



**Topic Test: OxfordAQA AS Further Mathematics**  
Pure Mathematics

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Date: \_\_\_\_\_

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Time: **90 minutes**

Marks: **75 marks**

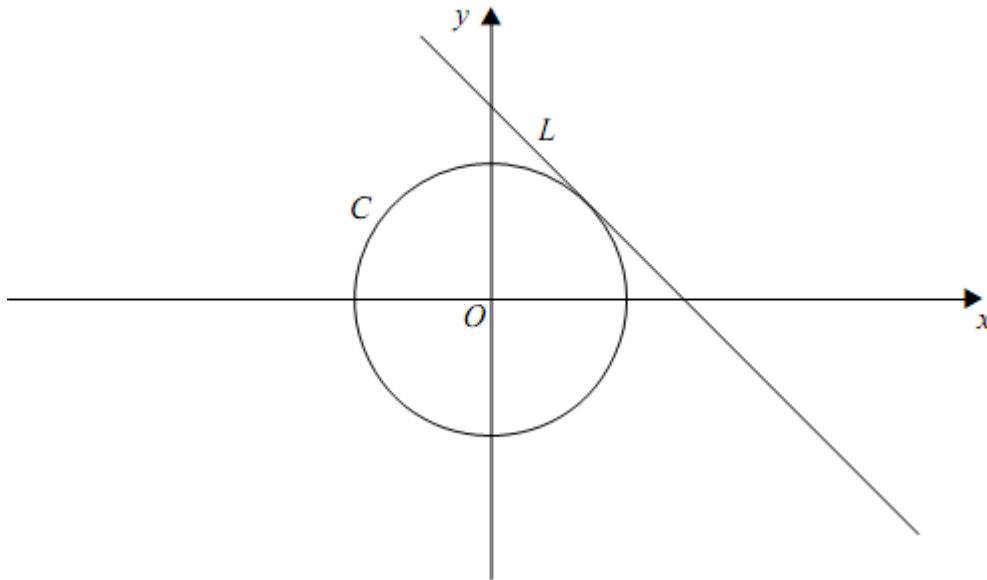
Comments:

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- 1** The diagram shows a circle  $C$  and a line  $L$ , which is the tangent to  $C$  at the point  $(1, 1)$ . The equations of  $C$  and  $L$  are

$$x^2 + y^2 = 2 \text{ and } x + y = 2$$

respectively.



The circle  $C$  is now transformed by a stretch with scale factor 2 parallel to the  $x$ -axis. The image of  $C$  under this stretch is an ellipse  $E$ .

- (a) **On the diagram below**, sketch the ellipse  $E$ , indicating the coordinates of the points where it intersects the coordinate axes.

**(4)**

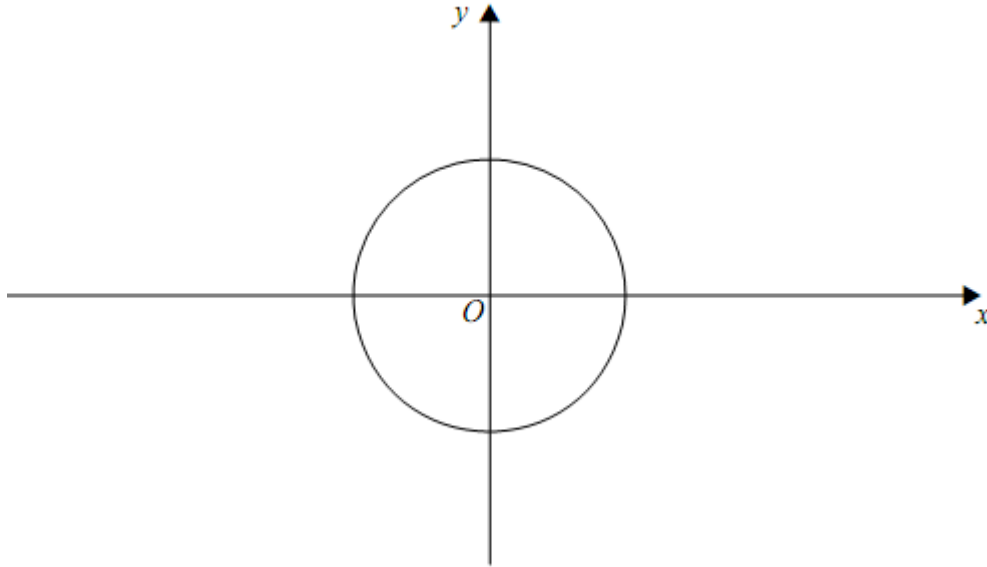
- (b) Find equations of:

- (i) the ellipse  $E$ ;

**(2)**

(ii) the tangent to  $E$  at the point  $(2, 1)$ .

(2)



(Total 8 marks)

2

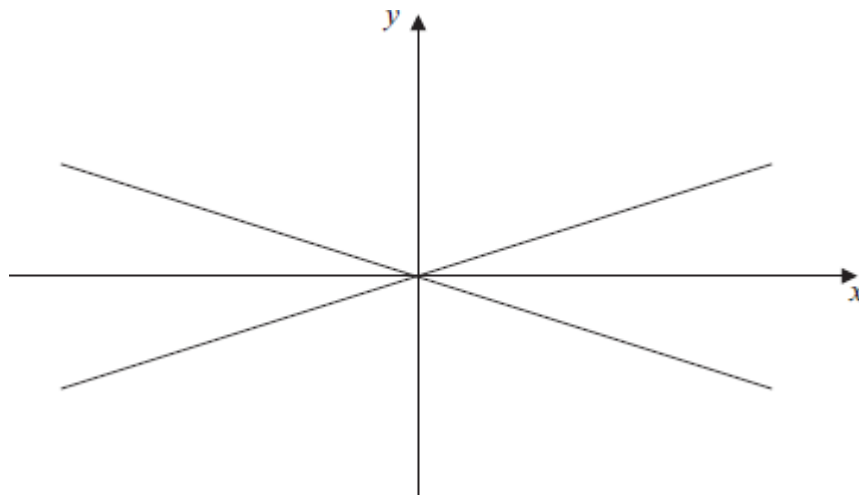
A hyperbola  $H$  has equation

$$\frac{x^2}{9} - y^2 = 1$$

(a) Find the equations of the asymptotes of  $H$ .

(1)

(b) The asymptotes of  $H$  are shown in the diagram below. On the same diagram, sketch the hyperbola  $H$ . Indicate on your sketch the coordinates of the points of intersection of  $H$  with the coordinate axes.



(3)

- (c) The hyperbola  $H$  is now translated by the vector  $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ .
- (i) Write down the equation of the translated curve. (2)
- (ii) Calculate the coordinates of the two points of intersection of the translated curve with the line  $y = x$ . (4)
- (d) From your answers to part (c)(ii), **deduce** the coordinates of the points of intersection of the original hyperbola  $H$  with the line  $y = x - 3$ . (2)
- (Total 12 marks)**

- 3**
- (a) It is given that  $z_1 = \frac{1}{2} - i$ .
- (i) Calculate the value of  $z_1^2$ , giving your answer in the form  $a + bi$ . (2)
- (ii) Hence verify that  $z_1$  is a root of the equation
- $$z^2 + z^* + \frac{1}{4} = 0$$
- (2)
- (b) Show that  $z_2 = \frac{1}{2} + i$  also satisfies the equation in part (a)(ii). (2)
- (c) Show that the equation in part (a)(ii) has two equal **real** roots. (2)
- (Total 8 marks)**

**4**

Two loci,  $L_1$  and  $L_2$ , in an Argand diagram are given by

$$L_1 : |z + 6 - 5i| = 4\sqrt{2}$$

$$L_2 : \arg(z + i) = \frac{3\pi}{4}$$

The point  $P$  represents the complex number  $-2 + i$ .

- (a) Verify that the point  $P$  is a point of intersection of  $L_1$  and  $L_2$ . (2)
- (b) Sketch  $L_1$  and  $L_2$  on one Argand diagram. (6)

- (c) The point  $Q$  is also a point of intersection of  $L_1$  and  $L_2$ . Find the complex number that is represented by  $Q$ .

(2)

(Total 10 marks)

5

The equation

$$2x^2 + 3x - 6 = 0$$

has roots  $\alpha$  and  $\beta$ .

- (a) Write down the value of  $\alpha + \beta$  and the value of  $\alpha\beta$ .

(2)

- (b) Hence show that  $\alpha^3 + \beta^3 = -\frac{135}{8}$ .

(3)

- (c) Find a quadratic equation, with integer coefficients, whose roots are

$$\alpha + \frac{\alpha}{\beta^2} \text{ and } \beta + \frac{\beta}{\alpha^2}.$$

(6)

(Total 11 marks)

6

- (a) Show that  $\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{A}{(5r-2)(5r+3)}$ , stating the value of the constant  $A$ .

(2)

- (b) Hence use the method of differences to show that

$$\sum_{r=1}^n \frac{1}{(5r-2)(5r+3)} = \frac{n}{3(5n+3)}$$

(4)

- (c) Find the value of

$$\sum_{r=1}^{\infty} \frac{1}{(5r-2)(5r+3)}$$

(1)

(Total 7 marks)

7

- (a) A curve has equation  $y = 2x^2 - 5x$ .

The point  $P$  on the curve has coordinates  $(1, -3)$ .

The point  $Q$  on the curve has  $x$ -coordinate  $1 + h$ .

- (i) Show that the gradient of the line  $PQ$  is  $2h - 1$ .

(3)

- (ii) Explain how the result of part (a)(i) can be used to show that the tangent to the curve at the point  $P$  is parallel to the line  $x + y = 0$ .

(2)

- (b) For the improper integral  $\int_1^{\infty} x^{-4}(2x^2 - 5x) dx$ , either show that the integral has a finite value and state its value, or explain why the integral does not have a finite value.

(3)

(Total 8 marks)

8

Show that only one of the following improper integrals has a finite value, and find that value:

(a)  $\int_8^{\infty} x^{-\frac{2}{3}} dx$ ;

(b)  $\int_8^{\infty} x^{-\frac{4}{3}} dx$ .

(Total 5 marks)

9

Find the general solution of the equation

$$\sin\left(4x - \frac{2\pi}{3}\right) = -\frac{1}{2}$$

giving your answer in terms of  $\pi$ .

(Total 6 marks)

## Mark schemes

**1**

- (a) Sketch of ellipse  
*centred at origin*

M1

Correct relationship to circle

A1

Coords  $(\pm 2\sqrt{2}, 0)$   $(0, \pm \sqrt{2})$

*Accept  $\sqrt{8}$  for  $2\sqrt{2}$ ;*

*B1 for any 2 of  $x = \pm 2\sqrt{2}$ ,  $y = \pm \sqrt{2}$   
allow B1 if all correct except for use of  
decimals (at least one DP)*

B2,1

4

- (b) (i) Replacing  $x$  by  $\frac{x}{2}$   
*or by  $2x$*

M1

$$E \text{ is } \left(\frac{x}{2}\right)^2 + y^2 = 2$$

*OE*

A1

2

- (ii) Tangent is  $\frac{x}{2} + y = 2$

*M1 for complete valid method*

M1A1

2

**[8]**

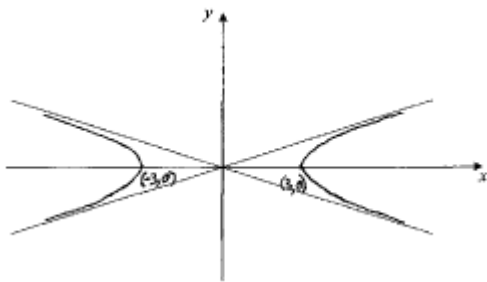
**2**

- (a)  $y = \pm \frac{1}{3}x$   
*ACF Need both*

B1

1

(b)



2-branch curve with branches in correct regions above and below  $x$ -axis

B1

Curve approaching asymptotes

B1

Coords  $(\pm 3, 0)$ , as **only** points of intersection with coordinate axes, indicated. Condone  $-3$  and  $+3$  marked on  $x$ -axis at points of intersection as  $(\pm 3, 0)$  indicated.

B1

3

(c) (i)  $\frac{(x+3)^2}{9} - y^2 = 1$   
Replacing  $x$  by either  $x + 3$  or  $x - 3$

M1

ACF

A1

2

(ii)  $\frac{(x+3)^2}{9} - x^2 = 1$

Substitution into c's (c)(i) eqn of  $y = x$  to eliminate  $y$  or of  $x = y$  to eliminate  $x$

M1

$$x^2 + 6x + 9 = 9(x^2 + 1)$$

Correct expansion of  $(x \pm 3)^2$  equated to  $9(x^2 + 1)$  OE ft; [OE in  $y$ ]

A1F

$$8x^2 - 6x = 0 \quad (8x^2 = 6x)$$

*Ft on error (x - 3) for (x + 3) in (c)(i) which gives*

$$8x^2 + 6x = 0 \quad (8x^2 = -6x) \text{ [OE in y]}$$

A1F

Points are (0,0),  $(\frac{3}{4}, \frac{3}{4})$

*Both. ACF*

A1

4

- (d) *Adding 3 to c's (c)(ii) **two** x-coords keeping y-coordinates unchanged.*

M1

Points are (3,0),  $(3\frac{3}{4}, \frac{3}{4})$

*Ft on c's (c)(ii) coordinates for the **two** points  
If not deduced then MOA0*

A1F

2

[12]

3

(a) (i)  $z_1^2 = \frac{1}{4} - i + i^2 = -\frac{3}{4} = i$

*M1 for use of  $i^2 = -1$*

M1A1

2

(ii)  $\text{LHS} = -\frac{3}{4} - i + \frac{1}{2} + i + \frac{1}{4} = 0$

*AG; M1 for  $z^*$  correct*

M1A1

2

(b)  $\text{LHS} = -\frac{3}{4} + i + \frac{1}{2} - i + \frac{1}{4} = 0$

*AG; M1 for correct*

M1A1

2

(c)  $z \text{ real} \Rightarrow z^* = z$

*Clearly stated*

M1

Discr't zero or correct factorisation

AG

A1

2

[8]

4

(a)  $|4 - 4i| = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

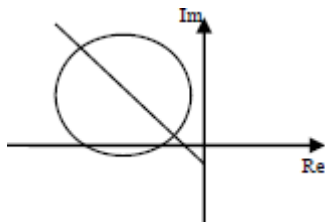
*verification that  $|-2+i+6-5i| = 4\sqrt{2}$*

B1

$$\arg(-2 + 2i) = \pi - \tan^{-1}(1) = \frac{3\pi}{4}$$

*verification that  $\arg(z + i) = \frac{3\pi}{4}$*

B1



2

(b) Circle

*freehand circle sketched*

M1

Centre at  $-6 + 5i$

*clear from diagram or centre stated*

A1

Cutting Re axis but not cutting Im axis

A1

“Straight” line

*freehand line*

M1

Half line from  $0 - i$

*not horizontal or vertical but end point at  $0 - i$  must be clear from diagram / stated*

A1

gradient  $-1$  (approx)

*making  $45^\circ$  to negative Re axis and positive Im axis*

A1

6

(c) Calculation based on fact that  $L_2$  passes through centre of  $L_1$

*idea of vector  $\begin{bmatrix} -4 \\ 4 \end{bmatrix}$  from centre*

M1

$Q$  represents  $-10 + 9i$

*must write as a complex number*

A1

2

[10]

5

(a)  $\alpha + \beta = -\frac{3}{2}$

OE

B1

$\alpha\beta = -3$

OE

B1

2

(b)  $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

*Using correct identity for  $\alpha^3 + \beta^3$  in terms of  $\alpha + \beta$  and  $\alpha\beta$ .*

M1

$$= \left(-\frac{3}{2}\right)^3 - 3(-3)\left(-\frac{3}{2}\right)$$

*with ft / or correct substitution*

A1F

$$= -\frac{27}{8} - \frac{27}{2} = -\frac{135}{8}$$

CSO AG. Correct evaluation of each of  $(-1.5)^3$  and  $-3(-3)(-1.5)$  must be seen before the printed answer is stated

A1

3

(c) 
$$\text{Sum} = \alpha + \frac{\alpha}{\beta^2} + \beta + \frac{\beta}{\alpha^2} = \alpha + \beta + \frac{\alpha^3 + \beta^3}{(\alpha\beta)^2} = -\frac{3}{2} + \frac{-135/8}{9}$$

Writing  $\alpha + \frac{\alpha}{\beta^2} + \beta + \frac{\beta}{\alpha^2}$  in a suitable form with ft/or correct substitution

M1

$$\text{Sum} = -\frac{27}{8}$$

PI OE exact value eg - 3.375 (A0 if  $\alpha\beta = 3$  used to get  $(\alpha\beta)^2 = 9$ )

A1

$$\text{Product} = \alpha\beta + \frac{\beta}{\alpha} + \frac{\alpha}{\beta} + \frac{1}{\alpha\beta} = \alpha\beta + \frac{\alpha^2 + \beta^2}{\alpha\beta} + \frac{1}{\alpha\beta} \quad (*)$$

$$\text{Now } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta (= \frac{9}{4} + 6)$$

(\*) OE with correct identity for  $\alpha^2 + \beta^2$  used in (c). Subst of values not required but PI by correct value of Product

M1

$$\text{Product} = -3 - \frac{1}{3} \left( \frac{9}{4} + 6 \right) - \frac{1}{3} = -\frac{73}{12}$$

PI OE exact value

A1

$$x^2 - Sx + P (= 0)$$

Using correct general form of LHS of eqn **with** ft substitution of c's S and P values.

M1

Eqn is  $24x^2 + 81x - 146 = 0$

*OE but integer coefficients and '= 0' needed*

A1

6

[11]

**6** (a)  $\frac{1}{5r-2} - \frac{1}{5r+3} = \frac{5r+3-(5r-2)}{(5r-2)(5r+3)}$

*condone omission of brackets for M1*

M1

$$= \frac{5}{(5r-2)(5r+3)}$$

$A = 5$

A1cso

2

(b) Attempt to use method of differences

*at least 2 terms of correct form seen*

M1

$$k \left\{ \frac{1}{3} - \frac{1}{5n+3} \right\}$$

*correct cancellation leaving correct two fractions*

A1

$$k \left\{ \frac{(5n+3)-3}{3(5n+3)} \right\}$$

*attempt to write with common denominator*

m1

$$S_n = \frac{1}{5} \left\{ \frac{(5n+3)-3}{3(5n+3)} \right\} = \frac{n}{3(5n+3)}$$

**AG**  $k = \frac{1}{5}$  used correctly throughout

A1cso

4

(c)  $S_\infty = \frac{1}{15}$

B1

1

[7]

7

(a) (i)  $y = 2x^2 - 5x$

$$y_Q = 2(1 + h)^2 - 5(1 + h) = 2 + 4h + 2h^2 - 5 - 5h$$

$$= 2h^2 - h - 3$$

$y_Q = 2(1 + h)^2 - 5(1 + h)$  with correct expansion of brackets P1

B1

$$\text{Grad.} = \frac{y_Q - y_P}{x_Q - x_P} = \frac{2(1 + h)^2 - 5(1 + h) - (-3)}{1 + h - 1}$$

Use of correct formula for gradient

M1

$$= \frac{2h^2 - h - 3 - (-3)}{h} = \frac{2h^2 - h}{h} = 2h - 1$$

CSO

A1

3

(ii) As  $h \rightarrow 0$ , (grad of  $PQ \rightarrow$  grad of tangent at  $P$ ) (ie) gradient (of tangent at  $P$ ) = -1

$h = 0$  scores E0

E1

Now **gradient** of  $x + y = 0$  (or  $y = -x$ ) is also **-1**  $\Rightarrow$  tangent at  $P$  is **parallel** to line  $x + y = 0$

Dep on  $h \rightarrow 0$  or  $h = 0$  being used earlier

E1

2

(b)  $I = \int_1^\infty x^{-4}(2x^2 - 5x) dx = \int_1^\infty (2x^{-2} - 5x^{-3}) dx$

$$I = \left[ -2x^{-1} - 5 \frac{x^{-2}}{-2} \right]_1^\infty$$

At least one term correct

M1

As  $x \rightarrow \infty$ ,  $x^{-1} \rightarrow 0$  and  $x^{-2} \rightarrow 0$

OE Ft on  $kx^{-n}$  provided M1 awarded

E1

$$I = 0 - (-2 + 5/2) = -\frac{1}{2}$$

$(I =) -\frac{1}{2}$  Dep on both terms integrated correctly in the M1 line

A1

3

[8]

**8** (a)  $\int x^{-\frac{1}{3}} dx = 3x^{\frac{2}{3}} (+c)$

$kx^{\frac{1}{3}}, k \neq 0$  ie condone incorrect non-zero coefficient here

B1

(3) $x^{\frac{1}{3}} \rightarrow \infty$  as  $x \rightarrow \infty$ , so no finite value

E1

(b)  $\int x^{-\frac{1}{3}} dx = -3x^{\frac{2}{3}} (+c)$

$\lambda x^{-\frac{1}{3}}, \lambda \neq 0$

M1

$-3x^{\frac{2}{3}}$  OE

A1

$$\int_0^{\infty} x^{-\frac{1}{3}} dx = -3(0 - \frac{1}{2}) = \frac{3}{2}$$

CSO

A1

[5]

**9**  $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

OE; dec/deg penalised at 6th mark

B1

$$\sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$$

OE; ft wrong first value

B1F

Use of  $2n\pi$

*(or  $n\pi$ ) at any stage*

M1

Going from  $4x - \frac{2\pi}{3}$  to  $x$

*including division of all terms by 4*

m1

$$\text{GS } x = \frac{\pi}{8} + \frac{1}{2}n\pi \text{ or } x = -\frac{\pi}{24} + \frac{1}{2}n\pi$$

OE

A1A1

**[6]**