OXFORD INTERNATIONAL AQA EXAMINATIONS

Topic Test: OxfordAQA A Level Further Mathematics

Pure

Name: ____________________

Class: ____________________

Date: ____________________

Time: **158 minutes**

Marks: **131 marks**

Comments:

1 The roots of the equation

$$z^3 - 5z^2 + kz - 4 = 0$$

are α, β and γ.

(a) (i) Write down the value of $\alpha + \beta + \gamma$ and the value of $\alpha\beta\gamma$.

(2)

(ii) Hence find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$.

(2)

(b) The value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$ is -4.

(i) Explain why α, β and γ cannot all be real.

(1)

(ii) By considering $(\alpha\beta + \beta\gamma + \gamma\alpha)^2$, find the possible values of k.

(4)

(Total 9 marks)

2 (a) Express $-4 + 4\sqrt{3}\,\mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(3)

(b) (i) Solve the equation $z^3 = -4 + 4\sqrt{3}\,\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(4)

(ii) The roots of the equation $z^3 = -4 + 4\sqrt{3}\,\mathrm{i}$ are represented by the points P, Q and R on an Argand diagram.

Find the area of the triangle PQR, giving your answer in the form $k\sqrt{3}$, where k is an integer.

(3)

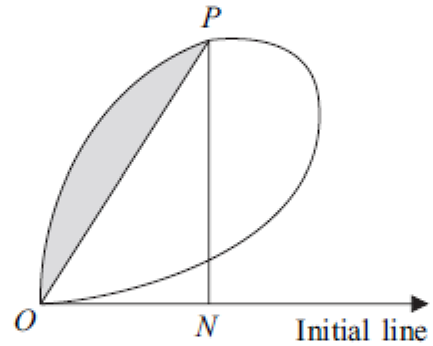
(c) By considering the roots of the equation $z^3 = -4 + 4\sqrt{3}\,\mathrm{i}$, show that

$$\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$$

(4)

(Total 14 marks)

3 The diagram shows a sketch of a curve.



The polar equation of the curve is

$$r = \sin 2\theta\sqrt{\left(2 + \frac{1}{2}\cos\theta\right)}, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P is the point of the curve at which $\theta = \frac{\pi}{3}$.

The perpendicular from P to the initial line meets the initial line at the point N.

(a) (i) Find the exact value of r when $\theta = \frac{\pi}{3}$.

(2)

(ii) Show that the polar equation of the line PN is $r = \frac{3\sqrt{3}}{8}\sec\theta$.

(2)

(iii) Find the area of triangle ONP in the form $\frac{k\sqrt{3}}{128}$, where k is an integer.

(2)

(b) (i) Using the substitution $u = \sin\theta$, or otherwise, find $\int \sin^n\theta\cos\theta \, d\theta$, where $n \geq 2$.

(2)

(ii) Find the area of the shaded region bounded by the line OP and the arc OP of the curve. Give your answer in the form $a\pi + b\sqrt{3} + c$, where a, b and c are constants.

(8)

(Total 16 marks)

4 (a) Prove by induction that, for all integers $n \geq 1$,

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots + \frac{2n+1}{n^2(n+1)^2} = 1 - \frac{1}{(n+1)^2}$$

(7)

(b) Find the smallest integer n for which the sum of the series differs from 1 by less than 10^{-5}.

(2)

(Total 9 marks)

5

(a) Given that $u = \sqrt{1 - x^2}$, find $\frac{du}{dx}$.

(2)

(b) Use integration by parts to show that

$$\int_0^{\frac{\sqrt{3}}{2}} \sin^{-1} x \, dx = a\sqrt{3}\pi + b$$

where a and b are rational numbers.

(6)

(Total 8 marks)

6

(a) It is given that $y = \ln(e^{3x} \cos x)$.

(i) Show that $\frac{dy}{dx} = 3 - \tan x$.

(3)

(ii) Find $\frac{d^4y}{dx^4}$.

(3)

(b) Hence use Maclaurin's theorem to show that the first three non-zero terms in the expansion, in ascending powers of x, of $\ln(e^{3x} \cos x)$ are $3x - \frac{1}{2}x^2 - \frac{1}{12}x^4$.

(3)

(c) Write down the expansion of $\ln(1 + px)$, where p is a constant, in ascending powers of x up to and including the term in x^2.

(1)

(d) (i) Find the value of p for which $\lim_{x \to 0} \left[\frac{1}{x^2} \ln\left(\frac{e^{3x} \cos x}{1 + px}\right)\right]$ exists.

(ii) Hence find the value of $\lim_{x \to 0} \left[\frac{1}{x^2} \ln\left(\frac{e^{3x} \cos x}{1 + px}\right)\right]$ when p takes the value found in part (d)(i).

(4)

(Total 14 marks)

7

(a) Show that

$$\frac{1}{4}(\cosh 4x + 2 \cosh 2x + 1) = \cosh^2 x \cosh 2x$$

(3)

(b) Show that, if $y = \cosh^2 x$, then

$$1 + \left(\frac{dy}{dx}\right)^2 = \cosh^2 2x$$

(3)

(c) The arc of the curve $y = \cosh^2 x$ between the points where $x = 0$ and $x = \ln 2$ is rotated through 2π radians about the x-axis. Show that the area S of the curved surface formed is given by

$$S = \frac{\pi}{256}(a \ln 2 + b)$$

where a and b are integers.

(7)
(Total 13 marks)

8 (a) Sketch the curve $y = \cosh x$.

(1)

(b) Solve the equation

$$6 \cosh^2 x - 7 \cosh x - 5 = 0$$

giving your answers in logarithmic form.

(6)
(Total 7 marks)

9 (a) By using an integrating factor, find the general solution of the differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = \ln x$$

(7)

(b) Hence, given that $y \to 0$ as $x \to 0$, find the value of y when $x = 1$.

(3)
(Total 10 marks)

10 It is given that the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

is $y = e^x(Ax + B)$. Hence find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 6e^x$$

(Total 5 marks)

11 The plane Π has equation $\mathbf{r} \cdot \begin{bmatrix} 12 \\ 15 \\ 16 \end{bmatrix} = 11$ and the point Q has coordinates (1, 1, –1).

(a) Show that Q is in Π.

(1)

(b) (i) Write down cartesian equations for the line l which passes through Q and is perpendicular to Π.

(2)

(ii) Deduce the direction cosines of l.

(2)

(c) The points M and N are on l, and each is 50 units from Π.

Find the coordinates of M and N.

(3)

(d) Given that the point P (5, 1, –4) is in Π, determine the area of triangle PMN.

(3)

(Total 11 marks)

12 The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

(a) Given that $\mathbf{A}^2 = \begin{bmatrix} p & -2 & -4 \\ 5 & 6 & 4 \\ 10 & q & 9 \end{bmatrix}$, find the value of p and the value of q.

(2)

(b) Given that $\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = \mathbf{0}$, prove that

$$\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I})$$

(2)

(c) Given that $\mathbf{A}^{-1} = \frac{1}{6}\begin{bmatrix} r & -2 & 2 \\ -1 & 5 & -2 \\ -2 & s & 2 \end{bmatrix}$, find the value of r and the value of s.

(2)

(d) Hence, or otherwise, find the solution of the system of equations

$$\begin{aligned} x \qquad - z &= k \\ x + 2y + z &= 5 \\ 2x + 2y + 3z &= 7 \end{aligned}$$

giving your answers in terms of k.

(3)

(Total 9 marks)

13 (a) Find the values of t for which the system of equations

$$\begin{aligned} tx + 2y + \quad 3z &= a \\ 2x + 3y - \quad tz &= b \\ 3x + 5y + (t + 1)z &= c \end{aligned}$$

does not have a unique solution.

(3)

(b) For the integer value of t found in part (a), find the relationship between a, b and c such that this system of equations is consistent.

(3)

(Total 6 marks)

1 (a) (i) $\alpha + \beta + \gamma = 5$

B1

$\alpha\beta\gamma = 4$

B1

2

(ii) $\alpha\beta\gamma^2 + \alpha\beta^2\gamma + \alpha^2\beta\gamma = \alpha\beta\gamma(\alpha + \beta + \gamma)$

M1

$= 5 \times 4 = 20$

FT their results from (a)(i)

A1√

2

(b) (i) If α, β, γ are all real then $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 \geq 0$

Hence α, β, γ cannot all be real

argument must be sound

E1

1

(ii) $\alpha\beta + \beta\gamma + \gamma\alpha = k$

$\sum \alpha\beta = k$ *PI*

B1

$(\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \sum \alpha^2\beta^2 + 2(\alpha\beta\gamma^2 + \alpha\beta^2\gamma + \alpha^2\beta\gamma)$

correct identity for $\left(\sum \alpha\beta\right)^2$

M1

$= -4 + 2(20)$

substituting their result from (a)(ii)

A1√

$k = \pm 6$

must see $k = \ldots$

A1 **cso**

4

[9]

2 (a) $r = 8$

B1

$\tan^{-1} \pm\frac{4\sqrt{3}}{4}$ or $\pm\frac{\pi}{3}$ seen

or $\frac{\pi}{6}$ *marked as angle to Im axis with "vector" in second quadrant on Arg diag*

M1

$\Rightarrow \theta = \frac{2\pi}{3}$

$-4+4\sqrt{3}\mathrm{i} = 8\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$

A1

3

(b) (i) modulus of each root = 2

B1√

use of De Moivre − dividing argument by 3

M1

$\Rightarrow \theta = \frac{4\pi}{9}, \frac{2\pi}{9}, \frac{8\pi}{9}$

A1 if 3 "correct" values not all in requested interval

$2\,\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{8\pi}{9}}$

A2

4

(ii) Area = $3 \times \frac{1}{2} \times PO \times OR \times \sin\frac{2\pi}{3}$

Correct expression for area of triangle PQR

M1

$= 3 \times \frac{1}{2} \times 2 \times 2 \times \sin\frac{2\pi}{3}$

correct values of lengths in formula

A1

$= 3\sqrt{3}$

A1cso

3

(c) Sum of roots (of cubic) = 0

must be stated explicitly

E1

Sum of 3 roots including Im terms

in form $r(\cos\theta + i\sin\theta)$

M1

$$2\left(\cos\frac{(-)4\pi}{9}+\cos\frac{2\pi}{9}+\cos\frac{8\pi}{9}\right)$$

isolating real terms; correct and with "2"

A1

$e^{-i\frac{4\pi}{9}} = \cos\frac{4\pi}{9} - i\sin\frac{4\pi}{9}$ seen earlier

or $\cos\frac{-4\pi}{9} = \cos\frac{4\pi}{9}$ *explicitly stated to earn final A1 mark*

$\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$

AG

A1**cso**

4

[14]

3 (a) (i) $r=\sin\frac{2\pi}{3}\sqrt{\left(2+\frac{1}{2}\cos\frac{\pi}{3}\right)} = \frac{\sqrt{3}}{2}\times\sqrt{\frac{9}{4}} = \frac{3\sqrt{3}}{4}$

M1; A1

2

(ii) $x = ON = (3\sqrt{3})/8$

Polar eqn of PN is $r\cos\theta = ON$

M1

$r = \frac{3\sqrt{3}}{8}\sec\theta$

AG Be convinced

A1

2

(iii) Area $\Delta\, ONP = 0.5 \times r_N \times r_P \times \sin(\pi/3)$

OE With correct or ft from (a)(i) (ii), values for r_P and r_N.

M1

$= \frac{1}{2} \times \frac{3\sqrt{3}}{8} \times \frac{3\sqrt{3}}{4} \times \frac{\sqrt{3}}{2} = \frac{27\sqrt{3}}{128}$

Be convinced

A1

2

(b) (i) $\int \sin^n \theta \cos \theta \, d\theta = \int u^n du$

PI

M1

$= \frac{\sin^{n+1} \theta}{n+1}$ $(+c)$

A1

2

(ii) Area of shaded region bounded by line OP and arc

$OP = \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2 2\theta(2 + \frac{1}{2}\cos\theta)d\theta$

Use of $\frac{1}{2}\int r^2 \, d\theta$

M1

Correct limits

B1

$\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1 - \cos 4\theta)\, d\theta + \frac{1}{4}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 4\sin^2 \theta \cos^2 \theta \cos \theta \, d\theta$

2 $\sin^2 2\theta = \pm 1 \pm \cos 4\theta$

M1

$\sin^2 2\theta \cos \theta = 4\sin^2 \theta \cos^2 \theta \cos \theta$

B1

Correct integration of 0.5(1 − cos 4θ)

A1

$= \left[\frac{\theta}{2} - \frac{\sin 4\theta}{8}\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sin^2 \theta - \sin^4\theta) \cos \theta d\theta$

Writing 2nd integrand in a suitable form to be able to use (b)(i) OE PI

m1

$$= \left[\frac{\theta}{2} - \frac{\sin 4\theta}{8} + \frac{\sin^3 \theta}{3} - \frac{\sin^5 \theta}{5} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

Last two terms OE

A1

$$= \frac{\pi}{12} - \frac{21\sqrt{3}}{160} + \frac{2}{15}$$

CSO

A1

8

[16]

4 (a) Assume true for $n = k$

Then $\sum_{r=1}^{k+1} \frac{2r+1}{r^2(r+1)^2} = 1 - \frac{1}{(k+1)^2} + \frac{2k+3}{(k+1)^2(k+2)^2}$

M1A0 if no LHS

M1A1

$= 1 - \frac{1}{(k+1)^2}\left(1 - \frac{2k+3}{(k+2)^2}\right)$

attempt to factorise or put over a common denominator

m1

$= 1 - \frac{1}{(k+1)^2}\left(\frac{k^2+2k+1}{(k+2)^2}\right)$

any correct combination starting 1−

A1

$= 1 - \frac{1}{(k+2)^2}$

A1

True for $n = 1$ LHS = RHS = $\frac{3}{4}$

B1

Method of induction set out properly

must score all 6 previous marks for this mark

E1

7

(b) $(n + 1)^2 > 10^2$ or $\frac{1}{(n+1)^2} > 10^{-5}$

Condone equals

M1

$n + 1 > 316.2$

$n > 315.2$

$n = 316$

A1

2

[9]

5

(a) $\frac{du}{dx} = \frac{1}{2}(1 - x^2)^{-\frac{1}{2}}$

B1

$\times (-2x)$

B1

2

(b) $\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1 - x^2}} dx$

A1 for each part of the integration by parts

M1

A1A1

$\int -\frac{x}{\sqrt{1 - x^2}} dx = \sqrt{1 - x^2}$ used

ft sign error in $\frac{du}{dx}$

A1F

$\frac{\sqrt{3}}{2}\frac{\pi}{3} + \sqrt{1 - \frac{3}{4}} - 1$

substitution of limits

m1

$\frac{1}{6}\sqrt{3}\pi - \frac{1}{2}$

CAO

A1

6

[8]

6 (a) (i) $y = \ln(e^{3x}\cos x) = \ln e^{3x} + \ln\cos x = 3x + \ln\cos x$

B1

$\frac{dy}{dx} = 3 + \frac{1}{\cos x} \times (-\sin x)$

Chain rule for derivative of ln cos x

M1

$\frac{dy}{dx} = 3 - \tan x$

CSO AG

A1

3

(ii) $\frac{d^2y}{dx^2} = -\sec^2 x$; $\frac{d^3y}{dx^3} = -2\sec x(\sec x \tan x)$

M1 for $d/dx\{[f(x)]^2\} = 2f(x)f'(x)$

B1; M1

$\frac{d^4y}{dx^4} = -4\sec x(\sec x \tan x)\tan x - 2\sec^4 x$

ACF

A1

3

(b) Maclaurin's Thm:

$$y(0)+x\,y'(0)+\frac{x^2}{2!}\,y''(0)+\frac{x^2}{3!}\,y'''(0)+\frac{x^4}{4!}\,y^{(iv)}(0)$$

$y(0) = \ln 1 = 0$; $y'(0) = 3$; $y''(0) = -1$; $y'''(0) = 0$; $y^{(iv)}(0) = -2$

Mac. Thm with attempt to evaluate at least two derivatives at x = 0

M1

$\ln(e^{3x}\cos x) = 0 + 3x + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{-2}{4!}x^4 \ldots$

At least 3 of 5 terms correctly obtained.
Ft one miscopy in (a)

A1F

$= 3x - \frac{1}{2}x^2 - \frac{1}{12}x^4$

CSO AG Be convinced

A1

3

(c) $\{\ln(1+px)\} = px - \frac{1}{2}p^2x^2$

accept $(px)^2$ for p^2x^2; ignore higher powers;

B1

1

(d) (i) $\left[\frac{1}{x^2}\{\ln\left(e^{3x}\cos x\right) - \ln(1+px)\}\right] =$

$$\left[\frac{1}{x^2}\left\{3x - \frac{1}{2}x^2 - O(x^4) - \left(px - \frac{1}{2}p^2x^2 + O(x^3)\right)\right\}\right]$$

Law of logs and expansions used;

M1

For $\lim_{x \to 0}\left[\frac{1}{x^2}\ln\left(\frac{e^{3x}\cos x}{1+px}\right)\right]$ to exist, $p = 3$

p = 3 convincingly found

A1

(ii) …… $\lim_{x \to 0}\left[\left(\frac{3-p}{x}\right)-\frac{1}{2}+\frac{p^2}{2}-O(x)\right]$

Divide throughout by x^2 before taking limit. (m1 can be awarded before or after the A1 above)

m1

Value of limit = $-\frac{1}{2}+\frac{p^2}{2}$ = 4

Must be convincingly obtained

A1

4

[14]

7 (a) Use of cosh 2x = 2cosh^2 x − 1

or cosh 4x = 2cosh^2 2x − 1

M1

RHS = $\frac{1}{2}$ cosh 2x + $\frac{1}{2}$ cosh^2 2x

A1

= $\frac{1}{4}$ (1 + 2cosh 2x + cosh 4x)

A1

3

If substituted for both cosh 4x and cosh 2x in LHS M1 only, until corrected
If RHS is put in terms of e^x
M1 for correct substitution
A1 for correct expansion
A1 for correct result

(b) $\frac{dy}{dx}$ = 2cosh $x$sinh x = sinh 2x

allow A1 for 1 + $\left(\frac{dy}{dx}\right)^2$ = 1 − 4cosh^2 x + 4cosh^4 x

Incorrect form for cosh^2 x in terms of cosh 2x M1 only

M1A1

Or

$y = \left(\frac{e^x + e^{-x}}{2}\right)^2 = \frac{e^{2x} + 2 + e^{-2x}}{4}$

$\frac{dy}{dx} = \frac{2e^{2x} - 2e^{x}}{4}$

(M1)

= sinh $2x$

(A1)

$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \sinh^2 2x = \cosh^2 2x$

AG

A1

3

(c) $S = 2\pi \int_{(0)}^{(\ln 2)} \cosh^2 x \cosh 2x \, dx$

allow even if limits missing

M1A1

$= 2\pi \int_0^{\ln 2} \frac{1}{4}(1 + 2\cosh 2x + \cosh 4x)dx$

m1

$= \frac{2\pi}{4}\left[x + \frac{2\sinh 2x}{2} + \frac{\sinh 4x}{4}\right]$

Integrated correctly

A1

Correct use of limits $a = 128$, $b = 495$

accept correct answers written down with no working.
Only one A1 if 2π not used

m1
A1,A1

7

[13]

8 (a) Sketch of $y = \cosh x$

approximately correct with minimum point above the x−axis, symmetrical about y-axis

B1

1

(b) Attempt to factorise

or complete square or use (correct unsimplified) formula

M1

$(3\cosh x - 5)(2\cosh x + 1) = 0$

A1

$\cosh x \neq -\frac{1}{2}$

indicated or stated (not merely neglected)

E1

$x = \ln\left(\frac{5}{3} + \sqrt{\frac{25}{9} - 1}\right)$

evidence of use of formula. Must see −1 or equivalent

M1

$= \pm \ln 3$

ft incorrect factorisation

A1F

A1 for ±

A1F

6

Alternative:

$3\left(\frac{e^x + e^{-x}}{2}\right) = 5$

$3e^{2x} - 10e^x + 3 = 0$

(M1)

$(3e^x - 1)(e^x - 3) = 0$

Correct factors

(A1F)

$x = \ln\frac{1}{3}$ or $\ln 3$

for both

(A1F)

NB if cosh x $\frac{e^x + e^{-x}}{2}$ used initially, M0 until quartic in e^x is factorised

M1 for $e^x - 3$ is a factor A1 if correct

M1 for $3e^x - 1$ is a factor A1 if correct

A1 for $x = \pm \ln 3$

E1 for showing remaining quadratic has no real roots

[7]

9 (a) IF is $\exp\left(\int \frac{2}{x}\mathrm{d}x\right)$

and with integration attempted

M1

$= e^{2\ln x}$

PI

A1

$= x^2$

A1

$\frac{\mathrm{d}}{\mathrm{d}x}\left[yx^2\right] = x^2 \ln x$

LHS; PI

M1

$\Rightarrow yx^2 = \int (\ln x) \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^3}{3}\right)$

Attempt integration by parts in correct direction to integrate $x^p \ln x$

M1

$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\mathrm{d}x$

RHS

A1

$yx^2 = \frac{x^3}{3}\ln x - \frac{x^3}{9} + A$

$\{y = \frac{x}{3}\ln x - \frac{x}{9} + Ax^{-2}\}$

A1

3

(b) Now, as $x \to 0$, $x^k \ln x \to 0$

Must be stated explicitly for a value of $k > 0$

E1

As $x \to 0$, $y \to 0 \Rightarrow A = 0$

Const of int = 0 must be convincing

B1

$yx^2 = \frac{x^3}{3} \ln x - \frac{x^3}{9}$

When $x = 1$, $y = -\frac{1}{9}$

ft on one slip but must have made a realistic attempt to find A

B1F

3

[10]

10 PI: $y_{PI} = kx^2e^x$

M1

$y'_{PI} = 2kxe^x + kx^2e^x$

$y''_{PI} = 2ke^x + 4kxe^x + kx^2e^x$

Product rule used in finding both derivatives

m1

$2ke^x + 4kxe^x + kx^2e^x - 4kxe^x - 2kx^2e^x + kx^2e^x = 6e^x$

Subst. into DE

m1

$2k = 6$; $k = 3$; $y_{PI} = 3x^2e^x$

CSO

A1

(GS: $y =$) $e^x(Ax + B) + 3x^2e^x$

$e^x(Ax + B) + kx^2e^x$, ft c's k.

B1F

[5]

11

(a) $\begin{bmatrix}1\\1\\-1\end{bmatrix}\bullet\begin{bmatrix}12\\15\\16\end{bmatrix} = 12 + 15 - 16 = 11$ shown

B1

1

(b) (i) eqn., or use, of line incorporating

p.v. $\begin{bmatrix}1\\1\\-1\end{bmatrix}$ and d.v. $\begin{bmatrix}12\\15\\16\end{bmatrix}$

M1

$$\frac{x-1}{12} = \frac{y-1}{15} = \frac{z+1}{16}$$

A1

2

(ii) $\sqrt{12^2 + 15^2 + 16^2} = 25$

FT

B1✓

$$\frac{12}{25}, \frac{15}{25}, \frac{16}{25}$$

FT

B1✓

2

(c) Use of $\mathbf{r} = \begin{bmatrix}1\\1\\-1\end{bmatrix} + \lambda\begin{bmatrix}12\\15\\16\end{bmatrix}$ with $\lambda = \pm 2$

M1

(25, 31, 31) and (– 23, – 29, – 33)

A1 A1

3

(d) **Method 1**

$PQ = 5$

B1

Area $\Delta PMN = \frac{1}{2} \times PQ \times MN = 250$

(Since Q = midpt.MN)

M1 A1

3

Method 2

$\overline{PM} = \begin{bmatrix} 20 \\ 30 \\ 35 \end{bmatrix}, \overline{PN} = \begin{bmatrix} -28 \\ -30 \\ -29 \end{bmatrix}$

Attempted

(M1)

$\overline{PM} \times \overline{PN} = (\pm)20 \begin{bmatrix} 9 \\ -20 \\ 12 \end{bmatrix}$ attempted

$||$gm. or Δ

(M1)

within an area formula
Area ΔPMN = 250

CAO

(A1)

[11]

12 (a) $\mathbf{A}^2 = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -2 & -4 \\ 5 & 6 & 4 \\ 10 & 10 & 9 \end{bmatrix}$

$p = -1$

p-value

B1

$q = 10$

q-value

B1

2

(b) $\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I} = 0$ multiply by $\mathbf{A}^{-1}$

$(\mathbf{A}^3 - 6\mathbf{A}^2 + 11\mathbf{A} - 6\mathbf{I})\mathbf{A}^{-1} = (0)\mathbf{A}^{-1}$

Multiplication by $\boldsymbol{A}^{-1}$

M1

$\mathbf{A}^3\mathbf{A}^{-1} - 6\mathbf{A}^2\mathbf{A}^{-1} + 11\mathbf{A}\mathbf{A}^{-1} - 6\mathbf{I}\mathbf{A}^{-1} = 0$

$\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I} - 6\mathbf{A}^{-1} = 0$

$6\mathbf{A}^{-1} = \mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I}$

$\mathbf{A}^{-1} = \frac{1}{6}(\mathbf{A}^2 - 6\mathbf{A} + 11\mathbf{I})$

AG

A1

2

(c) $\mathbf{A}^{-1} = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$

$r = 4$

r-value

B1

$s = -2$

s-value

B1

2

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4 & -2 & 2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}\begin{bmatrix} k \\ 5 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4k-10+14 \\ -k+25-14 \\ -2k-10+14 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 4k+4 \\ 11-k \\ 4-2k \end{bmatrix}$

use of $\boldsymbol{A}^{-1}\boldsymbol{v}$ *− one row correct*

M1

$x = \frac{2k+2}{3},\ y = \frac{11-k}{6},\ z = \frac{2-k}{3}$

correct solution for one variable

A1

all correct CAO

A1

Alternative:
If solving equations by elimination, M1 A1 for correct solution for one variable, A1 all correct

3

[9]

13 (a) $\begin{vmatrix} t & 2 & 3 \\ 2 & 3 & -t \\ 3 & 5 & t+1 \end{vmatrix} = 8t^2 - 7t - 1 = 0$

Attempt at det. of coefft. mtx. (or equivalent)

M1

Equating to zero and solving a quadratic eqn. in t

M1

$t = 1, -\frac{1}{8}$

A1

3

(b) $\begin{array}{rcrcrcl} & & x + 2y + 3z & = & a \\ t = 1 \Rightarrow & & 2x + 3y - z & = & b \\ & & 3x + 5y + 2z & = & c \end{array}$

FT any integer value found

B1√

E.g. ① + ② – ③ $\Rightarrow a + b = c$

M1 A1

3

[6]