

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9231 FURTHER MATHEMATICS

9231/21

Paper 2, maximum raw mark 100

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3	Use conservation of momentum, e.g.:	$mv_A + 9mv_B = mu$	M1	7	10
	Use Newton's law of restitution (consistent signs):	$v_B - v_A = eu$	M1		
	Relate v_A to v_B using K.E. (A.E.F.):	$\frac{1}{2}mv_A^2 + \frac{1}{2}9mv_B^2 = \frac{1}{2}mu^2$	M1		
	Combine two eqns to find v_A and v_B e.g.:	$v_A = (1 - 9e)u/10, v_B = (1 + e)u/10$			
		or $v_A, v_B = -u/2, u/6$ [or $7u/10, u/30$]	M1 A1		
	Use in 3rd eqn to find e , e.g.:	$(1 - 9e)^2 + 9(1 + e)^2 = 50$			
	(A0 if finally $\pm \frac{2}{3}$)	$90e^2 = 40, e = \frac{2}{3}$	M1 A1		
	Use Newton's law of restitution with	$v_C = 2v_B', \text{ e.g.: } v_C - v_B' = ev_B, v_B' = \frac{2}{3}v_B$	B1		
		$[v_B = u/6, v_B = u/9, v_C = 2u/9]$		3	
	Use conservation of momentum to find k :	$9mv_B' + kmv_C = 9mv_B$			
		$9v_B' + 2kv_B' = 13.5v_B', k = 9/4$	M1 A1		

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4	(i)	Use conservation of energy at lowest point: Use $F = ma$ radially at lowest point: Eliminate v^2 to find R [$v^2 = 2.3 ga$]:	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga$ $R - mg = mv^2/a$ $R = mu^2/a + 3mg = 3.3mg$	B1 B1 B1	3	10
	(ii)	Use conservation of energy at B to find V_B : (A.E.F.)	$\frac{1}{2}mV_B^2 = \frac{1}{2}mu^2 + mga \sin \theta$ $V_B^2 = (0.3 + 0.5)ga, V_B = \sqrt{(0.8ga)}$ or $2\sqrt{(ga/5)}$ or $0.894\sqrt{(ga)}$	M1A1 A1		
	(iii)	Use vertical component v_B of speed V_B at B : Find height h reached above B : Find height h reached above level of O :	$v_B = V_B \cos \theta [= \frac{1}{4}\sqrt{15} V_B = \sqrt{(3/4 ga)}]$ $h = v_B^2/2g = 3a/8$ $h - a \sin \theta = 3a/8 - \frac{1}{4}a = a/8$ A.G.	M1 M1 A1 A1		
5	Find MI of components about A : (M1 for BC or CD)		Glass $\frac{1}{3}M(5a)^2 + 25a^2 = 20Ma^2$ $AB \quad M\{\frac{1}{3}(4a)^2 + (4a)^2\} = 64Ma^2/3$ $AD \quad \frac{1}{3}M\{\frac{1}{3}(3a)^2 + (3a)^2\} = 4Ma^2$ $BC \quad \frac{1}{3}M\{\frac{1}{3}(3a)^2 + 73a^2\} = 76Ma^{2/3}$ $CD \quad M\{\frac{1}{3}(4a)^2 + 52a^2\} = 172Ma^{2/3}$	M1 A1 B1 B1 M1 A1 A1	8	13
	Find total MI about A : (OR can first find total MI about centre of mass) State or imply total mass acts at mid-point of AC		$I = 128Ma^2$ A.G.	A1		
	Use eqn of circular motion to find $d^2\theta/dt^2$: Approximate $\sin \theta$ by θ and substitute for I :		$I d^2\theta/dt^2 = [-] (49Mg/15) 5a \sin \theta$ $d^2\theta/dt^2 = -(49g/384a) \theta$	M1 A1 A1		
	Find period $T = 2\pi/\omega$ with $\omega = \sqrt{(49g/384a)}$:		$T = 2\pi\sqrt{(384a/49g)}$ or $(16\pi/7)\sqrt{(6a/g)}$ or $17.6\sqrt{(a/g)}$ (A.E.F.)	B1		

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6		State or find the expected value of X : using $p = \frac{1}{4}$:	$E(X) = 1/p = 1/\frac{1}{4} = 4$	B1	1	5
	(i)	Find $P(X = 4)$:	$P(X = 4) = (\frac{3}{4})^3 \frac{1}{4} = 27/256$ or 0.105	M1 A1	2	
	(ii)	Find $P(X < 6)$:	$P(X < 6) = 1 - (\frac{3}{4})^5$ or $\{1 + \frac{3}{4} + (\frac{3}{4})^2 + (\frac{3}{4})^3 + (\frac{3}{4})^4\} \frac{1}{4}$ $= 781/1024$ or 0.763	M1 A1	2	
		S.R. Using $p = \frac{1}{2}$ can earn B0 M1 A0 M0 A0				
7	(i)	State probability density function of T :	$f(t) = 0.001 \exp(-0.001 t) \quad (t \geq 0)$ [$= 0$ (otherwise or $t < 0$)]	B1	1	8
	(ii)	Find $P(T > 2000)$: S.R. $1 - e^{-2} = 0.865$ earns B1 only (max 1/3) State inequality for t (lose A1 if $=$ or $\leq$): Solve for $t_{\max}$: (Omitting power 10 earns 0/4; using $1 - (\exp(-0.001t))^{10}$ can earn M1 A0 M1 A0 only)	$P(t > 2000) = 1 - F(2000)$ $= 1 - (1 - e^{-2}) = e^{-2}$ or 0.135 $(\exp(-0.001t))^{10} \geq [or >] 0.9$ $t_{\max} = (\ln 0.9) / (-0.01) = 10.5$	M1 M1 A1 M1 A1 M1 A1	3 4	

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8	State hypotheses (B0 for $\bar{\chi} \dots$): Estimate both popln. variances using two samples: (allow use of biased: $\sigma_{X,60}^2 = 236$ or 15.36^2) (allow use of biased: $\sigma_{Y,50}^2 = 265$ or 16.28^2) Estimate population variance for combined sample: (allow $\sigma_{X,60}^2/60 + \sigma_{Y,50}^2/50$: 9.233 or 3.039^2) Calculate value of z (to 2 d.p., either sign): State or use correct tabular z – value (to 2 d.p.): (or can compare 6 with e.g. 2.326 $s = 7.13$ or 7.07) Correct conclusion (A.E.F, $\checkmark$ on z – values): S.R. Assuming equal population variances: Find pooled estimate of common variance s^2: Calculate value of z (to 2 d.p., either sign): Tabular value; conclusion	$H_0: \mu_X = \mu_Y, H_1: \mu_X \neq \mu_Y$ $S_X^2 = (626\,220 - 6060^2/60) / 59$ [= 240 or 15.49^2] And $s_Y^2 = (464\,500 - 4750^2/50) / 49$ [= 270.4 or 16.44^2] $s^2 = s_X^2/60 + s_Y^2/50$ = 9.408 or 3.067^2 $z = (101 - 95) / s$ = 6/3.067 = 1.96 (or 1.97) $z_{0.99} = 2.326$ or 2.33 (allow 2.36) [Accept H_0] Claims are the same Hypotheses; Explicit assumption : $s^2 = (626\,220 - 6060^2/60 +$ $464\,500 - 4750^2/50) / 108$ $z = 6 / s\sqrt{(1/60+1/50)} = 1.97$ = 253.8 or 15.93^2 As above)	B1 M1 A1 M1 A1 M1 A1 B1 B1$\checkmark$ (B1; B1) (M1 A1) (M1 A1) (A1) (B1; B1$\checkmark$)	9
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9	Find expected frequency p :	$p = 200 \int_2^3 (1/x \ln 8) dx$ $= (200 / \ln 8) [\ln x]_2^3$ $= 200 \times 0.1950 = 39.00$ A.G. $q = 21.46$ or 21.45	M1A1 M1A1	4	
	Find q by similar method <i>or</i> by using total of 200:				
10	State (at least) null hypothesis:	H_0 : $f(x)$ fits data (A.E.F.)	B1	6	10
	Calculate χ^2 (to 3 s.f.):	$\chi^2 = 0.202 + 0.923 + 0.678 + 0.584$ $+ 1.134 + 4.134 + 3.644 = 11.3$	M1A1		
	State or use correct tabular χ^2 value (to 3 s.f.):	$\chi_{6, 0.95}^2 = 12.59$	B1		
	Valid method for reaching conclusion:	Accept H_0 if $\chi^2 \leq$ tabular value	M1		
	Conclusion consistent with correct values (A.E.F):	Distribution fits observations	A1		
10	Find correlation coefficient r :	$r = (73\,527 - 866 \times 639 / 10) / \sqrt{\{(121\,276 - 866^2 / 10)(55\,991 - 639^2 / 10)\}}$ (A.E.F.; A0 if only 3 s.f. clearly used)	M1 A1 A1 *A1	4	
		$= 18\,189.6 / \sqrt{(46\,280.4 \times 15\,158.9)}$ $= 0.687$			
10	State both hypotheses (B0 for $r \dots$):	$H_0: \rho = 0, H_1: \rho \neq 0$	B1	4	
	State or use correct tabular two-tail r -value:	$r_{10, 5\%} = 0.632$	*B1		
	Valid method for reaching conclusion:	Reject H_0 if $ r >$ tabular value	M1		
	Correct conclusion (A.E.F, dep *A1, *B1):	There is non-zero correlation	A1		
10	Calculate gradient p in $x - \bar{x} = p(y - \bar{y})$:	$p = 18\,189.6 / 15\,158.9 = 1.20$	B1	3	11
	Find regression line of x on y :	$x = 86.6 + 1.20(y - 63.9)$ $= 1.20y + 9.92$	M1 A1		

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11 A	(i)	Use Pythagoras to find AB : Find $\angle SAB$:	$AB = \sqrt{(4a^2 + 12a^2)} = 4a$ $\angle CAB = \sin^{-1} 2a\sqrt{3}/4a$ or $\cos^{-1} 2a/4a$ or $\tan^{-1} 2a\sqrt{3}/2a$ $= 60^\circ$ so $\angle SAB = 30^\circ$	A.G.	M1 A1		
	(ii)	<i>EITHER</i> Resolve vertically and horizontally, e.g.: (F_A may be in either direction) Eliminate $N_B + F_A$ to find N_A : <i>OR</i>	$\frac{1}{2} N_A + \frac{1}{2} \sqrt{3} N_B + \frac{1}{2} \sqrt{3} F_A = W$ and $\frac{1}{2} \sqrt{3} N_A = \frac{1}{2} N_B + \frac{1}{2} F_A$ $N_A = \frac{1}{2} W$	A.G.	M1 A1 A1	4	
	(iii)	Resolve in dirn. PQ to find N_A : Second resolution, e.g. in dirn. PS : Take moments, e.g. about A : (A1 for each side of eqn) Solve to find N_B : Use N_B to find F_A :	$N_A = \frac{1}{2} W$ $N_B + F_A = \frac{1}{2} \sqrt{3} W$ $\frac{1}{2} \sqrt{3} W \times 3a/2 + \frac{1}{2} W \times (2\sqrt{3} - 3)a$ $= N_B \times 2a$ $N_B = \{(7\sqrt{3} - 6)/8\} W$ $F_A = \sqrt{3} N_A - N_B$ or $\frac{1}{2} \sqrt{3} W - N_B$ $= \{3(2 - \sqrt{3})/8\} W$ (A.E.F.)	A.G.	(M1 A1) (A1) M1 A1 A1 M1 A1 M1 A1	3	
						7	14

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B	Estimate population variance:	$s_P^2 = (236 \cdot 0 - 42 \cdot 8^2 / 8) / 7$			
	(allow biased here: 0.8775 or 0.9367 ²)	$= 351 / 350$ or 1.003 or 1.001 ²	M1		
	Find confidence interval (allow z in place of t) e.g.:	$42 \cdot 8 / 8 \pm t \sqrt{s_P^2 / 8}$	M1		
	Use correct tabular t -value:	$t_{7, 0.975} = 2.365$	A1		
	Evaluate C.I. correct to 2 d.p.:	5.35 ± 0.84 or [4.51, 6.19]	A1	4	
	Formulate inequality for k (or equality for $k_{\max}$):	$(5.35 - k) / \sqrt{s_P^2 / 8} \geq [\text{or } >] t$	M1		
	Use correct tabular t -value:	$t_{7, 0.9} = 1.415$	A1		
	Solve for $k_{\max}$ (A0 if = or $\leq$ was used for k above):	$5.35 - k \geq 0.50, k_{\max} = 4.85$	A1	3	
	State hypotheses (B0 for $\bar{x} \dots$), e.g.:	$H_0: \mu_P = \mu_Q, H_1: \mu_P > \mu_Q$	B1		
	State assumption (A.E.F.):	Normal distns. for [P and] Q and equal variances	B1		
	Estimate (pooled) common variance:	$s^2 = (7 \times 1.003 + 11 \times 1.962) / 18$ $= 1.589$ or 1.261 ²	M1 A1		
	Calculate value of t (to 3 s.f.):	$t = (5.35 - 4.60) / (s \sqrt{(1/8 + 1/12)})$ $= 1.30$	M1 A1		
	Correct conclusion (A.E.F., $\sqrt{h}$ on t):	$t < t_{18, 0.9} = 1.33$ so Q 's mean is not less than P 's	B1 $\sqrt{h}$	7	14