C1, IYGB, PAPER T

1. $y = \frac{1}{2x^2} + \frac{4}{3x^3} = \frac{1}{2}x^{-2} + \frac{4}{3}x^{-3}$

$$\frac{dy}{dx} = -x^{-3} - 4x^{-4} = -\frac{1}{x^3} - \frac{4}{x^4}$$

$$\frac{d^2y}{dx^2} = 3x^{-4} + 16x^{-5} = \frac{3}{x^4} + \frac{16}{x^5}$$

THUS

$$x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} + 6y = x^2\left(\frac{3}{x^4} + \frac{16}{x^5}\right) + 6x\left(-\frac{1}{x^3} - \frac{4}{x^4}\right) + 6\left(\frac{1}{2x^2} + \frac{4}{3x^3}\right)$$

$$= \cancel{\frac{3}{x^2}} + \cancel{\frac{16}{x^3}} - \cancel{\frac{6}{x^2}} - \cancel{\frac{24}{x^3}} + \cancel{\frac{3}{x^2}} + \cancel{\frac{8}{x^3}}$$

$$= 0$$

AS REQUIRED

2. a) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$\Rightarrow \frac{7}{2} = \frac{8-1}{k-2}$

$\Rightarrow \frac{7}{2} = \frac{7}{k-2}$

$\Rightarrow k - 2 = 2$

$\Rightarrow k = 4$

b) $y - y_0 = m(x - x_0)$

USING $(2,1)$ & $m = \frac{7}{2}$

$y - 1 = \frac{7}{2}(x-2)$

$2y - 2 = 7x - 14$

$2y = 7x - 12$

c) EQUATION OF l_2 IS $x = 2$

$\therefore C(2,y) \quad B(4,8)$

• $|BC|^2 = 2\sqrt{2}$

$\Rightarrow \sqrt{(2-4)^2 + (y-8)^2} = 2\sqrt{2}$

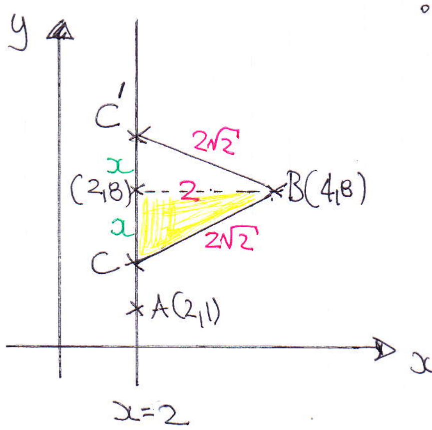
$\Rightarrow \sqrt{4 + (y-8)^2} = \sqrt{8}$

$\Rightarrow 4 + (y-8)^2 = 8$

$\Rightarrow (y-8)^2 = 4$

$\Rightarrow y - 8 = \begin{cases} 2 \\ -2 \end{cases} \quad \therefore y = \begin{cases} 10 \\ 6 \end{cases}$

$\therefore (2,10)$ OR $(2,6)$



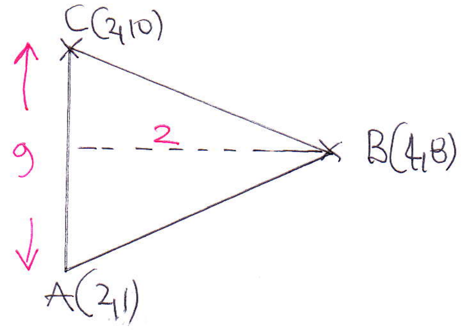
ALSO PYTHAGORAS FROM THIS PICTURE

$x^2 + 2^2 = (2\sqrt{2})^2$

$x^2 + 4 = 8$

$x = \pm 2$

d) LARGEST AREA OCCURS WHEN $C(2,10)$



$$\text{AREA} = \frac{1}{2} \times 9 \times 2 = 9$$

3. $25y^3 = 128(4x^2+1)^2$

WHEN $y=8$

$\Rightarrow 25 \times 8^3 = 128(4x^2+1)^2$

$\Rightarrow 25 \times 8^3 = 2 \times 8^2(4x^2+1)^2$

$\Rightarrow 25 \times 8 = 2(4x^2+1)^2$

$\Rightarrow 25 \times 4 = (4x^2+1)^2$

$\Rightarrow 100 = (4x^2+1)^2$

$\Rightarrow 4x^2+1 = \left\langle \begin{matrix} 10 \\ -10 \end{matrix} \right.$

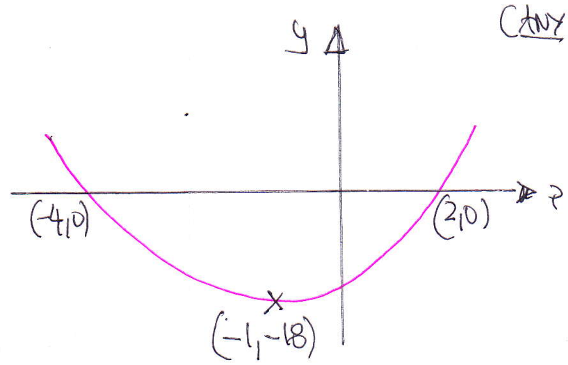
$\Rightarrow 4x^2 = \left\langle \begin{matrix} 9 \\ \cancel{-11} \end{matrix} \right.$

$\Rightarrow x^2 = \frac{9}{4}$

$\Rightarrow x = \pm \frac{3}{2}$

4. a) $-3f(x+2)$ CONSISTS OF:
- REFLECTION IN THE x AXIS
- VERTICAL STRETCH BY SCALE FACTOR 3
- TRANSLATION "HORIZONTALLY" BY 2 STEPS TO THE "LEFT"

(ANY ORDER IS OK)



b) EQUATION $y=f(x)$ MUST BE $y=k(x+2)(x-4)$

$$\left(0,\tfrac{16}{3}\right) \Rightarrow \frac{16}{3} = k\times 2\times(-4)$$

$$\frac{16}{3} = -8k$$

$$k = -\frac{2}{3}$$

$\therefore\ y = -\frac{2}{3}(x+2)(x-4)$

c) $f(x) = -\frac{2}{3}(x+2)(x-4)$

$3f(x) = -2(x+2)(x-4)$

$-3f(x) = 2(x+2)(x-4)$

$-3f(x+2) = -2\big[(x+2)+2\big]\big[(x+2)-4\big] = -2(x+4)(x-2)$

$\therefore$ when $x=0$ $y = -2\times 4\times(-2) = -16$

I.E $(0,-16)$

5. $$\frac{4}{\sqrt{3}+\sqrt{2}+1} = \frac{4(\sqrt{3}-\sqrt{2}-1)}{(\sqrt{3}+\sqrt{2}+1)(\sqrt{3}-\sqrt{2}-1)} = \frac{4(\sqrt{3}-\sqrt{2}-1)}{(\sqrt{3})^2-(\sqrt{2}+1)^2}$$

DIFFERENCE OF SQUARES

$$= \frac{4(\sqrt{3}-\sqrt{2}-1)}{3-(2+2\sqrt{2}+1)} = \frac{4(\sqrt{3}-\sqrt{2}-1)}{-2\sqrt{2}} = \frac{2(\sqrt{3}-\sqrt{2}-1)}{-\sqrt{2}}$$

$$= \frac{2(1+\sqrt{2}-\sqrt{3})}{\sqrt{2}} = \frac{2\sqrt{2}(1+\sqrt{2}-\sqrt{3})}{\sqrt{2}\sqrt{2}}$$

$$= \frac{\cancel{2}\sqrt{2}(1+\sqrt{2}-\sqrt{3})}{\cancel{2}} = \sqrt{2}+2-\sqrt{6}$$

AS REQUIRED

C1, IYGB, PAPER T

6. TRANSLATION IN THE POSITIVE x DIRECTION

$$x \longmapsto x-k \qquad (k>0)$$

$$\frac{(x-k)^2}{5} + \frac{y^2}{4} = 1 \quad \& \quad y = x-5$$

SOLVE SIMULTANEOUSLY

$$\Rightarrow \frac{(x-k)^2}{5} + \frac{(x-5)^2}{4} = 1$$

$$\Rightarrow 4(x-k)^2 + 5(x-5)^2 = 20$$

$$\Rightarrow 4(x^2-2kx+k^2) + 5(x^2-10x+25) = 20$$

$$\Rightarrow \left.\begin{array}{l} 4x^2 - 8kx + 4k^2 \\ 5x^2 - 50x + 125 \end{array}\right\} = 20$$

$$\Rightarrow \boxed{9x^2 - (8k+50)x + (4k^2+105) = 0}$$

• IF TANGENT $b^2-4ac=0$

$$\Rightarrow [-(8k+50)]^2 - 4\times 9\times(4k^2+105) = 0$$

$$\Rightarrow (8k+50)^2 - 36(4k^2+105) = 0$$

$$\Rightarrow 4(4k+25)^2 - 36(4k^2+105) = 0$$

$$\Rightarrow (4k+25)^2 - 9(4k^2+105) = 0$$

$$\Rightarrow 16k^2 + 200k + 625 - 36k^2 - 945 = 0$$

$$\Rightarrow 0 = 20k^2 - 200k + 320$$

$$\Rightarrow k^2 - 10k + 16 = 0$$

$$\Rightarrow (k-2)(k-8) = 0$$

$$k = \begin{cases} 2 \\ 8 \end{cases}$$

THESE CAN BE USED IN

$$9x^2 - (8k+50)x + (4k^2+105) = 0$$

- IF $k=2$

$9x^2-66x+121=0$

$(3x-11)^2=0$

$x=\frac{11}{3}$

$y=\frac{11}{3}-5=-\frac{4}{3}$

- IF $k=8$

$9x^2-114x+361=0$

$(3x-19)^2=0$

$x=\frac{19}{3}$

$y=\frac{19}{3}-5=\frac{4}{3}$

$\therefore \left(\frac{11}{3},-\frac{4}{3}\right)$ OR $\left(\frac{19}{3},\frac{4}{3}\right)$

7. $A=\frac{3}{2}xy+2yz+2xz=\frac{3}{2}\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{4}{3}\right)^{\frac{1}{3}}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}$

$=\frac{3}{2}\left(\frac{4}{3}\right)^{\frac{2}{3}}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}$

$=2\times\left(\frac{3}{4}\right)\left(\frac{4}{3}\right)^{\frac{2}{3}}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}+2\left(\frac{4}{3}\right)^{\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}$

$=2\times\left(\frac{3}{4}\right)^{1}\left(\frac{3}{4}\right)^{-\frac{2}{3}}+2\left(\frac{3}{4}\right)^{-\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}+2\left(\frac{3}{4}\right)^{-\frac{1}{3}}\left(\frac{3}{4}\right)^{\frac{2}{3}}$

$=2\left(\frac{3}{4}\right)^{\frac{1}{3}}+2\left(\frac{3}{4}\right)^{\frac{1}{3}}+2\left(\frac{3}{4}\right)^{\frac{1}{3}}$

$=6\times\left(\frac{3}{4}\right)^{\frac{1}{3}}=6\times\frac{3^{\frac{1}{3}}}{4^{\frac{1}{3}}}=6\times\frac{3^{\frac{1}{3}}}{4^{\frac{1}{3}}}\times\frac{4^{\frac{2}{3}}}{4^{\frac{2}{3}}}$

$=\frac{6\times3^{\frac{1}{3}}\times4^{\frac{2}{3}}}{4}=\frac{3}{2}\times3^{\frac{1}{3}}\times4^{\frac{2}{3}}$

$=\frac{3}{2}\times3^{\frac{1}{3}}\times\left(4^2\right)^{\frac{1}{3}}=\frac{3}{2}\times3^{\frac{1}{3}}\times16^{\frac{1}{3}}$

$=\frac{3}{2}\times48^{\frac{1}{3}}=\frac{3}{2}\times\sqrt[3]{48}=\frac{3}{2}\sqrt[3]{8}\sqrt[3]{6}$

$=\frac{3}{2}\times2\times\sqrt[3]{6}=3\sqrt[3]{6}$

AS REQUIRED

8. • SUPPOSE THE PROGRESSION HAS k TERMS

• SUM OF FIRST 20 IS 610

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$610 = \frac{20}{2}\left[2a+19\times5\right]$$
$$610 = 10\left[2a+95\right]$$
$$61 = 2a+95$$
$$-34 = 2a$$
$$\boxed{a=-17}$$

• THE LAST TERM OF THE PROGRESSION IS u_k

$$u_k = a+(k-1)d$$
$$u_k = -17+(k-1)\times5$$
$$u_k = -17+5k-5$$
$$\boxed{u_k = 5k-22}$$

NOW ONE OF TWO APPROACHES

METHOD A

CONSIDER THE SEQUENCE OF TERMS

$1^{st}, 2^{nd}, 3^{rd}, \ldots, (k-21)^{th}, (k-20)^{th}, \underbrace{(k-19)^{th}, \ldots, k^{th}}_{\text{LAST 20}}$

• $u_{k-19} = a+((k-19)-1)\times d$

$u_{k-19} = -17+(k-20)\times5$

$\boxed{u_{k-19} = 5k-117}$ ← "FIRST" TERM OF THE "LAST" 20

"FIRST" TERM OF THE "LAST" 20 IS $5k-117$
"LAST" TERM OF THE "LAST" 20 IS $5k-22$

USING $S_n = \frac{n}{2}\left[a+L\right]$

$$7410 = \frac{20}{2}\left[(5k-117)+(5k-22)\right]$$
$$741 = 10k-139$$
$$880 = 10k$$
$$k = 88$$

METHOD B

REMODEL THE SEQUENCE IN REVERSE SO THE FIRST TERM OF THE LAST 20 TERMS BACKWARDS IS THE SAME AS THE LAST TERM OF THE ENTIRE PROGRESSION.

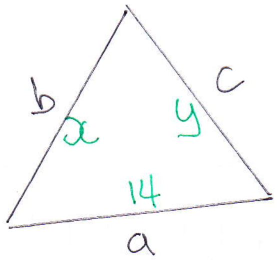
$a = 5k-22$
$d = -5$
$n = 20$
$S = 7410$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$7410 = \frac{20}{2}\left[2(5k-22)+19\times(-5)\right]$$
$$7410 = 10\left[10k-44-95\right]$$
$$741 = 10k-139$$
$$880 = 10k$$
$$k = 88$$

AS BEFORE

9. NON CALCULUS APPROACH

$$s = \tfrac{1}{2}(a+b+c)$$
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$



$s = \frac{1}{2} \times \text{PERIMETER} = \frac{1}{2} \times 36$

ie $\boxed{s = 18}$

NOW $x + y + 14 = 36$

$\boxed{x + y = 22}$

THUS $A = \sqrt{18(18-14)(18-x)(18-y)}$

$A = \sqrt{18 \times 4 \times (18-x)(18-(22-x))}$

$A = \sqrt{72(18-x)(x-4)}$

$A^2 = 72(18-x)(x-4)$

$A^2 = -72(x-18)(x-4)$

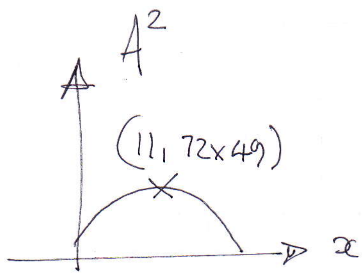
$A^2 = -72(x^2 - 22x + 72)$

$A^2 = -72\left[(x-11)^2 - 121 + 72\right]$

$A^2 = -72\left[(x-11)^2 - 49\right]$

$A^2 = 72\left[49 - (x-11)^2\right]$

$A^2 = 72 \times 49 - 72(x-11)^2$



THUS $A^2_{(MAX)}$ IS 72×49 (OCCURING WHEN $x = 11$)

$\Rightarrow A_{(MAX)} = \sqrt{72 \times 49}$

$\Rightarrow A_{(MAX)} = \sqrt{36 \times 49 \times 2}$

$\Rightarrow A_{(MAX)} = 6 \times 7 \times \sqrt{2}$

$\Rightarrow A_{(MAX)} = 42\sqrt{2}$ // AS REQUIRED

C4, IYGB, PAPER T

10. a) $\left.\begin{array}{l} u_n = 1+\left(\frac{1}{3}\right)^n \\ u_{n+1} = 1+\left(\frac{1}{3}\right)^{n+1}\end{array}\right\} \Rightarrow \left.\begin{array}{l} u_n - 1 = \left(\frac{1}{3}\right)^n \\ u_{n+1} - 1 = \left(\frac{1}{3}\right)^n \times \left(\frac{1}{3}\right)\end{array}\right\} \Rightarrow$

$\left.\begin{array}{l} u_n - 1 = \left(\frac{1}{3}\right)^n \\ 3(u_{n+1}-1) = \left(\frac{1}{3}\right)^n\end{array}\right\} \Rightarrow$

$$3(u_{n+1}-1) = u_n - 1$$
$$u_{n+1} - 1 = \tfrac{1}{3}u_n - \tfrac{1}{3}$$
$$u_{n+1} = \tfrac{1}{3}u_n + \tfrac{2}{3}$$

(WITH $u_1 = \frac{4}{3}$)

b) $U_{n+1} = 2U_n - 5$ HAS A "DOUBLING" FEATURE

(IN ANALOGY IN PART (a) $\frac{1}{3}$ APPEARS AS A "COMMON RATIO" IN THE Nth TERM & AS $\times\frac{1}{3}$ IN THE RECURRENCE)

- THUS TRY $\boxed{U_n = A \times 2^n + B}$ A, B CONSTANTS TO BE FOUND
- FROM THE RECURRENCE RELATION $U_1 = 6$ & $U_2 = 7$
- $\left.\begin{array}{l} 6 = A \times 2^1 + B \\ 7 = A \times 2^2 + B\end{array}\right\} \Rightarrow \left.\begin{array}{l} 6 = 2A + B \\ 7 = 4A + B\end{array}\right\} \Rightarrow 2A = 1$

$\boxed{A = \frac{1}{2}}$

&

$\boxed{B = 5}$

- $U_n = \frac{1}{2} \times 2^n + 5$

$U_n = 2^{n-1} + 5$

$U_{31} = 2^{30} + 5$

$\therefore U_{31} = 1073741829$

$\underbrace{2\times2}_{4}\underbrace{\times2}_{8}\underbrace{\times2}_{16}\underbrace{\times2}_{32}\underbrace{\times2}_{64}\underbrace{\times2}_{128}\underbrace{\times2}_{256}\underbrace{\;2}_{512}\underbrace{\;2}_{1024}$

```
   1024
 × 1024
   4096
  20480
1024000
1048576
```

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   1048576
 ×    1024
   4194304
  20971520
1048576000
1073741824
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