

CANDIDATE  
NAME

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CENTRE  
NUMBER

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CANDIDATE  
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**FURTHER MATHEMATICS**

**9231/11**

Paper 1

**October/November 2018**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

**DO NOT WRITE IN ANY BARCODES.**

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of a calculator is expected, where appropriate.

Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.













- 5 It is given that  $\lambda$  is an eigenvalue of the matrix  $\mathbf{A}$  with  $\mathbf{e}$  as a corresponding eigenvector, and  $\mu$  is an eigenvalue of the matrix  $\mathbf{B}$  for which  $\mathbf{e}$  is also a corresponding eigenvector.

(i) Show that  $\lambda + \mu$  is an eigenvalue of the matrix  $\mathbf{A} + \mathbf{B}$  with  $\mathbf{e}$  as a corresponding eigenvector. [2]

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The matrix  $\mathbf{A}$ , given by

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}$$

has  $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$  and  $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$  as eigenvectors.

(ii) Find the corresponding eigenvalues. [3]

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6 The curve  $C$  has equation

$$y = \frac{x^2 + ax - 1}{x + 1},$$

where  $a$  is constant and  $a > 1$ .

(i) Find the equations of the asymptotes of  $C$ . [3]

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(ii) Show that  $C$  intersects the  $x$ -axis twice. [1]

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(iii) Justifying your answer, find the number of stationary points on  $C$ . [2]

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(iv) Sketch  $C$ , stating the coordinates of its point of intersection with the  $y$ -axis. [3]









9 The curve  $C$  has polar equation

$$r = 5\sqrt{(\cot \theta)},$$

where  $0.01 \leq \theta \leq \frac{1}{2}\pi$ .

- (i) Find the area of the finite region bounded by  $C$  and the line  $\theta = 0.01$ , showing full working. Give your answer correct to 1 decimal place. [3]

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Let  $P$  be the point on  $C$  where  $\theta = 0.01$ .

- (ii) Find the distance of  $P$  from the initial line, giving your answer correct to 1 decimal place. [2]

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(iii) Find the maximum distance of  $C$  from the initial line.

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(iv) Sketch  $C$ .

[2]











**OR**

The curve  $C$  has equation

$$x^2 + 2xy = y^3 - 2.$$

- (i) Show that  $A(-1, 1)$  is the only point on  $C$  with  $x$ -coordinate equal to  $-1$ . [2]

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For  $n \geq 1$ , let  $A_n$  denote the value of  $\frac{d^n y}{dx^n}$  at the point  $A(-1, 1)$ .

- (ii) Show that  $A_1 = 0$ . [3]

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(v) Deduce the value of  $I_3$  in terms of  $I_1$ . [2]

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