OXFORD

INTERNATIONAL AQA EXAMINATIONS

Topic Test: Oxford AQA AS Mathematics

Pure Mathematics

Name: ______________________

Class: ______________________

Date: ______________________

Time: **94 minutes**

Marks: **78 marks**

Comments:

1 (a) Write $\sqrt[4]{x^3}$ in the form x^k.

(1)

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$.

(2)

(Total 3 marks)

2 (a) (i) Express $x^2 - 6x + 11$ in the form $(x - p)^2 + q$.

(2)

(ii) **Use the result from part (a)(i)** to show that the equation $x^2 - 6x + 11 = 0$ has no real solutions.

(2)

(b) A curve has equation $y = x^2 - 6x + 11$.

(i) Find the coordinates of the vertex of the curve.

(2)

(ii) Sketch the curve, indicating the value of y where the curve crosses the y-axis.

(3)

(iii) Describe the geometrical transformation that maps the curve with equation $y = x^2 - 6x + 11$ onto the curve with equation $y = x^2$.

(3)

(Total 12 marks)

3 The line AB has equation $3x - 4y + 5 = 0$.

(a) The point with coordinates $(p, p + 2)$ lies on the line AB. Find the value of the constant p.

(2)

(b) Find the gradient of AB.

(2)

(c) The point A has coordinates (1, 2). The point $C(-5, k)$ is such that AC is perpendicular to AB. Find the value of k.

(3)

(d) The line AB intersects the line with equation $2x - 5y = 6$ at the point D. Find the coordinates of D.

(3)

(Total 10 marks)

4 A curve has equation $y = 2x^2 - x - 1$ and a line has equation $y = k(2x - 3)$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the curve and the line satisfies the equation

$$2x^2 - (2k + 1)x + 3k - 1 = 0$$

(1)

(b) The curve and the line intersect at two distinct points.

(i) Show that $4k^2 - 20k + 9 > 0$.

(3)

(ii) Find the possible values of k.

(4)

(Total 8 marks)

5 The gradient, $\frac{dy}{dx}$, of a curve at the point (x, y) is given by

$$\frac{dy}{dx} = 10x^4 - 6x^2 + 5$$

The curve passes through the point $P(1, 4)$.

(a) Find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$.

(3)

(b) Find the equation of the curve.

(5)

(Total 8 marks)

6 The curve with equation $y = 13 + 18x + 3x^2 - 4x^3$ passes through the point P where $x = -1$.

(a) Find $\frac{dy}{dx}$

(3)

(b) Show that the point P is a stationary point of the curve and find the other value of x where the curve has a stationary point

(3)

(c) (i) Find the value of $\frac{d^2y}{dx^2}$ at the point P.

(3)

(ii) Hence, or otherwise, determine whether P is a maximum point or a minimum point.

(1)

(Total 10 marks)

7 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{2^x}{x+1}\,dx$$

giving your answer to three significant figures.

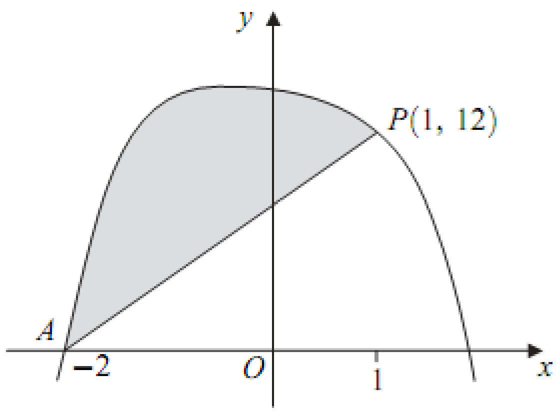
(4)

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule.

(1)

(Total 5 marks)

8 The curve sketched below passes through the point A (−2, 0).



The curve has equation $y = 14 - x - x^4$ and the point P (1, 12) lies on the curve.

(a) (i) Find the gradient of the curve at the point P.

(3)

(ii) Hence find the equation of the tangent to the curve at the point P, giving your answer in the form $y = mx + c$.

(2)

(b) (i) Find $\int_{-2}^{1}(14 - x - x^4)\,dx$.

(5)

(ii) Hence find the area of the shaded region bounded by the curve $y = 14 - x - x^4$ and the line AP.

(2)

(Total 12 marks)

9 (a) A geometric series begins 420 + 294 + 205.8 + ….

(i) Show that the common ratio of the series is 0.7.

(1)

(ii) Find the sum to infinity of the series.

(2)

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants.

(2)

(b) The first term of an arithmetic series is 240 and the common difference of the series is −8.

The nth term of the series is u_n.

(i) Write down an expression for u_n.

(1)

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$.

(4)

(Total 10 marks)

Mark schemes

1 (a) $\sqrt[4]{x^3} = x^{\frac{3}{4}}$

Accept k = ¾ OE

B1

1

(b) $\frac{1-x^2}{\sqrt[4]{x^3}} = \frac{1}{\sqrt[4]{x^3}} - \frac{x^2}{\sqrt[4]{x^3}} = x^{-k} - \frac{x^2}{\sqrt[4]{x^3}}$ [or $\frac{1}{\sqrt[4]{x^3}} - x^{2-k}$]

Split followed by at least one correct index law used to remove denominator.

M1

$= x^{-\frac{3}{4}} - x^{\frac{5}{4}}$

If incorrect, ft on c's non-integer k value answer to part (a), provided M1 has been awarded. Accept answer given in form of values for p and q.

A1F

2

[3]

2 (a) (i) $(x - 3)^2$

or p = 3 seen

M1

$(x - 3)^2 + 2$

A1

2

(ii) $(x - 3)^2 = -2$

FT their positive value of q

***not** use of discriminant*

M1

No (real) square root of −2 therefore equation has no real solutions

for graphical approach see below to see if SC1 can be awarded

A1cso

2

(b) (i) x = 'their' p *or* y = 'their' q

or x = 3 found using calculus

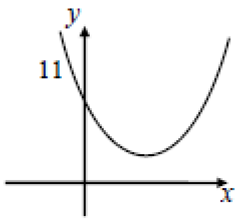
M1

Vertex is at (3, 2)

A1cao

2

(ii)



y intercept = 11 ***stated*** *or* ***marked on y-****axis*
(as y intercept of any graph)

B1

∪ shape (generous)

M1

above x-axis , vertex in first quadrant

crossing y-axis into second quadrant

A1

3

(iii) Translation

and no other transformation

E1

through $\begin{bmatrix} -3 \\ -2 \end{bmatrix}$

FT negative of BOTH 'their' vertex coords

M1

both *components* ***correct*** *for A1; may describe in words or use a column vector*

A1

3

[12]

3 (a) $3p - 4(p + 2) + 5 = 0$

condone omission of brackets or one sign error

M1

($\Rightarrow p =$) −3

A1

2

(b) $y = \frac{3}{4}x + \frac{5}{4}$

rearranging into form $y = \pm\frac{3}{4}x + c$

M1

(gradient AB =) $\frac{3}{4}$

condone slips in rearranging if gradient is correct.

A1

2

(c) (gradient AC =) $\frac{k-2}{-5-1}$

or $\frac{2-k}{1--5}$ *(condone one sign error)*

M1

"their" $\frac{(k-2)}{-6} \times \frac{3}{4} = -1$ *OE*

product of grads = −1 in terms of k

m1

($\Rightarrow k =$) 10

A1

3

(d) $3x - 4y + 5 = 0$ and $2x - 5y = 6$

$\Rightarrow P\,x = Q$ or $R\,y = S$

must use "correct" pair of equations ***and***
attempt to eliminate y (or x) (generous)

M1

$x = -7$

A1

$y = -4$

A1

$(-7, -4)$

3

[10]

4 (a) $2x^2 - x - 1 = 2kx - 3k$

$2x^2 - x - 1 - 2kx + 3k = 0$ OE

$\Rightarrow 2x^2 - (2k + 1)x + 3k - 1 = 0$

equated and multiplied out and all 5 terms on one side and = 0

AG *(correct with no trailing = signs etc)*

B1

1

(b) (i) $(2k + 1)^2 - 4 \times 2(3k - 1)$

clear attempt at $b^2 - 4ac$

M1

$(2k + 1)^2 - 4 \times 2(3k - 1) > 0$

discriminant > 0 which must appear before the printed answer

B1

$4k^2 + 4k + 1 - 24k + 8 > 0 \Rightarrow 4k^2 - 20k + 9 > 0$

AG *(all working correct with no missing brackets etc)*

A1cso

3

(ii) $4k^2 - 20k + 9 = (2k - 9)(2k - 1)$

correct factors or correct use of formula as far as

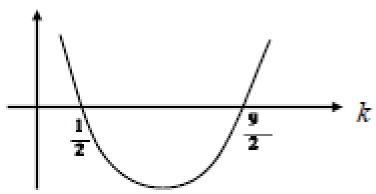
$$\frac{20 \pm \sqrt{256}}{8}$$

M1

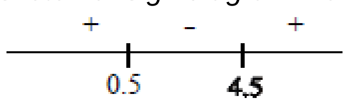
critical values are $\frac{1}{2}$ and $\frac{9}{2}$

condone $\frac{4}{8}$, $\frac{36}{8}$ *etc here but must combine sums of fractions*

A1



sketch or sign diagram including values



M1

$k < \frac{1}{2},\ k > \frac{9}{2}$

fractions must be simplified condone use of OR but not AND

A1

Take their final line as their answer.

4

[8]

5 (a) (Gradient = 10 − 6 + 5) = 9

correct gradient from sub x = 1 into $\frac{dy}{dx}$

B1

$y - 4 =$ "*their 9*" $(x - 1)$
or $y =$ "*their 9*" $x + c$ **and** attempt
to find c using $x = 1$ and $y = 4$

must attempt to use given expression for $\frac{dy}{dx}$ ***and*** *must be attempting tangent and not normal* ***and*** *coordinates must be correct*

M1

$y = 9x - 5$

condone $y = 9x + c, \ldots \quad c = -5$

A1

3

(b) $(y =)\ \frac{10}{5}x^5 - \frac{6}{3}x^3 + 5x + C$

one term correct

M1

another term correct

A1

all integration correct including $+ C$

A1

$4 = 2 - 2 + 5 + C \Rightarrow C = -1$

substituting both $x = 1$ *and* $y = 4$ ***and*** *attempting to find* C

m1

$y = 2x^5 - 2x^3 + 5x - 1$

must have $y = \ldots$ *and coefficients simplified*

A1cso

5

[8]

6

(a) $\frac{dy}{dx} = 18 + 6x - 12x^2$

one of these terms correct

M1

another term correct

A1

all correct (no + c etc)
(penalise + c once only in question)

A1

3

(b) $18 + 6x - 12x^2 = 0$

putting their $\frac{dy}{dx} = 0$,

PI by attempt to solve or factorise

M1

$6(3 - 2x)(x + 1)$ (= 0)

attempt at factors of ***their quadratic***
or use of quadratic equation formula

m1

$x = -1, \ x = \frac{3}{2}$ OE

must see both values unless x = –1
is verified separately

A1

3

If M1 not scored, award SC B1 for verifying

that x = –1 leads to $\frac{dy}{dx}$ *= 0 and a further*

SC B2 for finding $x = \frac{3}{2}$ *as other value*

(c) (i) $\frac{d^2y}{dx^2} = 6 - 24x$

FT their $\frac{dy}{dx}$ but $\frac{d^2y}{dx^2}$ must be correct if

3 marks earned in part (a)

B1√

When $x = -1$, $\frac{d^2y}{dx^2} = 6 - (24 \times -1)$

Sub $x = -1$ into their $\frac{d^2y}{dx^2}$

M1

$\frac{d^2y}{dx^2} = 30$

A1cso

3

(ii) Minimum point

must have a value in (c)(i)

FT "maximum" if their value of $\frac{d^2y}{dx^2} < 0$

E1√

1

[10]

7 (a) $h = 1$

$h = 1$ stated or used. (PI by x-values 0,1,2,3,4 provided no contradiction)

B1

$f(x) = \frac{2^x}{x+1}$

$\approx h / 2\{\ldots\}$

$\{.\} = f(0) + f(4) + 2[f(1) + f(2) + f(3)]$

OE summing of areas of the 'trapezia'…

M1

$\{.\} = 1 + \frac{16}{5} + 2\left(\frac{2}{2} + \frac{4}{3} + \frac{8}{4}\right)$

$= 1 + 3.2 + 2(1 + 1.33 \ldots + 2)$

OE Accept 1dp evidence. Can be implied by later correct work provided > 1 term or a single term which rounds to 6.43

A1

(I ≈) 0.5[4.2 + 2 × 4.333 …] = 6.43 (to 3sf)

CAO Must be 6.43

A1

(b) Increase the number of ordinates

OE eg increase the number of strips.

E1

1

[5]

8

(a) (i) $\frac{dy}{dx} = -1 - 4x^3$

one of these terms correct

M1

all correct (no + c)

A1

(When $x = 1$, grad =) –5

(Check that $\frac{dy}{dx}$ is actually correct!)

A1cso

3

(ii) $y - 12$ = 'their grad' $(x - 1)$

any form of equation through (1, 12) and attempt at c if using $y = mx + c$

M1

$y = -5x + 17$ (or $y = 17 - 5x$)

FT their gradient

Condone $y = -5x + c$, $c = 17$ etc

A1√

2

(b) (i) $14x - \frac{x^2}{2} - \frac{x^5}{5}$

one of these terms correct

M1

another term correct

A1

all correct (may have + c)

A1

$[\ ]_{-2}^{1} =$

$\left(14 - \frac{1}{2} - \frac{1}{5}\right) - \left(-28 - 2 + \frac{32}{5}\right)$

F(1) and F(–2) attempted

m1

= 36.9 OE

Condone recovery to this value

A1

5

(ii) Area $\Delta = \frac{1}{2} \times 3 \times 12$

Correct area of triangle unsimplified

M1

= 18

$\Rightarrow$ shaded area = 18.9

A1cso

2

[12]

9 (a) (i) $r = \frac{294}{420} = 0.7$

AG. Accept any valid justification to the given answer

B1

1

(ii) $\{S_\infty =\}\ \frac{a}{1-r} = \frac{420}{1-0.7}$

$\frac{a}{1-r}$ *used*

M1

$\{S_\infty =\}$ 1400

1400 NMS mark as 2 / 2 or 0 / 2

A1

2

(iii) nth term = 600 × $(0.7)^n$

If not B2 award B1 for 420 × $(0.7)^{n-1}$ OE

B2

2

(b) (i) $\{u_n =\} 248 - 8n$

Accept ACF

B1

1

(ii) $u_k = 0 \Rightarrow 8k = 248$

$248 - 8k = 0$ OE e.g. $240 + (k - 1)(-8) = 0$
ft if no recovery, on c's (b)(i) answer

M1

$k = 31$

A1

$\sum_{n=1}^{k} u_n = 240 + 232 + ... + 0 = \frac{k}{2}[240 + 0]$

For $\frac{k}{2}[240 + 0]$ or for $\frac{k}{2}$[c's $u_1 + 0$]

OE e.g. $\frac{k}{2}$[2 × c's $u_1 + (k - 1)(-8)$]

M1

$\sum_{n=1}^{k} u_n$ (= 15.5 × 240) = 3720

3720

A1

4

[10]