Cambridge Assessment International Education
Cambridge International Advanced Level

FURTHER MATHEMATICS **9231/12**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

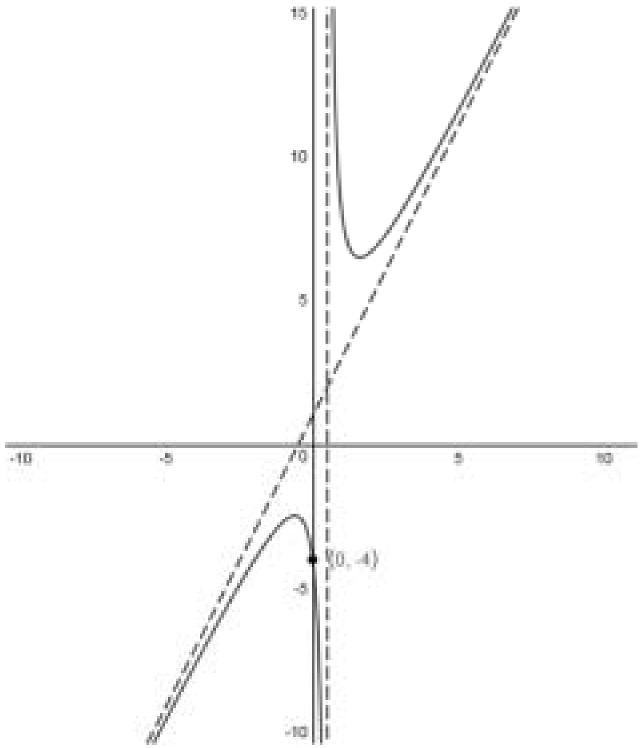
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$A=\int_0^1 x^a\,\mathrm{d}x=\left[\frac{x^{a+1}}{a+1}\right]_0^1=\frac{1}{a+1}$	**B1**	Finds area of region.
	$A\overline{x}=\int_0^1 xy\,\mathrm{d}x=\int_0^1 x^{a+1}\,\mathrm{d}x=\left[\frac{x^{a+2}}{a+2}\right]_0^1=\frac{1}{a+2}$	**M1 A1**	Finds $\int_0^1 xy\,\mathrm{d}x$.
	$2A\overline{y}=\int_0^1 y^2\,\mathrm{d}x=\int_0^1 x^{2a}\,\mathrm{d}x=\left[\frac{x^{2a+1}}{2a+1}\right]_0^1=\frac{1}{2a+1}$	**M1 A1**	Finds $\int_0^1 y^2\,\mathrm{d}x$.
	$(\overline{x},\overline{y})=\left(\frac{a+1}{a+2},\frac{a+1}{2(2a+1)}\right)$	**A1**	Both coordinates correct.
		6	

Question	Answer	Marks	Guidance
2	$\frac{dy}{dx} = \frac{a}{ax+1} = (-1)^0 \frac{0!a^1}{(ax+1)^1}$ so true for $n = 1$.	M1 A1	Proves base case.
	Assume that $\frac{d^k y}{dx^k} = (-1)^{k-1} \frac{(k-1)!a^k}{(ax+1)^k}$ for some positive integer k.	B1	States inductive hypothesis.
	Then $\frac{d^{k+1} y}{dx^{k+1}} = -ka(-1)^{k-1} \frac{(k-1)!a^k}{(ax+1)^{k+1}} = (-1)^k \frac{k!a^{k+1}}{(ax+1)^{k+1}}$ so true for $n = k+1$.	M1 A1	Differentiates k th derivative.
	By induction, true for every positive integer n.	A1	States conclusion.
		6	

Question	Answer	Marks	Guidance
3(i)	$I_{n+2}=\left[\frac{x^{-n-1}}{-n-1}\sin\pi x\right]_{\frac{1}{2}}^{1}-\pi\int_{\frac{1}{2}}^{1}\frac{x^{-n-1}}{-n-1}\cos\pi x\,\mathrm{d}x$	M1 A1	Integrates by parts.
	$=\frac{2^{n+1}}{n+1}+\frac{\pi}{n+1}\left(\left[\frac{x^{-n}}{-n}\cos\pi x\right]_{\frac{1}{2}}^{1}+\pi\int_{\frac{1}{2}}^{1}\frac{x^{-n}}{-n}\sin\pi x\,\mathrm{d}x\right)$	M1	Integrates by parts again.
	$=\frac{2^{n+1}}{n+1}+\frac{\pi}{n+1}\left(\frac{1}{n}-\frac{\pi}{n}I_n\right)$ $\Rightarrow(n+1)I_{n+2}=2^{n+1}+\pi\left(\frac{1}{n}-\frac{\pi}{n}I_n\right)$	M1	Uses I_n.
	$\Rightarrow n(n+1)I_{n+2}=2^{n+1}n+\pi-\pi^2I_n$	A1	AG
		5	
3(ii)	$2I_3=4+\pi-\pi^2I_1$ $12I_5=48+\pi-\frac{\pi^2}{2}\left(4+\pi-\pi^2I_1\right)$	M1	Substitutes I_3 into reduction formula.
	$\Rightarrow I_5=4+\frac{1}{24}\left(2\pi-4\pi^2-\pi^3+\pi^4I_1\right)$	A1	AEF, must be exact with fractions simplified.
		2	

Question	Answer	Marks	Guidance
4(i)	$x^2+1=(ax+b)(2x+1)+c$	**M1**	Uses that $2x+1$ is the quotient.
	$\Rightarrow a=\frac{1}{2},\ b=-\frac{1}{4}$	**A1 A1**	
		3	
4(ii)	$x=\frac{1}{2}$	**B1 FT**	
		1	

Question	Answer	Marks	Guidance
4(iii)		**B1**	Intersection (0,-4) given and asymptotes drawn.
		B1	Left branch correct.
		B1 FT	Right branch correct. Deduct at most one mark for poor forms at infinity.
		3	

Question	Answer	Marks	Guidance
5(i)	$\sum_{r=1}^{N}(5r+1)(5r+6)=25\sum_{r=1}^{N}r^2+35\sum_{r=1}^{N}r+6N$	M1	Expands.
	$25\left(\frac{1}{6}N(N+1)(2N+1)\right)+35\left(\frac{1}{2}N(N+1)\right)+6N$	M1	Substitutes formulae for $\sum r$ and $\sum r^2$.
	$=N\left(\frac{25}{6}(2N^2+3N+1)+\frac{35}{2}N+\frac{35}{2}+6\right)=\frac{1}{3}N(25N^2+90N+83)$	A1	Simplifies to the given answer (AG).
		3	
5(ii)	$\frac{1}{(5r+1)(5r+6)}=\frac{1}{5}\left(\frac{1}{5r+1}-\frac{1}{5r+6}\right)$	M1 A1	Finds partial fractions.
	$T_N=\frac{1}{5}\left(\frac{1}{6}-\frac{1}{11}+\frac{1}{11}-\frac{1}{16}+\cdots+\frac{1}{5N+1}-\frac{1}{5N+6}\right)$	M1	Expresses terms as differences.
	$\frac{1}{5}\left(\frac{1}{6}-\frac{1}{5N+6}\right)=\frac{1}{30}-\frac{1}{5(5N+6)}$	A1	At least 3 terms including last.
		4	
5(iii)	$\frac{S_N}{N^3}T_N\to\frac{25}{3}\times\frac{1}{30}=\frac{5}{18}$	M1 A1	Divides S_N by N^3 and takes limits as $N\to\infty$
		2	

Question	Answer	Marks	Guidance
6(i)	$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$	**B1**	
	$\overrightarrow{OC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & 2 & 7 \end{vmatrix} = \begin{pmatrix} -9 \\ -6 \\ 3 \end{pmatrix} = t\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	**M1 A1**	Finds direction of common perpendicular.
	$\dfrac{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{3^2 + 2^2 + 1^2}} = \dfrac{1}{\sqrt{14}} = 0.267$	**M1 A1**	Uses formula for shortest distance.
		5	
6(ii)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{vmatrix} = t\begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix}$	**M1 A1**	Finds normal to plane.
	$-(0) + 4(0) + 5(0) = 0$	**M1**	Uses point on plane.
	$-x + 4y + 5z = 0$	**A1**	AEF
		4	

Question	Answer	Marks	Guidance
7(i)	$\sqrt{-7y}(-7y)+2(-7y)+\sqrt{-7y}+7=0$ $\Rightarrow \sqrt{-7y}(-7y+1)=14y-7 \Rightarrow -7y(-7y+1)^2=(14y-7)^2$	M1	Uses given substitution and eliminates radical.
	$\Rightarrow 49y^3+14y^2-27y+7=0$	A1	AG
	$y=\frac{x^2}{-7}=\frac{x^2}{\alpha\beta\gamma}$	M1	Uses $\alpha\beta\gamma=-7$.
	So roots are $\frac{\alpha^2}{\alpha\beta\gamma}=\frac{\alpha}{\beta\gamma}, \frac{\beta^2}{\alpha\beta\gamma}=\frac{\beta}{\alpha\gamma}, \frac{\gamma^2}{\alpha\beta\gamma}=\frac{\gamma}{\alpha\beta}$	A1	AG
		4	
7(ii)	$\frac{\alpha}{\beta\gamma}+\frac{\beta}{\alpha\gamma}+\frac{\gamma}{\alpha\beta}=-\frac{2}{7}, \frac{1}{\gamma^2}+\frac{1}{\beta^2}+\frac{1}{\alpha^2}=-\frac{27}{49}$	B1	States sum of roots and $\alpha'\beta'+\alpha'\gamma'+\beta'\gamma'$.
	$\frac{\alpha^2}{\beta^2\gamma^2}+\frac{\beta^2}{\alpha^2\gamma^2}+\frac{\gamma^2}{\alpha^2\beta^2}=\left(-\frac{2}{7}\right)^2-2\left(-\frac{27}{49}\right)=\frac{58}{49}$	M1 A1	Uses $\alpha'^2+\beta'^2+\gamma'^2=(\alpha'+\beta'+\gamma')^2-2(\alpha'\beta'+\alpha'\gamma'+\beta'\gamma')$ AG
		3	
7(iii)	$49\left(\frac{\alpha^3}{\beta^3\gamma^3}+\frac{\beta^3}{\alpha^3\gamma^3}+\frac{\gamma^3}{\alpha^3\beta^3}\right)=-14\left(\frac{58}{49}\right)+27\left(-\frac{2}{7}\right)-21$	M1	Uses $49\alpha'^3=-14\alpha'^2+27\alpha'-7$.
	$\Rightarrow \frac{\alpha^3}{\beta^3\gamma^3}+\frac{\beta^3}{\alpha^3\gamma^3}+\frac{\gamma^3}{\alpha^3\beta^3}=-\frac{317}{343}$	A1	
		2	

Question	Answer	Marks	Guidance
8(i)	Eigenvalues of (upper diagonal matrix) $\mathbf{A}$ are $2, m$ and 1. (Or from characteristic equation: $(\lambda-2)(\lambda-m)(\lambda-1)=0$)	B1	
	$\lambda=2:\ \mathbf{e}_1=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & m-2 & 1\\ 0 & 0 & -1\end{vmatrix}=\begin{pmatrix}2-m\\0\\0\end{pmatrix}=t\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1 A1	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda=m:\ \mathbf{e}_2=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2-m & m & 1\\ 0 & 0 & 7\end{vmatrix}=\begin{pmatrix}7m\\7(m-2)\\0\end{pmatrix}=t\begin{pmatrix}m\\m-2\\0\end{pmatrix}$	A1	
	$\lambda=1:\ \mathbf{e}_3=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & m & 1\\ 0 & m-1 & 7\end{vmatrix}=\begin{pmatrix}6m+1\\-7\\m-1\end{pmatrix}=t\begin{pmatrix}6m+1\\-7\\m-1\end{pmatrix}$	A1	
	Thus $\mathbf{P}=\begin{pmatrix}1 & m & 6m+1\\ 0 & m-2 & -7\\ 0 & 0 & m-1\end{pmatrix}$ and $\mathbf{D}=\begin{pmatrix}2 & 0 & 0\\ 0 & m & 0\\ 0 & 0 & 1\end{pmatrix}$	M1 A1 FT	Or correctly matched permutations of columns. No follow through on two or more zero eigenvectors.
		7	

Question	Answer	Marks	Guidance
8(ii)	$\mathbf{M}^7\mathbf{P} = \mathbf{P}\mathbf{D}^7\mathbf{P}^{-1}\mathbf{P} = \mathbf{P}\mathbf{D}^7 = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix}\begin{pmatrix} 2^7 & 0 & 0 \\ 0 & m^7 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	M1 A1 FT	Applies $\mathbf{M}^7 = \mathbf{P}\mathbf{D}^7\mathbf{P}^{-1}$.
	$= \begin{pmatrix} 2^7 & m^8 & 6m+1 \\ 0 & m^8-2m^7 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$	A1	Order of columns might be swapped depending on **P**.
		3	

Question	Answer	Marks	Guidance
9(i)	Write $c = \cos\theta,\ s = \sin\theta$. $\cos 6\theta + i\sin 6\theta = (c+is)^6$	M1	Uses binomial theorem.
	$\Rightarrow \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1	
	$c^6 - 15c^4s^2 + 15c^2s^4 - s^6 = c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$	M1	Uses $c^2 = 1 - s^2$.
	$= c^6 - 15c^4(1-c^2) + 15c^2(1-2c^2+c^4) - (1-3c^2+3c^4-c^6)$	A1	
	$= 32c^6 - 48c^4 + 18c^2 - 1$	M1	Divides numerator and denominator by c^6.
	$\Rightarrow \sec 6\theta = \dfrac{1}{32c^6 - 48c^4 + 18c^2 - 1} = \dfrac{\sec^6\theta}{32 - 48\sec^2\theta + 18\sec^4\theta - \sec^6\theta}$	A1	AG
		6	

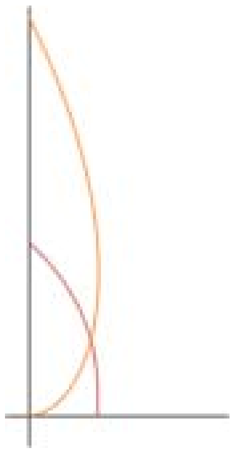
Question	Answer	Marks	Guidance
9(ii)	$x^6 = 2\left(32 - 48x^2 + 18x^4 - x^6\right) \Rightarrow \dfrac{x^6}{32 - 48x^2 + 18x^4 - x^6} = 2$	**M1 A1**	Relates with equation in part (i).
	$\sec 6\theta = 2 \Rightarrow \cos 6\theta = \dfrac{1}{2}$	**M1**	Solves $\cos 6\theta = \dfrac{1}{2}$.
	$x = \sec\dfrac{\pi}{18}$	**A1**	Gives one correct solution.
	$x = \sec q\pi$, $q = \dfrac{5}{18}, \dfrac{7}{18}, \dfrac{11}{18}, \dfrac{13}{18}, \dfrac{17}{18}$	**A1**	Gives five other solutions. Allow different values of q as long as all six solutions are found.
		5	

Question	Answer	Marks	Guidance
10(i)	$\begin{pmatrix}1 & 5 & 1\\ 1 & -2 & -2\\ 2 & 3 & \theta\end{pmatrix} \to \begin{pmatrix}1 & 5 & 1\\ 0 & -7 & -3\\ 0 & 0 & \theta+1\end{pmatrix}$	M1 A1	Reduces to echelon form. At least one row operation for M1.
	$r(\mathbf{A})=3$ if $\theta\neq-1$	A1	
	$r(\mathbf{A})=2$ if $\theta=-1$	B1	
		4	
10(ii)	$\begin{array}{llll} x & +5y & +z & = -1\\ & -7y & -3z & = 1\\ & & (\theta+1)z & = (\theta+1)\end{array}$	M1	Uses reduced form of augmented matrix or eliminates variables from scratch.
	$z=1,\ y=-\frac{4}{7},\ x=\frac{6}{7}$	A1 A1	One correct. All three correct.
		3	
10(iii)	$\begin{array}{llll} x & +5y & +z & = -1\\ & -7y & -3z & = 1\\ & & (\theta+1)z & = (\theta+1)\end{array}$ $z=t$	M1	Uses parameter.
	$y=-\frac{3t+1}{7},\ x=\frac{8t-2}{7}$	A1 A1	
		3	

Question	Answer	Marks	Guidance
10(iv)	$x+5y+z=-1$, $-7y-3z=1$, $(\theta+1)z=\phi+1$	M1	Uses reduced form of augmented matrix or eliminates variables from scratch
	$\theta=-1\Rightarrow\phi=-1$ so no solution (inconsistent).	A1	
		2	

Question	Answer	Marks	Guidance
11E(i)	$w=\cos y\Rightarrow\frac{\mathrm{d}w}{\mathrm{d}x}=-\sin y\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
	$\frac{\mathrm{d}^2w}{\mathrm{d}x^2}=-\sin y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\cos y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	B1	
	$\frac{\mathrm{d}^2w}{\mathrm{d}x^2}+2\frac{\mathrm{d}w}{\mathrm{d}x}+w=-\sin y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-\cos y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-2\sin y\frac{\mathrm{d}y}{\mathrm{d}x}+\cos y$	M1	Uses substitution to obtain w - x equation, AG.
	$=-\cos y\left(\mathrm{e}^{-2x}\sec y\right)=-\mathrm{e}^{-2x}$	A1	
		4	

Question	Answer	Marks	Guidance
11E(ii)	$m^2+2m+1=0 \Rightarrow m=-1$	**M1**	Finds CF.
	CF: $w=\left(Ax+B\right)\mathrm{e}^{-x}$	**A1**	
	PI: $w=k\mathrm{e}^{-2x} \Rightarrow w'=-2k\mathrm{e}^{-2x} \Rightarrow w''=4k\mathrm{e}^{-2x}$	**M1**	Forms PI and differentiates.
	$4k-4k+k=-1 \Rightarrow k=-1$	**A1**	
	$w=\left(Ax+B\right)\mathrm{e}^{-x}-\mathrm{e}^{-2x}$	**A1**	States general solution.
	$x=0$, $y=\frac{1}{3}\pi$, $w=\frac{1}{2} \Rightarrow B=\frac{3}{2}$	**B1**	Uses initial conditions to find constants.
	$w'=-\left(Ax+B\right)\mathrm{e}^{-x}+A\mathrm{e}^{-x}+2\mathrm{e}^{-2x}$	**M1**	Differentiates general solution.
	$x=0$, $y=\frac{1}{3}\pi$, $y'=\frac{\sqrt{3}}{3}$, $w'=-\frac{1}{2} \Rightarrow -\frac{1}{2}=-\frac{3}{2}+A+2 \Rightarrow A=-1$	**M1 A1**	Substitutes initial conditions.
	$y=\cos^{-1}\left(\left(\frac{3}{2}-x\right)\mathrm{e}^{-x}-\mathrm{e}^{-2x}\right)$	**A1**	States particular solution for y in terms of x.
		10	

Question	Answer	Marks	Guidance
11O(i)	$e^{2\alpha} - e^{-2\alpha} = 2\left(e^{\alpha} + e^{-\alpha}\right) \Rightarrow e^{\alpha} - e^{-\alpha} = 2$	**M1**	Sets equations equal and divides by $e^{\alpha} + e^{-\alpha}$.
	$e^{2\alpha} - 2e^{\alpha} - 1 = 0 \Rightarrow e^{\alpha} = 1 + \sqrt{2}$	**M1 A1**	Forms quadratic in e^{α}, AG.
	$\alpha = \ln\left(1 + \sqrt{2}\right)$	**A1**	Must be exact.
	$r = 2\left(1 + \sqrt{2} + \sqrt{2} - 1\right) = 4\sqrt{2}$	**M1 A1**	Substitutes to find r.
		6	
11O(ii)		**B1**	C_1 has correct shape.
		B1	C_2 has correct shape.
		B1	Intersection points positioned correctly.
		3	

Question	Answer	Marks	Guidance
11O(iii)	$2\int_0^{\ln(1+\sqrt{2})}\left(e^{\theta}+e^{-\theta}\right)^2\ d\theta-\frac{1}{2}\int_0^{\ln(1+\sqrt{2})}\left(e^{2\theta}-e^{-2\theta}\right)^2\ d\theta$ $=\int_0^{\ln(1+\sqrt{2})} 5+2e^{2\theta}+2e^{-2\theta}-\frac{1}{2}e^{4\theta}-\frac{1}{2}e^{-4\theta}\ d\theta$	M1 A1	Uses $\frac{1}{2}\int r^2\ d\theta$ to formulate correct area.
	$=\left[5\theta+e^{2\theta}-e^{-2\theta}-\frac{1}{8}e^{4\theta}+\frac{1}{8}e^{-4\theta}\right]_0^{\ln(1+\sqrt{2})}$	M1 A1	Expands and integrates.
	$=5\ln(1+\sqrt{2})+\left(1+\sqrt{2}\right)^2-\left(1+\sqrt{2}\right)^{-2}-\frac{1}{8}\left(\left(1+\sqrt{2}\right)^4-\left(1+\sqrt{2}\right)^{-4}\right)=5.82$	A1	
		5	