



Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

8 (a) Starting from the definitions of \tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2 x = \operatorname{sech}^2 x. \quad [3]$$

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(b) Using the substitution $u = \tanh x$, or otherwise, find $\int \operatorname{sech}^2 x \tanh^2 x \, dx$. [2]

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It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \tanh^2 x \, dx$.

(c) Show that, for $n \geq 2$,

$$(n + 1)I_n = \left(\frac{4}{5}\right)^3 \left(\frac{3}{5}\right)^{n-2} + (n - 2)I_{n-2}. \quad [5]$$

[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

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