# Topic Test: OxfordAQA A level Mathematics

Mechanics

Name: ______________________

Class: ______________________

Date: ______________________

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Time: **88 minutes**

Marks: **73 marks**

Comments:

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**1** A particle moves so that at time $t$ seconds its velocity $\mathbf{v}$ m $s^{-1}$ is given by

$$\mathbf{v} = (4t^3 - 12t + 3)\mathbf{i} + 5\mathbf{j} + 8t\mathbf{k}$$

(a) When $t = 0$, the position vector of the particle is $(-5\mathbf{i} + 6\mathbf{k})$ metres.

Find the position vector of the particle at time $t$.

**(4)**

(b) Find the acceleration of the particle at time $t$.

**(2)**

(c) Find the magnitude of the acceleration of the particle at time $t$. Do not simplify your answer.

**(2)**

(d) Hence find the time at which the magnitude of the acceleration is a minimum.

**(2)**

(e) The particle is moving under the action of a single variable force **F** newtons. The mass of the particle is 7 kg.

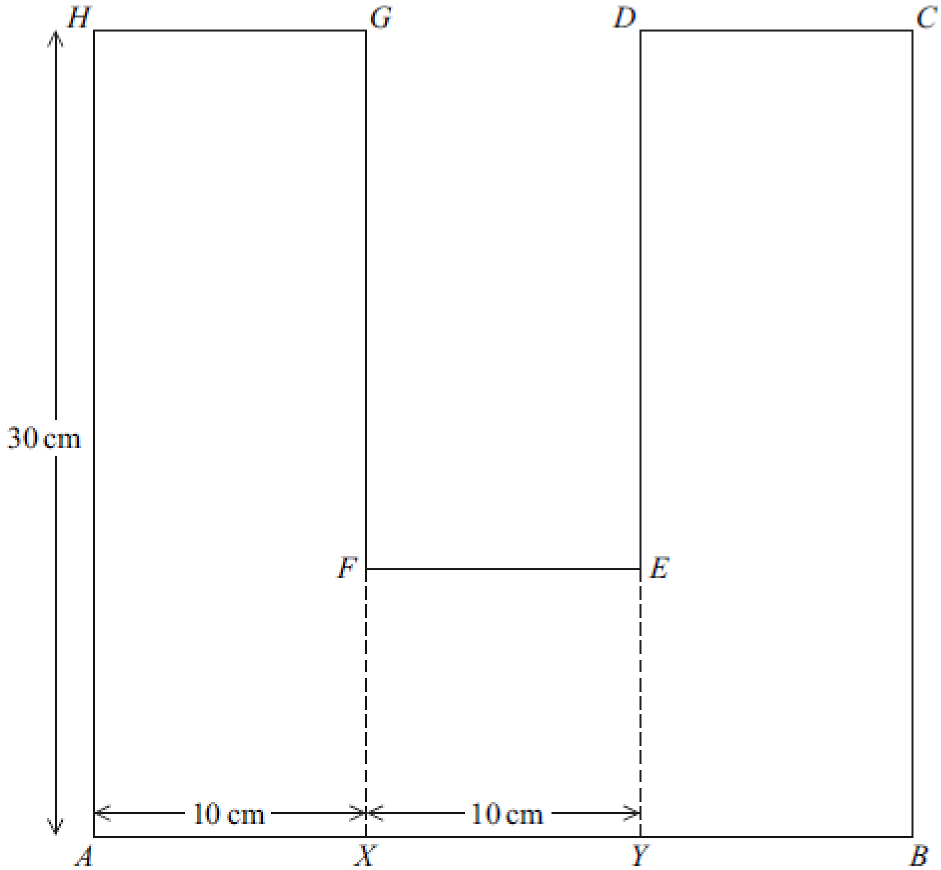
Find the minimum magnitude of **F**.

**(2)**

**(Total 12 marks)**

**2** The diagram shows a uniform lamina which is in the shape of two identical rectangles $AXGH$ and $YBCD$ and a square $XYEF$, arranged as shown.

The length of $AX$ is 10 cm, the length of $XY$ is 10 cm and the length of $AH$ is 30 cm.




(a) Explain why the centre of mass of the lamina is 15 cm from $AH$.

**(1)**

(b) Find the distance of the centre of mass of the lamina from $AB$.

**(3)**

(c) The lamina is freely suspended from the point $H$.

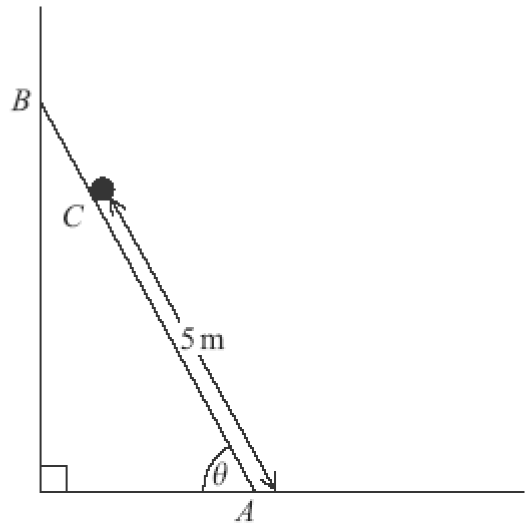
Find, to the nearest degree, the angle between $HG$ and the horizontal when the lamina is in equilibrium.

**(4)**

**(Total 8 marks)**

**3** A uniform ladder, of length 6 metres and mass 22 kg, rests with its foot, $A$, on a rough horizontal floor and its top, $B$, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the floor is $\theta$.

A man, of mass 90 kg, is standing at point $C$ on the ladder so that the distance $AC$ is 5 metres. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the horizontal floor is 0.6. The man may be modelled as a particle at $C$.




(a) Show that the magnitude of the frictional force between the ladder and the horizontal floor is 659 N, correct to three significant figures.

**(4)**

(b) Find the angle $\theta$.

**(5)**

**(Total 9 marks)**

**4** A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time $t$ seconds, the velocity of the particle, **v** m s$^{-1}$, is given by

$$\mathbf{v} = 4e^{-2t}\,\mathbf{i} + (6t - 3t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time $t$.

**(3)**

(b) The mass of the particle is 5 kg.

(i) Find an expression for the force **F** acting on the particle at time $t$.

**(2)**

(ii) Find the magnitude of **F** when $t = 0$ .

**(2)**

(c) Find the value of $t$ when **F** acts due west.

**(2)**

(d) When $t = 0$ , the particle is at the point with position vector (6**i** + 5**j**) m.

Find the position vector, **r** metres, of the particle at time $t$.

**(5)**

**(Total 14 marks)**

**5** A football is kicked from ground level with an initial velocity of 30 m $s^{-1}$ at an angle of 35° above the horizontal.

(a) Find the maximum height of the ball above ground level.

**(4)**

(b) Show that when the speed of the ball is 28 m $s^{-1}$, the magnitude of the vertical component of its velocity is 13.4 m $s^{-1}$, correct to three significant figures.

**(4)**

(c) Find the angle between the velocity of the ball and the horizontal when the speed of the ball is 28 m $s^{-1}$.

**(2)**

**(Total 10 marks)**

**6** When a car, of mass 1200 kg, travels at a speed of $v$ m $s^{-1}$, it experiences a resistance force of magnitude $30v$ newtons.

The car has a maximum constant speed of 48 m $s^{-1}$ on a straight horizontal road.

(a) Show that the maximum power of the car is 69 120 watts.

**(2)**

(b) The car is travelling along a straight horizontal road.

Find the maximum possible acceleration of the car when it is travelling at a speed of 40 m $s^{-1}$.

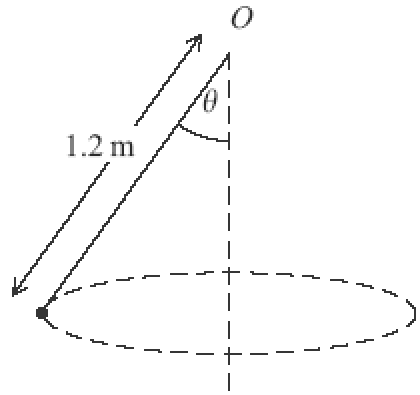
**(4)**

(c) The car starts to descend a hill on a straight road which is inclined at an angle of 3° to the horizontal. Find the maximum possible constant speed of the car as it travels on this road down the hill.

**(7)**

**(Total 13 marks)**

**7** A particle, of mass 4 kg, is attached to one end of a light inextensible string of length 1.2 metres. The other end of the string is attached to a fixed point $O$. The particle moves in a horizontal circle at a constant speed. The angle between the string and the vertical is $\theta$.




(a) Find the radius of the horizontal circle in terms of $\theta$.

**(1)**

(b) The angular speed of the particle is 5 radians per second. Find $\theta$.

**(6)**
**(Total 7 marks)**

# Mark schemes

**1** (a) $\mathbf{r} = \int \mathbf{v} \; dt$

*M1 for at least one term correct*

M1

$= (t^4 - 6t^2 + 3t)\mathbf{i} + 5t\mathbf{j} + 4t^2\mathbf{k} + \mathbf{c}$

*m1 for +* ***c***

A1m1

When $t = 0$, $\mathbf{r} = -5\mathbf{i} + 6\mathbf{k}$ $\therefore$ $\mathbf{c} = -5\mathbf{i} + 6\mathbf{k}$

$\therefore$ $\mathbf{r} = (t^4 - 6t^2 + 3t - 5)\mathbf{i} + 5t\mathbf{j} + (6 + 4t^2)\,\mathbf{k}$

A1

4

(b) $\mathbf{a} = (12t^2 - 12)\mathbf{i} + 8\mathbf{k}$

*M1 for either component*

M1A1

2

(c) Magnitude is $\left\{(12t^2 - 12)^2 + 64\right\}^{\frac{1}{2}}$

M1A1F

2

(d) Magnitude is a minimum when $12t^2 - 12$ is zero

*M1 for correct differentiation of correct expression in (c)*

M1

ie when $t = 1$

A1

2

(e) Minimum acceleration is 8

Using $F = ma$,

*$a$ could be a vector*

M1

$F = 7 \times 8 = 56$

*CAO*

A1

**2**

**[12]**

**2** (a) Symmetry

E1

**1**

(b) Moments about $AB$:

$300\sigma.15 + 100\sigma.5 + 300\sigma.15 = 700\sigma.\ x$

*(condone lack of σ.)*

*M1 needs correct total marks*

$x = \frac{9500}{700}$

M1A1

$= \frac{95}{7}$ or 13.6 cm

A1

**3**

(c) Distance from $HG$ is 16.4 cm

B1

$\tan\theta = \frac{15}{16.42857}$

*Seeing both 15, 16.4 and tan*

M1

= 0.913043

$\theta$ = 42.3974°

A1

$\theta$ = 42°

*[ 48° probably B1, M1]*

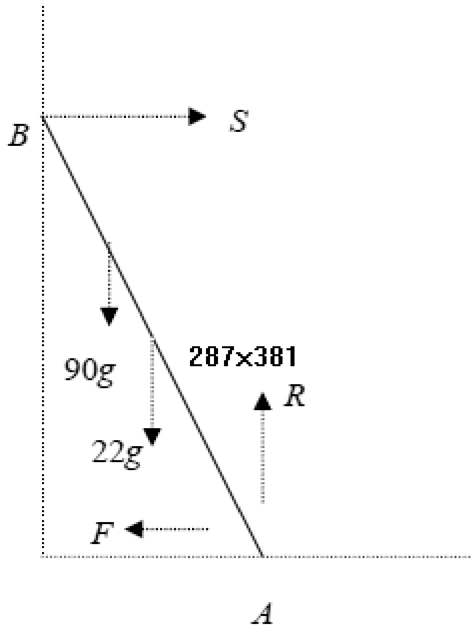
*NB* $\frac{13.6}{15}$ *etc ⇒ 42° no marks*

A1

**4**

**[8]**

**3** (a)




Resolve vertically:

$R = 22g + 90g$

$= 112g$

B1

Using $F = \mu R$:

M1

$F = 0.6R$

$F = 0.6 \times 112g$

*Needs 0.6 × 112g or 0.6 × 1097.6*
*NOT 0.6 × 1097 unless 658.56 seen*

A1

$= 67.2g$ or 658.56

$F = 659$ N

*AG (659 must be shown from correct working)*

A1

**4**

(b) Resolve horizontally: $S = F$

B1

Moments about $A$:

$90g \times 5 \times \cos\theta + 22g \times 3 \times \cos\theta$

*M1 (one term, force × distance × cos or sin)*

M1A1

$= 67.2g \times 6 \times \sin\theta$

A1

$450g + 66g = 403.2g \tan\theta$

$\tan\theta = \frac{516}{403.2}$

$\theta = 52.0°$

*accept 52*

***Alternative**: or moments about $B$:*

*M1 A2, 1 or 0 for four-term moment equation*

*+ M1 for rearranging etc (dep on 4 term)*

*+ A1 for answer*

A1

**5**

**[9]**

**4**

(a) $\mathbf{a} = \frac{dv}{dt}$

$\mathbf{a} = -8e^{-2t}\,\mathbf{i} + (6 - 6t)\mathbf{j}$

*M1: Differentiating with either of the two components correct. Do not need to see* **i** *or* **j**.

*A1: Correct* **i** *component.*

*A1: Correct* **j** *component.*

M1
A1
A1

**3**

(b) (i) Using $\mathbf{F} = m\mathbf{a}$

$\mathbf{F} = 5 \times \{-8e^{-2t}\,\mathbf{i} + (6 - 6t)\mathbf{j}\}$

$= -40e^{-2t}\,\mathbf{i} + (30 - 30t)\mathbf{j}$

*M1: Multiplying their acceleration by 5, even if not a vector.*

*A1: Correct expression.*

M1
A1
**2**

(ii) Magnitude of $\mathbf{F}$ is

$\{(-40)^2 + (30)^2\}^{\frac{1}{2}}$

*M1: Finding magnitude from two non-zero terms. Must add terms and square root. Condone* $\{(40)^2 + (30)^2\}^{\frac{1}{2}}$

M1

$= 50$

*A1: Correct answer only.*

*In this part, condone lack of negative signs in expression for force in (b) (i).*

A1
**2**

(c) When $\mathbf{F}$ acts due west, $\mathbf{j}$ component is zero

$30 - 30t = 0$

*M1: Putting* $\mathbf{j}$ *component equal to zero.*

M1

$t = 1$

*A1: Correct time.*

A1
**2**

(d) $\mathbf{r} = -2e^{-2t}\,\mathbf{i} + (3t^2 - t^3)\mathbf{j} + \mathbf{c}$

*M1: Integration with either of the two components correct.*
*Do not need to see* $\mathbf{i}$ *or* $\mathbf{j}$.

*A1: Correct* $\mathbf{i}$ *component.*

*A1: Correct* $\mathbf{j}$ *component.*

*Condone lack of +* $\mathbf{c}$

M1
A1
A1

When $t = 0$, $\mathbf{r} = 6\mathbf{i} + 5\mathbf{j}$ $\therefore \mathbf{c} = 8\mathbf{i} + 5\mathbf{j}$

*dM1: Finding* $\mathbf{c}$ *using* $6\mathbf{i} + 5\mathbf{j}$ *and* $e^0 = 1$.

dM1

$\therefore \mathbf{r} = (8 - 2e^{-2t})\mathbf{i} + (5 + \frac{1}{4}t^4 - t^3)\mathbf{j}$

*A1: Correct position vector.*

A1

**5**

**[14]**

**5**

(a) $0^2 = (30 \sin 35°)^2 + 2 \times (-9.8)s$

*M1: Equation to fine the max height, with = 0*

M1

*A1: Correct equation*

A1

*M1: Solving for the height*

M1

$$s = \frac{(30 \sin 35°)^2}{2 \times 9.8} = 15.1 \text{ m}$$

*A1: Correct height*

A1

**4**

(b) $28^2 = (30 \cos 35°)^2 + v_y^2$

*M1: Equation to find vertical component*

M1

*A1: Correct equation*

A1

$v_y = \sqrt{28^2 - (30\cos 35°)^2}$

*M1: Solving equation*

M1

$= 13.4198 = 13.4$ m s$^{-1}$ **AG**

*A1: Correct speed from correct working.*

A1

**4**

(c) $\tan\theta = \dfrac{13.4}{30\cos 35°}$

*M1: Expression to find the angle.*

M1

$\theta = 28.6°$

*A1: Correct angle.*

A1

**2**

**[10]**

**6** (a) Using power = force × velocity

Power = (30 × 48) × 48

M1

= 69120 watts

*AG*

A1

**2**

(b) When speed is 40 m $s^{-1}$,

max force exerted is $\frac{69120}{40}$

= 1728 N

B1

Accelerating force is ‘1728’ – 1200 N

M1

Using $F = ma$:

528 = 1200$a$

m1

$a$ = 0.44 m $s^{-2}$

A1

**4**

(c) Force exerted by engine is $\frac{69120}{v}$

B1

Force exerted by the engine
= 30$v$ – $mg$ sin 3

*(Use of cos3 delete A1, A1 of 3 A terms)*

M1

$$30v - 615.47(\text{or } 1200g\sin 3) = \frac{69120}{v}$$

*A2 All terms correct*
*A1 Two terms correct*

A1A1

$30v^2 - 615.47v - 69120 = 0$

*SC3 for* $30v^2 + 615.47v - 69120 = 0$

A1

$$v = \frac{615.47 \pm \sqrt{615.47^2 + 4 \times 30 \times 69120}}{2 \times 30}$$

M1

Speed is 59.3 m $s^{-1}$

A1

**7**

**[13]**

**7** (a) $r$ = 1.2 sin $\theta$

*1.2 cos $\theta$ 0 marks*

B1

**1**

(b) Resolve horiz: $T \sin \theta = m\omega^2 r$

*$T \cos \theta = m\omega^2 r$ etc M1 (+ second M1)*

M1A1

$T \sin \theta = 4 \times 5^2 \times 1.2 \sin \theta$

$T = 120$

A1

Resolve vert: $T \cos \theta = 4g$

*M1 for tan* $\theta = \dfrac{30 \sin\theta}{g}$

M1A1

$\cos \theta = 0.32666$

$\theta = 70.9°$ or $1.24^c$

A1

**6**

**[7]**