

CANDIDATE
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FURTHER MATHEMATICS

9231/23

Paper 2

May/June 2018

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be 10 m s^{-2} .

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.



- 1 A particle P is moving in a fixed circle of radius 0.8 m. At time t s its velocity is $(t^2 - t + 2) \text{ m s}^{-1}$. Find the magnitudes of the radial and the transverse components of the acceleration of P when $t = 2$. [3]

Radial component

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Transverse component

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- 2 Two uniform small spheres A and B have equal radii and masses $4m$ and m respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is at rest. The coefficient of restitution between the spheres is e .

- (i) Show that after the collision A moves with speed $\frac{1}{5}u(4 - e)$ and find the speed of B . [4]

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Sphere B continues to move until it collides with a fixed smooth vertical barrier which is perpendicular to the direction of motion of B . The coefficient of restitution between B and the barrier is $\frac{3}{4}e$. After this collision, the speeds of A and B are equal.

(ii) Find the value of e . [3]

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The spheres A and B now collide directly again.

(iii) Determine whether sphere B collides with the barrier for a second time. [2]

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(iii) Find the time taken by P to travel directly from A to B . [3]

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4 A uniform rod AB has length $2a$ and weight W . The end A rests on rough horizontal ground and the end B rests against a smooth vertical wall. The angle between the rod and the horizontal is θ , where $\tan \theta = \frac{4}{3}$. One end of a light inextensible rope is attached to a point C on the rod. The other end is attached to a point where the vertical wall and the horizontal ground meet. The rope is taut and perpendicular to the rod. The rope and rod are in a vertical plane perpendicular to the wall.

(i) Show that $AC = \frac{18}{25}a$. [2]

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The magnitude of the frictional force at A is equal to one quarter of the magnitude of the normal reaction force at A .

(ii) Show that the tension in the rope is $\frac{1}{4}W$. [6]

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6 A random sample of 15 observations of pairs of values of two variables gives a product moment correlation coefficient of 0.430.

(i) Test at the 10% significance level whether there is evidence of non-zero correlation between the variables. [4]

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A second random sample of N observations gives a product moment correlation coefficient of 0.615. Using a 5% significance level, there is evidence of positive correlation between the variables.

(ii) Find the least possible value of N , justifying your answer. [2]

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7 The probability that a driver passes an advanced driving test has a fixed value p for each attempt. A driver keeps taking the test until he passes. The random variable X denotes the number of attempts required for the driver to pass. The variance of X is 3.75.

(i) Show that $15p^2 + 4p - 4 = 0$ and hence find the value of p . [4]

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(ii) Find $P(X = 5)$. [1]

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(iii) Find $P(3 \leq X \leq 7)$. [2]

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8 For a random sample of 6 observations of pairs of values (x, y) , the equation of the regression line of y on x is $y = bx + 1.306$, where b is a constant. The corresponding equation of the regression line of x on y is $x = 0.6331y + d$, where d is a constant. The values of x from the sample are

2.3 2.8 3.7 p 6.1 6.4

and the sum of the values of y is 46.5. The product moment correlation coefficient is 0.9797.

(i) Find the value of b correct to 3 decimal places. [2]

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(ii) Find the value of p . [4]

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(iii) Use the equation of the regression line of x on y to estimate the value of x when $y = 8.5$. [3]

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OR

A scientist carries out an experiment to investigate the quantity X , which takes the values 0, 1, 2, 3, 4, 5 or 6. He believes that the values taken by X follow a binomial distribution. He conducts 250 trials. His results are summarised in the following table.

x	0	1	2	3	4	5	6
Observed frequency	22	83	72	53	17	3	0

- (i) Show that unbiased estimates of the mean and variance for these results are 1.876 and 1.266 respectively, correct to 3 decimal places. By evaluating the mean and variance of the distribution $B(6, 0.313)$, explain why X could have this distribution. [4]

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The expected frequencies corresponding to the distribution $B(6, 0.313)$ are shown in the following table.

x	0	1	2	3	4	5	6
Observed frequency	22	83	72	53	17	3	0
Expected frequency	26.3	71.9	81.8	49.7	17.0	3.1	0.2

- (ii) Show how the expected frequency for $x = 4$ is calculated. [2]

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