

CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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FURTHER MATHEMATICS

9231/13

Paper 1

May/June 2018

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.



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2 (i) Verify that

$$\frac{n(e-1)+e}{n(n+1)e^{n+1}} = \frac{1}{ne^n} - \frac{1}{(n+1)e^{n+1}}. \quad [1]$$

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Let $S_N = \sum_{n=1}^N \frac{n(e-1)+e}{n(n+1)e^{n+1}}$.

(ii) Express S_N in terms of N and e . [2]

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4 The curve C has equation

$$y = \frac{x^2 + 7x + 6}{x - 2}.$$

- (i) Find the coordinates of the points of intersection of C with the axes. [2]

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- (ii) Find the equation of each of the asymptotes of C . [3]

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(iii) Sketch *C*.

[3]

5 It is given that \mathbf{e} is an eigenvector of the matrix \mathbf{A} with corresponding eigenvalue λ .

(i) Show that \mathbf{e} is an eigenvector of \mathbf{A}^3 and state the corresponding eigenvalue. [3]

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It is given that

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix}.$$

(ii) Find a matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that

$$\mathbf{A}^3 + \mathbf{I} = \mathbf{PDP}^{-1},$$

where \mathbf{I} is the 2×2 identity matrix. [5]

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6 The equation

$$9x^3 - 9x^2 + x - 2 = 0$$

has roots α, β, γ .

(i) Use the substitution $y = 3x - 1$ to show that $3\alpha - 1, 3\beta - 1, 3\gamma - 1$ are the roots of the equation

$$y^3 - 2y - 7 = 0. \tag{2}$$

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The sum $(3\alpha - 1)^n + (3\beta - 1)^n + (3\gamma - 1)^n$ is denoted by S_n .

(ii) Find the value of S_3 . [2]

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7 The lines l_1 and l_2 have vector equations

$$\mathbf{r} = a\mathbf{i} + 9\mathbf{j} + 13\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

respectively. It is given that l_1 and l_2 intersect.

(i) Find the value of the constant a . [3]

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The point P has position vector $3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$.

(ii) Find the perpendicular distance from P to the plane containing l_1 and l_2 . [4]

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(iii) Find the perpendicular distance from P to l_2 . [4]

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8 The curves C_1 and C_2 have polar equations, for $0 \leq \theta \leq \pi$, as follows:

$$C_1: r = a,$$

$$C_2: r = 2a|\cos \theta|,$$

where a is a positive constant. The curves intersect at the points P_1 and P_2 .

(i) Find the polar coordinates of P_1 and P_2 . [2]

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(ii) In a single diagram, sketch C_1 , C_2 and their line of symmetry. [3]

(ii) Deduce the set of values of x for which the infinite series

$$(u_1 - 3)x + (u_2 - 3)x^2 + \dots + (u_r - 3)x^r + \dots$$

is convergent.

[2]

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(iii) Use the result given in part (i) to find surds a and b such that

$$\sum_{n=1}^N \ln(u_n - 3) = N^2 \ln a + N \ln b.$$

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(ii) Express \mathbf{v}_4 as a linear combination of \mathbf{v}_1 , \mathbf{v}_2 and \mathbf{v}_3 . [2]

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(iii) Write down a basis for V . [1]

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Let $\mathbf{M} = \begin{pmatrix} 1 & -2 & 0 & 0 \\ 2 & -5 & -3 & -2 \\ 0 & 5 & 15 & 10 \\ 2 & 6 & 18 & 8 \end{pmatrix}$.

(iv) Find the general solution of $\mathbf{M}\mathbf{x} = \mathbf{v}_1 + \mathbf{v}_2$. [6]

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The set of elements of \mathbb{R}^4 which are not solutions of $\mathbf{M}\mathbf{x} = \mathbf{v}_1 + \mathbf{v}_2$ is denoted by W .

(v) State, with a reason, whether W is a vector space. [2]

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