\alpha = 1.176^c$ (**4**)

(*b*) $5\sin x\cos x - 12\sin^2 x$

$= 2.5(\sin 2x) + 6\left(-2\sin^2 x + 1\right) - 6$

$= 2.5\sin 2x + 6\cos 2x - 6$ **(4)**

(*c*) $= 6.5\sin(2x + 1.176) - 6$

max. value $6.5 - 6 = 0.5$ **(2)**

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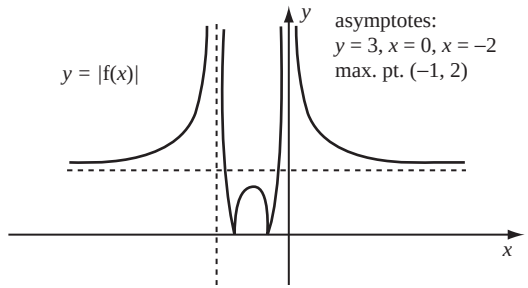
7. (*a*)



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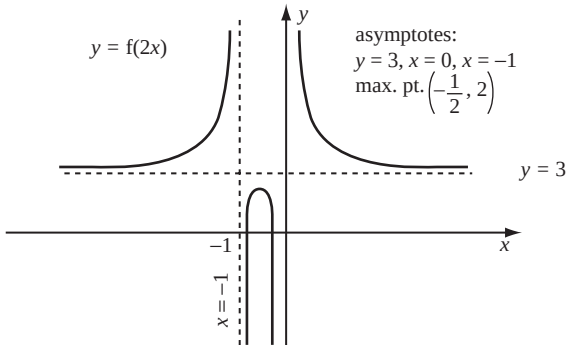
(*b*)



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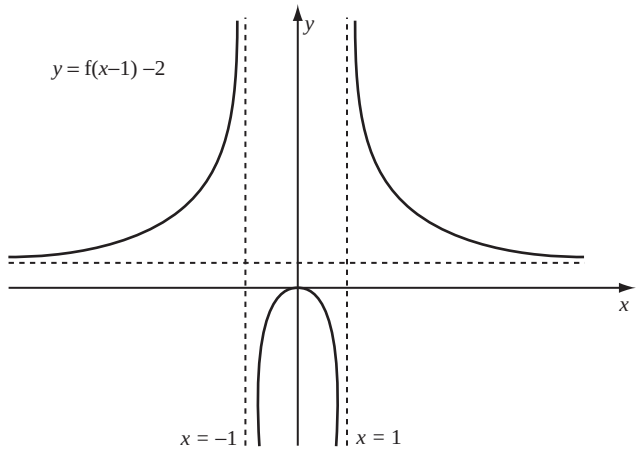
(*c*)



asymptotes $y = 1 \qquad x = 1 \qquad x = -1$ max. pt. $(0, 0)$

(*d*) $x = 0$ **(2)**

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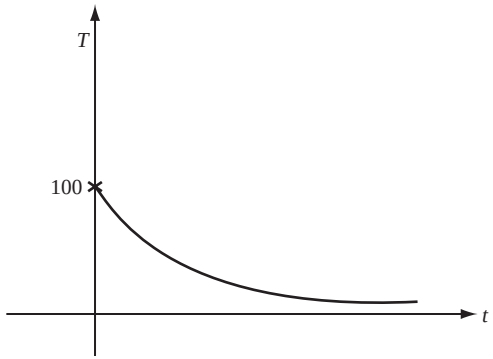
8. (*a*)



(3)

(*b*) $T = 15 + 85e^{-\frac{1}{2}} = 15 + 85 \times 0.606 = 66.6°$ (1 d.p.) **(2)**

(*c*) $40 = 15 + 85\,e^{-\frac{t}{8}} \quad \because \quad e^{-\frac{t}{8}} = \dfrac{25}{85}$

$e^{\frac{t}{8}} = 3.4$

$\therefore \quad t = 8 \ln 3.4$

$t = 9.79$ mins.

$= 9$ mins. 47 secs. (**3**)

(*d*) $\dfrac{dT}{dt} = -\dfrac{85}{8} \cdot e^{-\frac{t}{8}} : -\dfrac{85}{8} \cdot e^{-\frac{t}{8}} = -1.7$

$t = 8 \ln 6.25 = 14.7$ mins. (1 d.p.). **(4)**

$T = 15 + 85 \times 0.16$

$T = 28.6$

(*e*) $15°C$ **(1)**