



Topic Test: OxfordAQA AS Further Mathematics

Mechanics

Name: _____

Class: _____

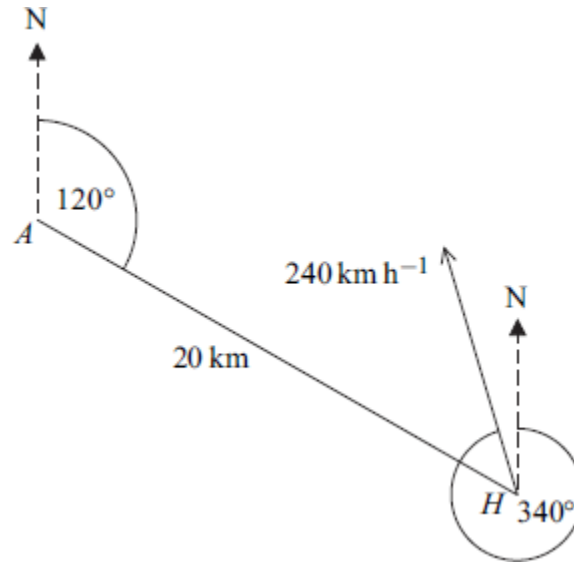
Date: _____

Time: **69 minutes**

Marks: **58 marks**

Comments:

- 1** From an aircraft A , a helicopter H is observed 20 km away on a bearing of 120° . The helicopter H is travelling horizontally with a constant speed 240 km h^{-1} on a bearing of 340° . The aircraft A is travelling with constant speed $v_A \text{ km h}^{-1}$ in a straight line and at the same altitude as H .



- (a) Given that $v_A = 200$:
- (i) find a bearing, to one decimal place, on which A could travel in order to intercept H ; (5)
- (ii) find the time, in minutes, that it would take A to intercept H on this bearing. (4)
- (b) Given that $v_A = 150$, find the bearing on which A should travel in order to approach H as closely as possible. Give your answer to one decimal place. (5)
- (Total 14 marks)**

- 2** The unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are directed due east, due north and vertically upwards respectively.

A helicopter, A , is travelling in the direction of the vector $-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ with constant speed 140 km h^{-1} . Another helicopter, B , is travelling in the direction of the vector $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ with constant speed 60 km h^{-1} .

- (a) Find the velocity of A relative to B . (5)
- (b) Initially, the position vectors of A and B are $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \text{ km}$ and $(-3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \text{ km}$ respectively, relative to a fixed origin.

Write down the position vector of A relative to B , t hours after they leave their initial positions.

(2)

(c) Find the distance between A and B when they are closest together.

(8)

(Total 15 marks)

3 A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u \text{ m s}^{-1}$, at which the liquid escapes is given by

$$u = C V \rho g$$

where $V \text{ m}^3$ is the volume of the liquid in the tank, $\rho \text{ kg m}^{-3}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C .

(Total 5 marks)

4 A car has mass m and travels up a slope which is inclined at an angle θ to the horizontal. The car reaches a maximum speed v at a height h above its initial position. A constant resistance force R opposes the motion of the car, which has a maximum engine power output P .

Neda finds a formula for P as

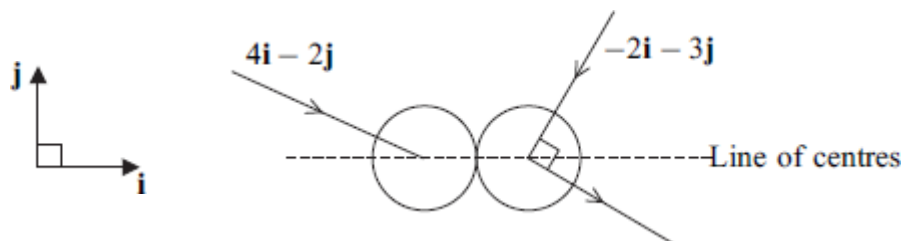
$$P = mgv \sin \theta + Rv + \frac{1}{2} mv^3 \frac{\sin \theta}{h}$$

where g is the acceleration due to gravity.

Given that the engine power output may be measured in newton metres per second, determine whether the formula is dimensionally consistent.

(Total 6 marks)

5 Two smooth spheres, A and B , have equal radii and masses 4 kg and 2 kg respectively. The sphere A is moving with velocity $(4\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ and the sphere B is moving with velocity $(-2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$ on the same smooth horizontal surface. The spheres collide when their line of centres is parallel to unit vector \mathbf{i} . The direction of motion of B is changed through 90° by the collision, as shown in the diagram.



(a) Show that the velocity of B immediately after the collision is $(\frac{9}{2}\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-1}$.

(4)

(b) Find the coefficient of restitution between the spheres.

(5)

(c) Find the impulse exerted on B during the collision. State the units of your answer.

(3)

(Total 12 marks)

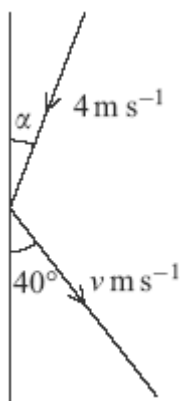
6

A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed 4 m s^{-1} and the angle between the sphere's direction of motion and the wall is α .

Immediately after the impact, the sphere is moving with speed $v \text{ m s}^{-1}$ and the angle between the sphere's direction of motion and the wall is 40° .

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



(a) Show that $\tan \alpha = \frac{3}{2} \tan 40^\circ$.

(3)

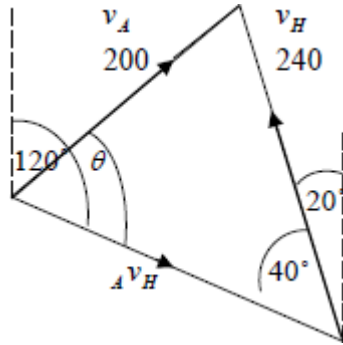
(b) Find the value of v .

(3)

(Total 6 marks)

Mark schemes

1 (a) (i)



Correct diagram with or without arrows.

B1

40° marked correctly, PI by correct method.

B1

$$\frac{\sin \theta}{240} = \frac{\sin 40}{200}$$

Correct sine rule allowing their angle opposite 200 in their diagram.

M1

$$\theta = 50.47483^\circ \text{ or } 50.5^\circ$$

AWRT 50.5°, PI by correct bearing

A1

$$\text{Bearing of } v_A = 069.5^\circ$$

Allow 69.5°

A1

5

$$(ii) \frac{{}_A v_H}{\sin(180^\circ - 40^\circ - 50.5^\circ)} =$$

$$\frac{200}{\sin 40^\circ} \text{ or } \frac{240}{\sin 50.5^\circ}$$

Allow using their angle from part (a)(i).

M1

$${}_A v_H = 311.13408 \text{ or } 311$$

FT their angle from part (a)(i)

A1F

$$\text{Time} = \frac{20}{311.13408}$$

PI by correct answer. Allow their v_H .

M1

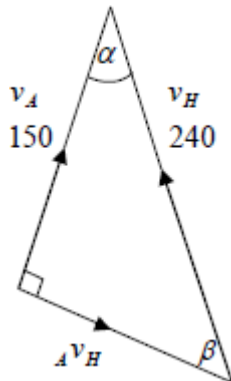
$$(\text{= } 0.0642809 \text{ hours}) = 3.86 \text{ min}$$

3sf required

A1F

4

(b)



Right-angled triangle with 240 and 150 marked.

M1

Correct orientation

A1

$$\cos \alpha = \frac{150}{240} \quad \text{or} \quad \sin \beta = \frac{150}{240}$$

M1

$$\alpha = 51.3^\circ \quad \text{or} \quad \beta = 38.7^\circ$$

PI by correct bearing

A1

$$\text{Bearing: } 031.3^\circ$$

Allow 31.3°

A1

5

[14]

2

(a) $u_A = \frac{(-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k})140}{\sqrt{(2)^2 + (3)^2 + (6)^2}} = -40\mathbf{i} + 60\mathbf{j} + 120\mathbf{k}$

Simplification not needed

M1 A1

$$u_B = \frac{(-2\mathbf{i} + \mathbf{j} + 2\mathbf{k})60}{\sqrt{(2)^2 + (1)^2 + (2)^2}} = 40\mathbf{i} - 20\mathbf{j} + 40\mathbf{k}$$

Simplification not needed

A1

$$\begin{aligned} {}_A u_B &= (-40\mathbf{i} + 60\mathbf{j} + 120\mathbf{k}) - (40\mathbf{i} - 20\mathbf{j} + 40\mathbf{k}) \\ &= -80\mathbf{i} + 80\mathbf{j} + 80\mathbf{k} \end{aligned}$$

Subtracting B from A

M1
A1F

5

(b) ${}_A r_B = (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) - (-3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) + t(-80\mathbf{i} + 80\mathbf{j} + 80\mathbf{k})$

A difference of initial p.v. + $t \times {}_A u_B$

or $(7\mathbf{i} - 8\mathbf{j}) + t(-80\mathbf{i} + 80\mathbf{j} + 80\mathbf{k})$

M1
A1F

2

$$(c) \quad {}_A r_B = (7 - 80t)\mathbf{i} + (-8 + 80t)\mathbf{j} + (80t)\mathbf{k}$$

B1F

$$s^2 = (7 - 80t)^2 + (-8 + 80t)^2 + (80t)^2$$

B1F

$$2s \frac{ds}{dt} = 2(7 - 80t)(-80) + 2(-8 + 80t)(80) + 2(80t)(80) = 0$$

Differentiation

M1

A1F

$$240t = 15$$

Solving

m1

$$t = 0.0625 \quad \text{or} \quad \frac{1}{16}$$

A1F

$$s^2 = (7 - 80 \times 0.0625)^2 + (-8 + 80 \times 0.0625)^2 + (80 \times 0.0625)^2$$

M1

$$s = 6.16\text{km} \quad \text{or} \quad \sqrt{38} \text{ km}$$

A1F

8

Alternative (Not in the specification)

$$A \text{ and } B \text{ are closest} \Rightarrow {}_A r_B \cdot {}_A v_B = 0$$

$$[(7 - 80t)\mathbf{i} + (-8 + 80t)\mathbf{j} + (80t)\mathbf{k}] \cdot$$

B1 M1

$$[-80\mathbf{i} + 80\mathbf{j} + 80\mathbf{k}] = 0$$

A1

$$-80(7 - 80t) + 80(-8 + 80t) + 80(80t) = 0$$

A1

$$240t = 15$$

M1

$$t = 0.0625$$

A1

[15]

3

LT^{-1}

For dimensions of u

B1

$LT^{-1} = M^\alpha L^\beta T^\gamma \times L^3 \times ML^{-3} \times LT^{-2}$

M1 for equation with five components

M1A1

$1 = \beta + 1$

$-1 = \gamma - 2$

$0 = \alpha + 1$

Forming and solving equations (PI)

m1

$\beta = 0, \alpha = -1, \gamma = 1$

The dimensions of C are $M^{-1}T$

A1F

5

Alternative:

LT^{-1}

For dimensions of u

(B1)

$LT^{-1} = C \times L^3 \times ML^{-3} \times LT^{-2}$

M1 for equation with five components

(M1A1)

$LT^{-1} = C \times L MT^{-2}$

(m1)

The dimensions of C are $M^{-1}T$

(A1F)

[5]**4**

$[P] = MLT^{-2} \cdot L \cdot T^{-1} = ML^2T^{-3}$

For correct unsimplified dimensions of quantities

B1

$$[mgv \sin \theta] = M \cdot LT^{-2} \cdot LT^{-1} = ML^2T^{-3}$$

*For correct unsimplified
dimensions of quantities*

B1

$$[Rv] = MLT^{-2} \cdot LT^{-1} = ML^2T^{-3}$$

*For correct unsimplified
dimensions of quantities*

B1

$$\left[\frac{1}{2}mv^3 \frac{\sin \theta}{h} \right] = M \cdot L^3T^{-3} \cdot L^{-1} = ML^2T^{-3}$$

*For correct unsimplified
dimensions of quantities*

B1

All simplifications correct

B1

The formula is dimensionally consistent

Dependent on the last B1

E1

[6]

5

(a) (Let $v_B = ai - bj$)

$$\frac{a}{b} = \frac{3}{2}$$

Allow sign error

M1

$$\frac{a}{b} = \frac{3}{2}$$

OE

A1

(Squares are smooth \Rightarrow j component \Rightarrow)

$$b = 3$$

B1

$$a = \frac{9}{2}$$

$$\left(v_B = \frac{9}{2} \mathbf{i} - 3 \mathbf{j} \right)$$

AG

A1

4

(b) (C.L.M. along the line of centres:)

$$4(4) - 2(2) = 4(v_A) + 2\left(\frac{9}{2}\right)$$

OE, No sign errors

M1

$$v_A = \frac{3}{4}$$

A1

(Restitution along the line of centres:)

$$e = \frac{-\frac{3}{4} + \frac{9}{2}}{4+2} \quad \text{OE}$$

M1 for correct terms, A0 for sign error

M1 A1

$$e = \frac{5}{8}$$

A1

5

(c) (I = Change in momentum of B along the line of centres)

$$= 2\left(\frac{9}{2}i\right) - 2(-2i)$$

Allow sign error and missing i

M1

$$= 13i$$

A0 for magnitude or $-13i$

A1

Ns or kg m s^{-1}

B1

3

[12]

6

(a) Parallel to the wall

$$4 \cos \alpha = v \cos 40^\circ$$

Correct trigonometric ratios

M1

Perpendicular to the wall

$$v \sin 40^\circ = \frac{2}{3} \times 4 \sin \alpha$$

Correct trigonometric ratios

M1

$$\tan \alpha = \frac{3}{2} \tan 40^\circ$$

AG

A1

3

(b) $\alpha = 51.5^\circ$

M1

$$v = \frac{4 \cos 51.5^\circ}{\cos 40^\circ}$$

M1

$$v = 3.25 \text{ m s}^{-1}$$

OE

A1

3

[6]