



**Topic Test: OxfordAQA A Level  
Mathematics**  
Pure Mathematics

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Date: \_\_\_\_\_

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Time: **140 minutes**

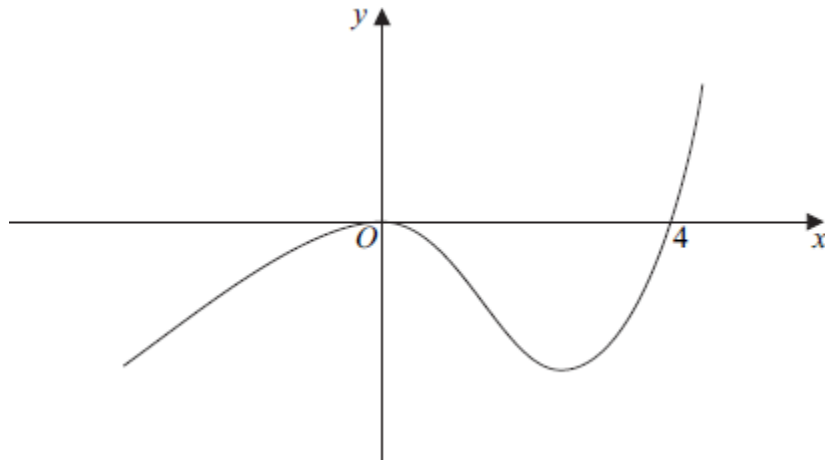
Marks: **118 marks**

Comments:

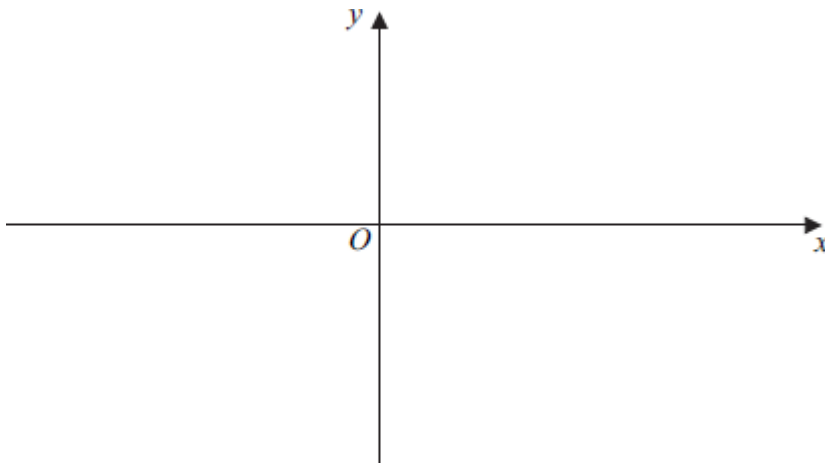
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**1**

The diagram shows a sketch of the curve with equation  $y = f(x)$ .



(a) On the axes below, sketch the curve with equation  $y = |f(x)|$ .

**(2)**

(b) Describe a sequence of two geometrical transformations that maps the graph of  $y = f(x)$  onto the graph of  $y = f(2x - 1)$ .

**(4)****(Total 6 marks)****2**

The polynomial  $f(x)$  is defined by  $f(x) = 2x^3 + x^2 - 8x - 7$ .

(a) Use the Remainder Theorem to find the remainder when  $f(x)$  is divided by  $(2x + 1)$ .

**(2)**

(b) The polynomial  $g(x)$  is defined by  $g(x) = f(x) + d$ , where  $d$  is a constant.

(i) Given that  $(2x + 1)$  is a factor of  $g(x)$ , show that  $g(x) = 2x^3 + x^2 - 8x - 4$ .

**(1)**

(ii) Given that  $g(x)$  can be written as  $g(x) = (2x + 1)(x^2 + a)$ , where  $a$  is an integer, express  $g(x)$  as a product of three linear factors.

**(1)**

(iii) Hence, or otherwise, show that  $\frac{g(x)}{2x^3 - 3x^2 - 2x} = p + \frac{q}{x}$ , where  $p$  and  $q$  are integers.

(3)

(Total 7 marks)

3

It is given that  $f(x) = \frac{7x - 1}{(1 + 3x)(3 - x)}$

(a) Express  $f(x)$  in the form  $\frac{A}{3 - x} + \frac{B}{1 + 3x}$ , where  $A$  and  $B$  are integers.

(3)

(b) (i) Find the first three terms of the binomial expansion of  $f(x)$  in the form  $a + bx + cx^2$ , where  $a$ ,  $b$  and  $c$  are rational numbers.

(7)

(ii) State why the binomial expansion cannot be expected to give a good approximation to  $f(x)$  at  $x = 0.4$ .

(1)

(Total 11 marks)

4

A curve is defined by the parametric equations

$$x = 8t^2 - t, \quad y = \frac{3}{t}$$

(a) Show that the cartesian equation of the curve can be written as  $xy^2 + 3y = k$ , stating the value of the integer  $k$ .

(2)

(b) (i) Find an equation of the tangent to the curve at the point  $P$ , where  $t = \frac{1}{4}$ .

(7)

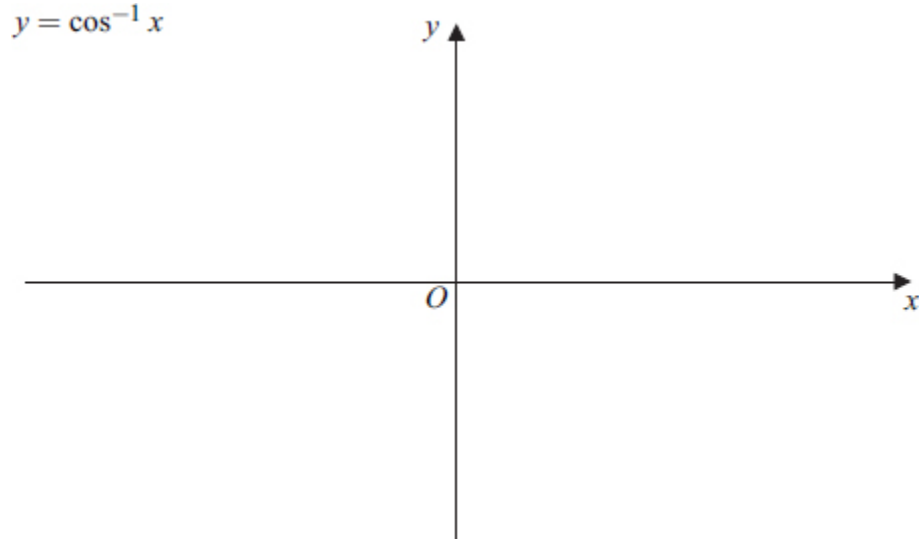
(ii) Verify that the tangent at  $P$  intersects the curve when  $x = \frac{3}{2}$ .

(2)

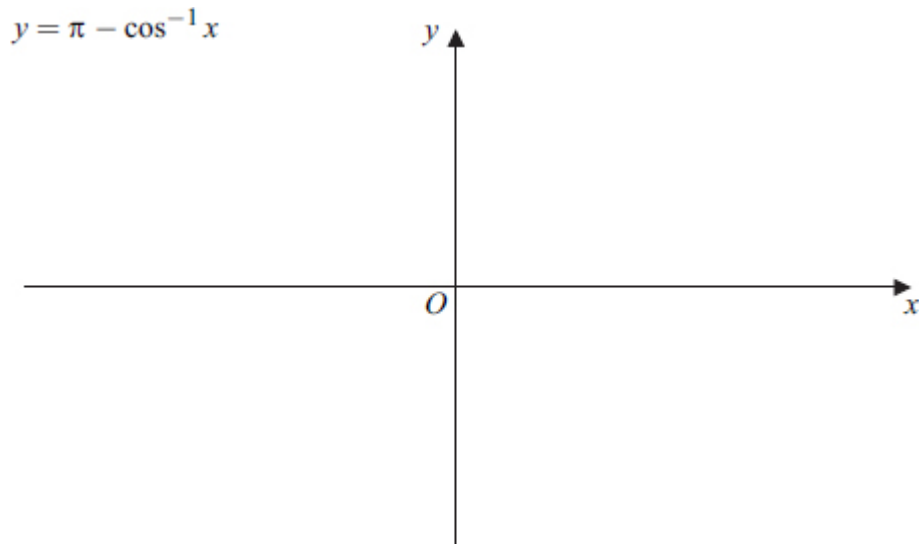
(Total 11 marks)

**5**

- (a) Sketch the graph of  $y = \cos^{-1} x$ , where  $y$  is in radians. State the coordinates of the end points of the graph.

**(2)**

- (b) Sketch the graph of  $y = \pi - \cos^{-1} x$ , where  $y$  is in radians. State the coordinates of the end points of the graph.

**(2)****(Total 4 marks)****6**

- (a) Express  $3 \cos x + 2 \sin x$  in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ , giving your value of  $\alpha$  to the nearest  $0.1^\circ$ .

**(3)**

- (ii) Hence find the minimum value of  $3 \cos x + 2 \sin x$  and the value of  $x$  in the interval  $0^\circ < x < 360^\circ$  where the minimum occurs. Give your value of  $x$  to the nearest  $0.1^\circ$ .

**(3)**

- (b) (i) Show that  $\cot x - \sin 2x = \cot x \cos 2x$  for  $0^\circ < x < 180^\circ$ .

**(3)**

- (ii) Hence, or otherwise, solve the equation

$$\cot x - \sin 2x = 0$$

in the interval  $0^\circ < x < 180^\circ$ .

(3)

(Total 12 marks)

7

A scientist is testing models for the growth and decay of colonies of bacteria.

For a particular colony, which is growing, the model is  $P = Ae^{\frac{1}{8}t}$ , where  $P$  is the number of bacteria after a time  $t$  minutes and  $A$  is a constant.

- (a) This growing colony consists initially of 500 bacteria. Calculate the number of bacteria, according to the model, after one hour. Give your answer to the nearest thousand.

(2)

- (b) For a second colony, which is decaying, the model is  $Q = 500\,000e^{-\frac{1}{8}t}$ , where  $Q$  is the number of bacteria after a time  $t$  minutes.

Initially, the growing colony has 500 bacteria and, at the same time, the decaying colony has 500 000 bacteria.

- (i) Find the time at which the populations of the two colonies will be equal, giving your answer to the nearest 0.1 of a minute.

(3)

- (ii) The population of the growing colony will exceed that of the decaying colony by 45 000 bacteria at time  $T$  minutes.

Show that

$$\left(e^{\frac{1}{8}T}\right)^2 - 90e^{\frac{1}{8}T} - 1000 = 0$$

and hence find the value of  $T$ , giving your answer to one decimal place.

(4)

(Total 9 marks)

8

- (a) Find  $\frac{dy}{dx}$  when

$$y = e^{3x} + \ln x$$

(2)

- (b) (i) Given that  $u = \frac{\sin x}{1 + \cos x}$ , show that  $\frac{du}{dx} = \frac{1}{1 + \cos x}$ .

(3)

(ii) Hence show that if  $y = \ln \left( \frac{\sin x}{1 + \cos x} \right)$ , then  $\frac{dy}{dx} = \operatorname{cosec} x$ .

(2)

(Total 7 marks)

9

A curve has equation  $y = x^3 \ln x$ .

(a) Find  $\frac{dy}{dx}$ .

(2)

(b) (i) Find an equation of the tangent to the curve  $y = x^3 \ln x$  at the point on the curve where  $x = e$ .

(3)

(ii) This tangent intersects the  $x$ -axis at the point  $A$ . Find the exact value of the  $x$ -coordinate of the point  $A$ .

(2)

(Total 7 marks)

10

(a) (i) By writing  $\ln x$  as  $(\ln x) \times 1$ , use integration by parts to find  $\int \ln x \, dx$ .

(4)

(ii) Find  $\int (\ln x)^2 \, dx$ .

(4)

(b) Use the substitution  $u = \sqrt{x}$  to find the exact value of  $\int_1^4 \frac{1}{x + \sqrt{x}} \, dx$ .

(7)

(Total 15 marks)

11

Solve the differential equation

$$\frac{dy}{dx} = y^2 x \sin 3x$$

given that  $y = 1$  when  $x = \frac{\pi}{6}$ . Give your answer in the form  $y = \frac{9}{f(x)}$ .

(Total 9 marks)

12

(a) Use Simpson's rule with 7 ordinates (6 strips) to find an estimate for

$$\int_0^3 4^x \, dx.$$

(4)

(b) A curve is defined by the equation  $y = 4^x$ . The curve intersects the line  $y = 8 - 2x$  at a single point where  $x = \alpha$ .

(i) Show that  $\alpha$  lies between 1.2 and 1.3.

(2)

(ii) The equation  $4^x = 8 - 2x$  can be rearranged into the form  $x = \frac{\ln(8 - 2x)}{\ln 4}$ .

Use the iterative formula  $x_{n+1} = \frac{\ln(8 - 2x_n)}{\ln 4}$  with  $x_1 = 1.2$  to find the values

of  $x_2$  and  $x_3$ , giving your answers to three decimal places.

(2)

(Total 8 marks)

13

The points  $A$  and  $B$  have coordinates  $(4, -2, 3)$  and  $(2, 0, -1)$  respectively.

The line  $l$  passes through  $A$  and has equation  $\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$ .

(a) (i) Find the vector  $\overrightarrow{AB}$ .

(2)

(ii) Find the acute angle between  $AB$  and the line  $l$ , giving your answer to the nearest degree.

(4)

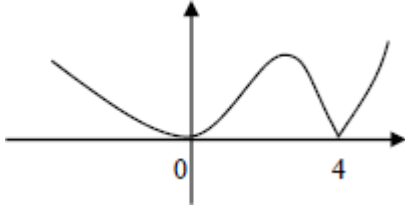
(b) The point  $C$  lies on the line  $l$  such that the angle  $ABC$  is a right angle. Given that  $ABCD$  is a rectangle, find the coordinates of the point  $D$ .

(6)

(Total 12 marks)

Mark schemes

**1** (a)



*reflection in the  $x$ -axis for the negative  $f(x)$  and remainder as given on sketch*

M1

*correct curvatures, correct cusp at  $x = 4$   
condone straight lines for  $x < 0$  and  $x > 4$   
4 marked on  $x$ -axis*

A1

2

(b) **Either**

1. Stretch

*1 and either 2 or 3*

M1

2.  $\parallel x$ -axis

3. by factor 0.5

*1, 2 and 3*

A1

(followed by) translation

E1

$$\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

B1

4

or  
translation

(E1)

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(B1)

(followed by) 1. Stretch  
1 and either 2 or 3

(M1)

2. || x-axis

3. by factor 0.5  
1, 2 and 3

(A1)

(4)

[6]

2

(a)  $f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) - 7$

Evaluate  $f\left(-\frac{1}{2}\right)$ , not long division.

M1

$$= -3$$

A1

2

(b) (i)  $g\left(-\frac{1}{2}\right) = 0 \Rightarrow -3 + d = 0$

$$\text{Or } \left(-\frac{1}{2}\right) + d = 0$$

$$d = 3 \Rightarrow g(x) = 2x^3 + 2x^2 - 8x - 7 + 3$$

$$g(x) = 2x^3 + 2x^2 - 8x - 4$$

All steps seen with conclusion

**AG**

B1

Allow verification with  $-\frac{1}{4} + \frac{1}{4} + 4 - 4 = 0$  seen,  
and conclusion; therefore factor

1

(ii)  $g(x) = 2x^3 + 2x^2 - 8x - 4 = (2x + 1)(x^2 - 4) = (2x + 1)(x + 2)(x - 2)$   
 $a = -4$

B1

1

(iii)  $2x^3 - 3x^2 - 2x = x(2x + 1)(x - 2)$

*Clear attempt to factorise denominator; 3 factors needed.*

M1

$$\frac{(2x+1)(x+2)(x-2)}{x(2x+1)(x-2)} = \frac{x+2}{x}$$

*At least one correct factor cancelled*

m1

$$\frac{g(x)}{2x^3 - 3x^2 - 2x} = 1 + \frac{2}{x}$$

*CSO part (a)(iii) NMS is 0 / 3*

A1

3

**Alternative:**

$$\frac{g(x)}{2x^3 - 3x^2 - 2x} = 1 + \frac{4x^2 - 6x - 4}{2x^3 - 3x^2 - 2x}$$

$$1 + \frac{\text{quadratic}}{2x^3 - 3x^2 - 2x}$$

(M1)

$$= 1 + \frac{2(2x^2 - 3x - 2)}{2x^3 - 3x^2 - 2x}$$

(A1)

$$= 1 + \frac{2}{x}$$

(A1)

(3)

[7]

3

(a)  $7x - 1 = A(1 + 3x) + B(3 - x)$

M1

$$x = 3 \quad x = -\frac{1}{3}$$

Use two values of  $x$  to find  $A$  **and**  $B$ .

Or solve  $A + 3B = -1$   $3A - B = 7$

Or cover up rule

m1

$$A = 2 \quad B = -1$$

A1

3

(b) (i) 
$$\frac{1}{1+3x} = (1+3x)^{-1} = 1 + (-1)3x + \frac{1}{2}(-1)(-2)(3x)^2$$

Condone missing brackets

M1

$$= 1 - 3x + 9x^2$$

A1

$$\frac{1}{3-x} = (3-x)^{-1} = \frac{1}{3} \left( 1 - \frac{x}{3} \right)^{-1}$$

B1

$$\left( 1 - \frac{x}{3} \right)^{-1} = 1 + (-1) \left( -\frac{x}{3} \right) + kx^2$$

Condone missing brackets

M1

$$= 1 + \frac{x}{3} + \frac{x^2}{9}$$

A1

$$\frac{7x-1}{3+8x-3x^2} = 2 \times \frac{1}{3} \times \left( 1 + \frac{x}{3} + \frac{x^2}{9} \right) - 1 \times (1-3x+9x^2)$$

Attempt to use PFs to combine expansions, or expand

$(7x-1)(3-x)^{-1}(1+3x)^{-1}$

and simplify to  $a + bx + cx^2$

M1

$$= -\frac{1}{3} + \frac{29}{9}x - \frac{241}{27}x^2$$

A1

7

(ii) 0.4 is outside the range of validity, because  $0.4 > \frac{1}{3}$ .

OE Accept  $0.4 > \frac{1}{3}$

B1

1

[11]

**4** (a)  $xy^2 + 3y = (8t^2 - t)\left(\frac{3}{t}\right)^2 + 3\left(\frac{3}{t}\right)$   
*Substitute and expand*

M1

$$= 72 - \frac{9}{t} + \frac{9}{t} = 72$$

$$k = 72$$

A1

2

**Alternative**

$$x = 8\left(\frac{3}{y}\right)^2 - \frac{3}{y}$$

*Eliminate t*

(M1)

$$xy^2 + 3y = 72$$

$$k = 72$$

(A1)

(2)

(b) (i)  $\frac{dx}{dt} = 16t - 1$   $\frac{dy}{dt} = -\frac{3}{t^2}$

B1B1

$$t = \frac{1}{4} \quad \frac{dy}{dx} = \frac{-\frac{3}{(\frac{1}{4})^2}}{16 \times \frac{1}{4} - 1}$$

Use chain rule  $\left(\frac{dy}{dx} = \frac{-3}{16t^3 - t^2}\right)$  and calculate gradient

using  $t = \frac{1}{4}$

M1

$$= -16$$

A1

$$t = \frac{1}{4} \quad x = \frac{8}{16} - \frac{1}{4} \quad y = \frac{3}{\frac{1}{4}}$$

Calculate  $x$  and  $y$  using  $t = \frac{1}{4}$

M1

$$x = \frac{1}{4} \quad y = 12$$

Both correct

A1

tangent  $y = -16x + 16$

ACF CSO

$$y - 12 = -16\left(x - \frac{1}{4}\right) \text{ ISW}$$

A1

7

**Alternative**

$$2xy \frac{dy}{dx} + y^2$$

Product rule attempted; two terms added, one with  $\frac{dy}{dx}$

(M1A1)

$$+ 3 \frac{dy}{dx} = 0$$

(B1)

$$t = \frac{1}{4} \quad x = \frac{8}{16} - \frac{1}{4} \quad y = \frac{3}{\frac{1}{4}}$$

Calculate  $x$  and  $y$  using  $t = \frac{1}{4}$

(M1)

$$x = \frac{1}{4} \quad y = 12$$

Both correct.

(A1)

$$\left( \frac{dy}{dx} = \frac{-y^2}{2xy + 3} \right) \frac{dy}{dx} = -16$$

Calculate gradient from candidate's expression.

m1

$$\text{tangent } y = -16x + 16$$

ACF CSO

$$y - 12 = -16 \left( x - \frac{1}{4} \right) \text{ ISW}$$

(A1)

(7)

### Alternative

$$x = \frac{72 - 3y}{y^2}$$

Correct expression for  $x$  from candidate's implicit equation.

Quotient rule attempted;  $y^4$  and two terms subtracted.

(M1)

$$\frac{dx}{dy} = \frac{y^2(-3) - (72 - 3y) \times 2y}{y^4}$$

(A1)

$$\left( \frac{dx}{dy} = \frac{3y - 144}{y^3} \right)$$

Numerator; first term; second term

(A1)

$$t = \frac{1}{4} \quad y = \frac{3}{\frac{1}{4}} = 12$$

(B1)

$$\frac{dx}{dy} = -\frac{1}{16} \quad \frac{dy}{dx} = -16$$

Use  $t = \frac{1}{4}$  to calculate  $y$

m1

$$x = \frac{1}{4} \quad x = \frac{8}{16} - \frac{1}{4} = \frac{1}{4}$$

Evaluate and invert.

(B1)

$$y = -16x + 16$$

(A1)

(7)

**Alternative** for  $\frac{dx}{dy}$

Use  $t = \frac{1}{4}$  to calculate  $x$

$$x = \frac{72}{y^2} - \frac{3}{y}$$

ACF CSO

(M1)

$$\frac{dx}{dy} = -\frac{144}{y^3} + \frac{3}{y^2}$$

(A1)

$$\left( \frac{dx}{dy} = \frac{3y - 144}{y^3} \right)$$

Correct expression for  $x$  from candidate's implicit equation and attempt derivatives

(A1)

$$(ii) \quad y = -16 \times \frac{3}{2} + 16 = -8$$

Substitute  $x = \frac{3}{2}$  into candidate's tangent; calculate  $y$

M1

$$\frac{3}{2}(-8)^2 + 3 \times (-8) = 96 - 24 = 72$$

$y = -8$  used to verify 72

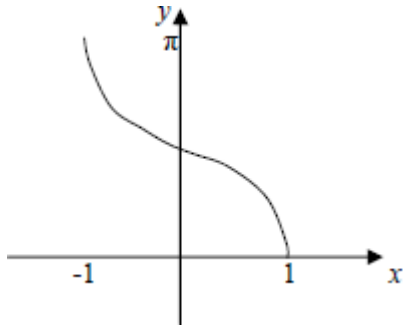
A1

2

[11]

5

(a)



Correct sketch of  $\cos^{-1} x$ .

B1

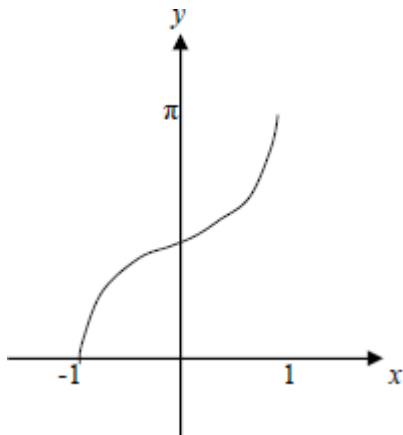
$(-1, \pi)$  and  $(1, 0)$

Stated

B1

2

(b)



Correct sketch of  $\pi - \cos^{-1} x$   
Must touch negative  $x$ -axis.

B1

$(-1, 0)$  and  $(1, \pi)$

Stated

B1

2

[4]

6

(a) (i)  $R = \sqrt{13}$   
 Accept 3.6 or better

B1

$$\tan \alpha = \frac{2}{3}$$

OE

M1

$$\alpha = 33.7^\circ$$

A1

3

(ii) minimum value =  $-\sqrt{13}$   
 Accept - 3.6 or better; ft R

B1ft

when  $x - \alpha = \cos^{-1}(-1)$   
 NMS 0 / 2

M1

$$x = 213.7^\circ$$

Calculus used 0 / 2

A1

3

(b) (i)  $\text{LHS} = \frac{\cos x}{\sin x} - 2 \sin x \cos x$   
 Express  $\cot x - \sin 2x$  in terms of  $\sin x$  and  $\cos x$ ; ACF

M1

$$= \frac{\cos x}{\sin x} (1 - 2 \sin^2 x)$$

Factor out  $\frac{\cos x}{\sin x}$  and  $1 - 2 \sin^2 x$

m1

$$= \cot x \cos 2x$$

All correct

A1

3

**Alternatives:**

$$\text{RHS} = \cot x \cos 2x$$

$$= \frac{\cos x}{\sin x}(1 - 2\sin^2 x)$$

*Express  $\cot x \cos 2x$  in terms of  $\cos x$  and  $\sin x$ ,  
 $\cos 2x$  ACF*

(M1)

$$= \frac{\cos x}{\sin x} - 2\sin x \cos x$$

*$\cos 2x = 1 - 2\sin^2 x$  and multiply out and simplify.*

(m1)

$$= \cot x - \sin 2x$$

*All correct.*

(A1)

(3)

$$\cot x(1 - \cos 2x) - \sin 2x = 0$$

$$\frac{\cos x}{\sin x}(1 - (1 - 2\sin^2 x)) - 2\sin x \cos x = 0$$

*Rearrange to expression = 0 and factor out  $\cot x$ ;*

*Express  $\cot x$ ,  $\cos 2x$  and  $\sin 2x$  in terms of  $\sin x$  and  
 $\cos x$ , ACF*

(M1)

$$\frac{\cos x}{\sin x}(2\sin^2 x) - 2\sin x \cos x = 0$$

*$\cos 2x = 1 - 2\sin^2 x$  used*

(m1)

$$2\sin x \cos x - 2\sin x \cos x = 0$$

*Simplified, with all correct*

(A1)

(3)

(ii)  $\cot x - \sin 2x = 0$

$$\cot x \cos 2x = 0$$

$$\cot x = 0 \quad \text{or} \quad \cos 2x = 0$$

*Both equations correct*

M1

$$2x = 90^\circ \text{ (270}^\circ\text{)}$$

*Condone missing 270°*

m1

$$x = 90^\circ, 45^\circ, 135^\circ$$

*All correct*

A1

3

**Alternatives:**

$$\cot x - \sin 2x = \frac{\cos x}{\sin x} - 2\sin x \cos x = 0$$

$$\cos x \left( \frac{1}{\sin x} - 2\sin x \right) = 0$$

$$\cos x = 0 \quad \text{or} \quad 1 - 2\sin^2 x = 0$$

*Both equations*

(M1)

$$\sin x = (\pm) \frac{1}{\sqrt{2}}$$

(m1)

$$x = 90^\circ, 45^\circ, 135^\circ$$

(A1)

(3)

[12]

**7** (a)  $P = 500e^{\frac{1}{8} \times 60}$

*Must use  $t = 60$*

M1

= 904 000

*Nearest thousand required 904000 only*

A1

2

(b) (i)  $\left(e^{\frac{1}{8}t}\right)^2 = \frac{500000}{500}$

M1

$$t = 8 \ln\sqrt{1000}$$

*OE Take logs correctly leading to expression for t.*

M1

$$t = 27.6 \text{ (minutes)}$$

*Accept 27.631*

A1

3

**Alternative**

$$e^{\frac{1}{8}t} = 1000e^{-\frac{1}{8}t} \Rightarrow e^{\frac{1}{4}t} = \frac{500000}{500}$$

(M1)

$$t = 4 \ln 1000$$

*Take logs correctly leading to expression for t.*

(M1)

$$t = 27.6 \text{ (minutes)}$$

(A1)

(3)

**Alternative**

$$e^{\frac{1}{8}t} = 1000e^{-\frac{1}{8}t} \Rightarrow \ln\left(e^{\frac{1}{8}t}\right) = \ln 1000 + \ln\left(e^{-\frac{1}{8}t}\right)$$

*Take logs correctly.*

(M1)

$$t = 4 \ln 1000$$

(M1)

$$t = 27.6 \text{ (minutes)}$$

(A1)

27.6 as final answer NMS 3/3

27.6 following wrong working AO (FIW) but could still score M mark(s)

(3)

$$(ii) \quad 500e^{\frac{1}{8}t} - 500000e^{\frac{1}{8}t} = 45000$$

Set up equation; condone one error; allow in  $t$ .

Condone inequality.

M1

$$\times \frac{e^{\frac{1}{8}t}}{500} \Rightarrow \left( e^{\frac{1}{8}t} \right)^2 - 1000 = 90e^{\frac{1}{8}t}$$

$$\left( e^{\frac{1}{8}t} \right)^2 - 90e^{\frac{1}{8}t} - 1000 = 0$$

Multiply by  $\frac{e^{\frac{1}{8}t}}{500}$  and rearrange to AG, be convinced.

A1

$$e^{\frac{1}{8}t} = 100 \quad (e^{\frac{1}{8}t} = -10 \text{ rejected})$$

Solve quadratic equation (retaining positive root).

M1

$$t = 36.8 \text{ (minutes)}$$

CAO

A1

M1 for solve quadratic equation. Let  $x = e^{\frac{1}{8}t}$  solve quadratic equation

$x^2 - 90x - 1000 = 0$  by inspection,  $x = 100$  seen;

factors  $(x - 100)(x + 10)$  with 100 and 10 seen;

complete square  $x = 45 \pm \sqrt{3025}$  all correct.

formula  $\frac{90 \pm \sqrt{90^2 + 4000}}{2}$  all correct.

Final answer must have  $t = 36.8$  for A1

4

[9]

8

(a)  $\left(\frac{dy}{dx} = 3e^{3x} + \frac{1}{x}\right)$

B1 for one term correct

B1

B1 all correct

B1

2

(b) (i)  $\left(\frac{du}{dx} = \frac{\pm \cos x(1 + \cos x) \pm \sin x(\sin x)}{(1 + \cos x)^2}\right)$

clear attempt at quotient / product rule  
condone poor use of brackets

M1

$$\frac{\cos x(1 + \cos x) - \sin x(-\sin x)}{(1 + \cos x)^2}$$

any correct form seen

A1

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2} = \frac{\cos x + 1}{(1 + \cos x)^2} = \frac{1}{1 + \cos x}$$

AG be convinced

correct use of brackets and correct notation used  
throughout (eg A0 if  $\cos x^2$  etc seen)

A1cso

3

(ii)  $\left(\frac{dy}{dx} = \frac{1 + \cos x}{\sin x} \times \frac{1}{1 + \cos x}\right)$  OE

correct use of chain rule

M1

$$= \frac{1}{\sin x}$$

= cosec x

AG, must see  $= \frac{1}{\sin x}$  and no errors seen;

condone incorrect use of brackets only if penalised in part (b)(i)

A1

2

[7]

9

(a)  $\left(\frac{dy}{dx} =\right) x^3 \times \frac{1}{x} + 3x^2 \ln x$

$px^3 \times \frac{1}{x} + qx^2 \ln x$

where  $p$  and  $q$  are integers

$p = 1, q = 3$

M1

A1

2

(b) (i)  $\left(\frac{dy}{dx} =\right) e^2 + 3e^2 \ln e \quad (= 4e^2)$

Substituting  $e$  for  $x$  in their  $\frac{dy}{dx}$ , but must have scored M1 in (a)

M1

$y = e^3 \ln e (= e^3)$

B1

$y - e^3 = 4e^2 (x - e)$

OE **but** must have evaluated  $\ln e$  (twice) for this mark (must be in exact form, but condone numerical evaluation after correct equation)

A1

3

(ii)  $-e^3 = 4e^2 (x - e)$  or  $4e^2 x = 3e^3$  OE

Correctly substituting  $y = 0$  into a correct tangent equation in (b)(i)

M1

$x = \frac{3}{4} e$

CSO;

ignore subsequent decimal evaluation

A1

2

[7]

10

(a) (i)

$$\left. \begin{array}{l} u = \ln x \quad \frac{dv}{dx} = 1 \\ \frac{du}{dx} = \frac{1}{x} \quad v = x \end{array} \right\}$$

$$\frac{d \ln x}{dx} \& \int dx \text{ attempted}$$

M1

All correct

A1

$$\left( \int \ln x \, dx = \right) x \ln x - \int x \times \frac{1}{x} (dx)$$

Correct substitution of their terms into parts

m1

$$= x \ln x - x + C$$

All correct (**constant needed**)

A1

4

(ii)

$$\left. \begin{array}{l} u = (\ln x)^2 \quad \frac{dv}{dx} = 1 \\ \frac{du}{dx} = (2 \ln x) \frac{1}{x} \quad v = x \end{array} \right\}$$

$$\frac{d(\ln x)^2}{dx} \& \int dx \text{ attempted}$$

M1

All correct

A1

$$\left( \int (\ln x)^2 \, dx = \right) x(\ln x)^2 - \int x \times \frac{2}{x} \ln x (dx)$$

OE correct substitution of their terms into parts

m1

$$= x(\ln x)^2 - 2(x \ln x - x) + C \quad \text{OE}$$

All correct (constant needed) including correct use of brackets. **Do not penalise missing constant if already penalised in part (i)**

ISW

A1

4

$$(b) \quad \frac{du}{dx} = \frac{1}{2\sqrt{x}} \quad \text{or} \quad \frac{1}{2}x^{-\frac{1}{2}}$$

$$(dx = 2u \, du)$$

$$u = \sqrt{x}$$

B1

$$\int_{(1)}^{(4)} \frac{1}{x + \sqrt{x}} \, dx = \int_{(1)}^{(2)} \frac{1}{u^2 + u} 2u \, (du)$$

All in terms of  $u$  including attempt at replacing  $dx$  (not simply writing  $du$ ), condone missing limits and  $du$

M1

Integrand correct unsimplified

A1

$$= 2 \int_{(1)}^{(2)} \frac{1}{u+1} \, (du)$$

A1

$$= 2 \ln(u+1) \Big|_{(1)}^{(2)}$$

$$\text{FT their } \int \frac{k}{u+1} \, (du)$$

A1F

$$= 2 \ln(2+1) - 2 \ln(1+1) \quad \text{or} \quad 2 \ln(\sqrt{4}+1) - 2 \ln(\sqrt{1}+1)$$

correct use of correct limits on  $k \ln(u+1)$  or  $k \ln(\sqrt{x}+1)$

A1F

$$= 2 \ln \frac{3}{2} \quad \text{or} \quad \ln \frac{9}{4} \quad \text{or} \quad 2 \ln 3 - 2 \ln 2$$

OE ISW

A1

7

[15]

11

$$\int \frac{dy}{y^2} = \int x \sin 3x \, dx$$

Correct separation and notation;  
condone missing integral signs

B1

$$\int \frac{dy}{y^2} = -\frac{1}{y}$$

B1

$$\int x \sin 3x \, dx = x \left( -\frac{1}{3} \cos 3x \right)$$

$$u = x \quad \frac{dv}{dx} = \sin 3x$$

Use parts  $\frac{du}{dx} = 1$   $v = k \cos 3x$  with correct substitution into  
formula

$$-\int -\frac{1}{3} \cos 3x$$

A1

$$= -\frac{1}{3} x \cos 3x \, dx + \frac{1}{9} \sin 3x$$

CAO

A1

$$-\frac{1}{y} = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C$$

$$-1 = -\frac{1}{3} \times \frac{\pi}{6} \cos \left( \frac{\pi}{2} \right) + \frac{1}{9} \sin \left( \frac{\pi}{2} \right) + C$$

Use  $x = \frac{\pi}{6}$   $y = 1$  to find  $C$

M1

$$C = -\frac{10}{9}$$

CAO

A1

$$-\frac{1}{y} = -\frac{1}{9} (3x \cos 3x \, dx - \sin 3x + 10)$$

And invert to  $-y = -\frac{9}{(\dots)}$

m1

$$y = \frac{9}{3x \cos 3x - \sin 3x + 10}$$

CSO, condone first B1 not given

A1

Second M1 finding  $C$ ; substitute  $x = \frac{\pi}{6}$   $y = 1$  into  $f(y) = px \cos 3x + q \sin 3x + C$  and evaluate using radians.

Must calculate a value of  $C$ .

m1 for reaching form  $\pm \frac{k}{y} = \frac{1}{9}(Px \cos 3x + Q \sin 3x + R)$  where  $P$  and  $Q$  are  $\pm 3$  or  $\pm \frac{1}{3}$  or  $\pm 1$  and

inverting to  $\pm \frac{y}{k} = \frac{9}{(Px \cos 3x + Q \sin 3x + R)}$

[9]

**12** (a)

| $x$            | $y$ |
|----------------|-----|
| 0              | 1   |
| $\frac{1}{2}$  | 2   |
| 1              | 4   |
| $1\frac{1}{2}$ | 8   |
| 2              | 16  |
| $2\frac{1}{2}$ | 32  |
| 3              | 64  |

all 7x values correct (and no extra)  
(PI by 7 correct y values)

B1

5 or more correct y values, exact  $\left(4^{\frac{1}{2}}, 4^1 \dots\right)$  or evaluated (in table or in formula)

B1

$$A = \frac{1}{3} \times \frac{1}{2} [65 + 4 \times 42 + 2 \times 20]$$

correct substitution of their 7 y-values into Simpson's rule

M1

$$= \frac{91}{2} \text{ or } 45.5 \text{ or } \frac{273}{6}$$

CAO

A1

(b) (i)  $f(x) = 4^x + 2x - 8$  or  $g(x) = 8 - 2x - 4^x$

$$\left. \begin{aligned} f(1.2) &= -0.3 \text{ or } g(1.2) = 0.3 \\ f(1.3) &= 0.7 \text{ or } g(1.3) = -0.7 \end{aligned} \right\}$$

*attempt at evaluating  $f(1.2)$  and  $f(1.3)$*

AWRT  $\pm 0.3$  and  $\pm 0.7$

condone  $f(1.2) < 0$ ,  $f(1.3) > 0$  if  $f$  is defined

**alternative method**

$$\left. \begin{aligned} 4^{1.2} &= 5.3, 8 - 2 \times 1.2 = 5.6 \\ 4^{1.3} &= 6.1, 8 - 2 \times 1.3 = 5. \end{aligned} \right\} \text{ M1}$$

M1

change of sign

$$\therefore 1.2 < a < 1.3$$

( $f(x)$  must be defined and all working correct)

*at 1.2 LHS < RHS*

*at 1.3 LHS > RHS*

$$\therefore 1.2 < a < 1.3 \quad \text{A1}$$

A1

2

(ii)  $(x_2 =) 1.243$

B1

$(x_3 =) 1.232$

*these values only*

B1

2

[8]

13

(a) (i)  $\overline{AB} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} - \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix}$

$\pm (\overline{OB} - \overline{OA})$  implied by two correct components

M1

*Allow as  $(-2, 2, -4)$*

A1

2

$$(ii) \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} \cdot \overline{AB} = -2 + 10 + 8 = 16$$

M1

*ft on  $\overline{AB}$*

A1ft

$$\cos\theta = \frac{16}{\sqrt{24}\sqrt{30}}$$

*Correct formula for  $\cos\theta$  with consistent vectors and correct moduli, in form  $\sqrt{a^2 + b^2 + c^2}$*

M1

$$\theta = 53^\circ$$

*CSO Accept  $53.4^\circ$ ,  $53.40^\circ$*

A1

4

$$(b) \overline{AB} \cdot \overline{BC} = \begin{bmatrix} -2 \\ 2 \\ -4 \end{bmatrix} \cdot \left( \begin{bmatrix} 4+p \\ -2+5p \\ 3-2p \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} \right)$$

*SC B1  $90^\circ$  following  $sp = 0$*

*Set up scalar product.  $\mu = p$  at C. Any letter for p.*

M1

$$\overline{BC} = \begin{bmatrix} 2+p \\ -2+5p \\ 4-2p \end{bmatrix}$$

*Clear attempt to find  $\overline{BC}$  in terms of p.*

*$\overline{BC}$  or  $\overline{CB}$  correct*

B1

$$-4 - 2p - 4 + 10p - 16 + 8p = 0$$

m1

$$16p = 24 \quad p = \frac{3}{2}$$

*Expand scalar product and solve for p;  
(= 0 possibly implied)*

A1

$$\overline{OD} = \overline{OA} + \overline{BC} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \begin{bmatrix} \frac{7}{2} \\ \frac{11}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{15}{2} \\ \frac{7}{2} \\ 4 \end{bmatrix}$$

Correct vector expression to find  $\overline{OD}$  written in components

m1

$$D \text{ is at } \left( \frac{15}{2}, \frac{7}{2}, 4 \right)$$

CAO; condone column vector

6

A1

**Alternative** for last 2 marks

$$\overline{OD} = \overline{OC} + \overline{BA} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + \frac{3}{2} \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$$

(M1)

$$D \text{ is at } \left( \frac{15}{2}, \frac{7}{2}, 4 \right)$$

(A1)

**NB**  $p = \frac{3}{2}$  can come from wrong working where candidate uses  $\overline{OC}$  in place of  $\overline{BC}$ .

This is M0 and scores no further marks, (unless they happen to find and go on to use it correctly).

(6)

[12]