

CANDIDATE  
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**FURTHER MATHEMATICS**

**9231/22**

Paper 2

**May/June 2019**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value is necessary, take the acceleration due to gravity to be  $10 \text{ m s}^{-2}$ .

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **25** printed pages and **3** blank pages.



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- 1 A particle  $P$  moves along an arc of a circle with centre  $O$  and radius 2 m. At time  $t$  seconds, the angle  $POA$  is  $\theta$ , where  $\theta = 1 - \cos 2t$ , and  $A$  is a fixed point on the arc of the circle.

(i) Show that the magnitude of the radial component of the acceleration of  $P$  when  $t = \frac{1}{6}\pi$  is  $6 \text{ m s}^{-2}$ . [2]

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(ii) Find the magnitude of the transverse component of the acceleration of  $P$  when  $t = \frac{1}{6}\pi$ . [2]

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3 Three uniform small spheres  $A$ ,  $B$  and  $C$  have equal radii and masses  $2m$ ,  $4m$  and  $m$  respectively. The spheres are moving in a straight line on a smooth horizontal surface, with  $B$  between  $A$  and  $C$ . The coefficient of restitution between each pair of spheres is  $e$ . Spheres  $A$  and  $B$  are moving towards each other with speeds  $2u$  and  $u$  respectively. The first collision is between  $A$  and  $B$ .

(i) Find the velocities of  $A$  and  $B$  after this collision. [3]

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Sphere  $C$  is moving towards  $B$  with speed  $\frac{4}{3}u$  and now collides with it. As a result of this collision,  $B$  is brought to rest.

(ii) Find the value of  $e$ . [4]

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**(iii)** Find the total kinetic energy lost by the three spheres as a result of the two collisions. [3]

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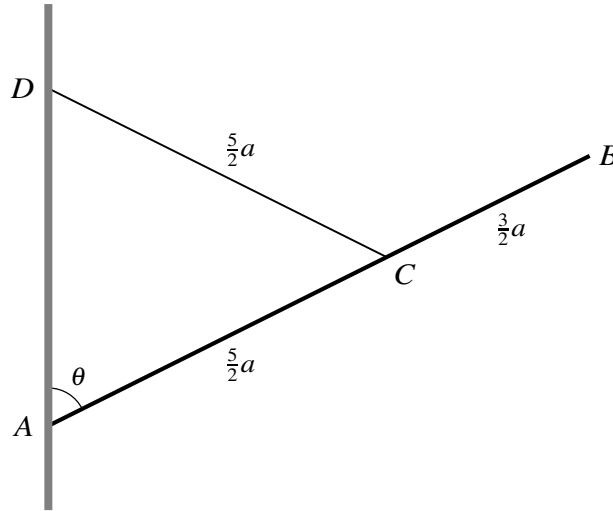
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A uniform rod  $AB$  of length  $4a$  and weight  $W$  rests with the end  $A$  in contact with a rough vertical wall. A light inextensible string of length  $\frac{5}{2}a$  has one end attached to the point  $C$  on the rod, where  $AC = \frac{5}{2}a$ . The other end of the string is attached to a point  $D$  on the wall, vertically above  $A$ . The vertical plane containing the rod  $AB$  is perpendicular to the wall. The angle between the rod and the wall is  $\theta$ , where  $\tan \theta = 2$  (see diagram). The end  $A$  of the rod is on the point of slipping down the wall and the coefficient of friction between the rod and the wall is  $\mu$ .

Find, in either order, the tension in the string and the value of  $\mu$ .

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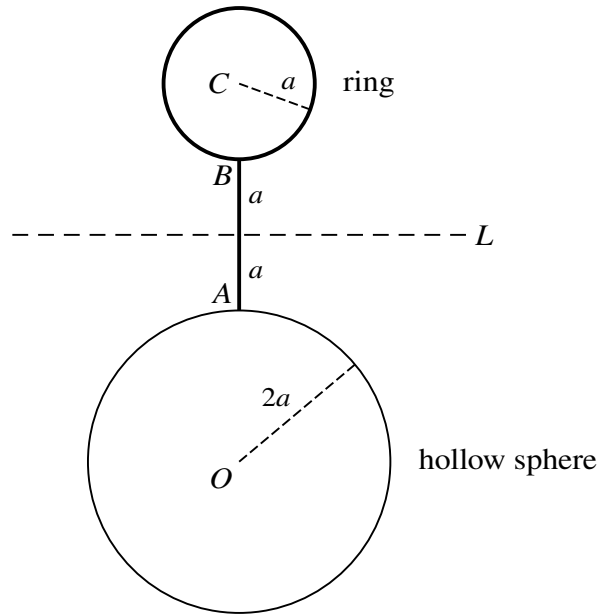
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A thin uniform rod  $AB$  has mass  $kM$  and length  $2a$ . The end  $A$  of the rod is rigidly attached to the surface of a uniform hollow sphere with centre  $O$ , mass  $kM$  and radius  $2a$ . The end  $B$  of the rod is rigidly attached to the circumference of a uniform ring with centre  $C$ , mass  $M$  and radius  $a$ . The points  $C, B, A, O$  lie in a straight line. The horizontal axis  $L$  passes through the mid-point of the rod and is perpendicular to the rod and in the plane of the ring (see diagram). The object consisting of the rod, the ring and the hollow sphere can rotate freely about  $L$ .

- (i) Show that the moment of inertia of the object about  $L$  is  $\frac{3}{2}(8k + 3)Ma^2$ . [6]

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**6** A fair six-sided die is thrown until a 3 or a 4 is obtained. The number of throws taken is denoted by the random variable  $X$ .

(i) State the mean value of  $X$ . [1]

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(ii) Find the probability that obtaining a 3 or a 4 takes exactly 6 throws. [1]

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(iii) Find the probability that obtaining a 3 or a 4 takes more than 4 throws. [2]

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- 9 A random sample of 50 observations of the continuous random variable  $X$  was taken and the values are summarised in the following table.

Interval	$0 \leq x < 0.8$	$0.8 \leq x < 1.6$	$1.6 \leq x < 2.4$	$2.4 \leq x < 3.2$	$3.2 \leq x < 4$
Observed frequency	18	16	8	6	2

It is required to test the goodness of fit of the distribution with probability density function  $f$  given by

$$f(x) = \begin{cases} \frac{3}{16}(4 - x)^{\frac{1}{2}} & 0 \leq x < 4, \\ 0 & \text{otherwise.} \end{cases}$$

The relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$0 \leq x < 0.8$	$0.8 \leq x < 1.6$	$1.6 \leq x < 2.4$	$2.4 \leq x < 3.2$	$3.2 \leq x < 4$
Expected frequency	14.22	12.54	10.59	8.18	4.47

- (i) Show how the expected frequency for  $1.6 \leq x < 2.4$  is obtained. [3]

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(ii) Find the value of  $c$ .

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(iii) Find the value of the product moment correlation coefficient.

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**OR**

A farmer grows two different types of cherries, Type *A* and Type *B*. He assumes that the masses of each type are normally distributed. He chooses a random sample of 8 cherries of Type *A*. He finds that the sample mean mass is 15.1 g and that a 95% confidence interval for the population mean mass,  $\mu$  g, is  $13.5 \leq \mu \leq 16.7$ .

- (i) Find an unbiased estimate for the population variance of the masses of cherries of Type *A*. [3]

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The farmer now chooses a random sample of 6 cherries of Type *B* and records their masses as follows.

12.2    13.3    16.4    14.0    13.9    15.4

- (ii) Test at the 5% significance level whether the mean mass of cherries of Type *B* is less than the mean mass of cherries of Type *A*. You should assume that the population variances for the two types of cherry are equal. [9]

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