OXFORD
INTERNATIONAL
AQA EXAMINATIONS

Topic Test: OxfordAQA AS Mathematics

Statistics

Name: ____________________

Class: ____________________

Date: ____________________

Time: **77 minutes**

Marks: **62 marks**

Comments:

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1 Gary and his neighbour Larry work at the same place.

On any day when Gary travels to work, he uses one of three options: his car only, a bus only or both his car and a bus. The probability that he uses his car, either on its own or with a bus, is 0.6. The probability that he uses both his car and a bus is 0.25.

(a) Calculate the probability that, on any particular day when Gary travels to work, he:

(i) does not use his car;

(1)

(ii) uses his car only;

(2)

(iii) uses a bus.

(3)

(b) On any day, the probability that Larry travels to work with Gary is 0.9 when Gary uses his car only, is 0.7 when Gary uses both his car and a bus, and is 0.3 when Gary uses a bus only.

Calculate the probability that, on any particular day when Gary travels to work, Larry travels with him.

(4)

(Total 10 marks)

2 Roger is an active retired lecturer. Each day after breakfast, he decides whether the weather for that day is going to be fine (F), dull (D) or wet (W). He then decides on only one of four activities for the day: cycling (C), gardening (G), shopping (S) or relaxing (R). His decisions from day to day may be assumed to be independent.

The table shows Roger's probabilities for each combination of weather and activity.

		Weather		
		Fine (F)	**Dull (D)**	**Wet (W)**
Activity	**Cycling (C)**	0.30	0.10	0
	Gardening (G)	0.25	0.05	0
	Shopping (S)	0	0.10	0.05
	Relaxing (R)	0	0.05	0.10

(a) Find the probability that, on a particular day, Roger decided:

(i) that it was going to be fine and that he would go cycling;

(ii) on either gardening or shopping;

(iii) to go cycling, given that he had decided that it was going to be fine;

(iv) **not** to relax, given that he had decided that it was going to be dull;

(v) that it was going to be fine, given that he did **not** go cycling.

(9)

(b) Calculate the probability that, on a particular Saturday and Sunday, Roger decided that it was going to be fine and decided on the same activity for both days.

(3)

(Total 12 marks)

3 In a computer game, players try to collect five treasures. The number of treasures that Isaac collects in one play of the game is represented by the discrete random variable X.

The probability distribution of X is defined by

$$P(X = x) = \begin{cases} \dfrac{1}{x+2} & x = 1, 2, 3, 4 \\ k & x = 5 \\ 0 & \text{otherwise} \end{cases}$$

(a) (i) Show that $k = \dfrac{1}{20}$.

(2)

(ii) Calculate the value of $E(X)$.

(2)

(iii) Show that $\text{Var}(X) = 1.5275$.

(3)

(iv) Find the probability that Isaac collects more than 2 treasures.

(2)

(b) The number of points that Isaac scores for collecting treasures is Y where

$$Y = 100X - 50$$

Calculate the mean and the standard deviation of Y.

(4)

(Total 13 marks)

4 (a) The number of text messages, N, sent by Peter each month on his mobile phone never exceeds 40.

When $0 \le N \le 10$, he is charged for 5 messages.
When $10 < N \le 20$, he is charged for 15 messages.
When $20 < N \le 30$, he is charged for 25 messages.
When $30 < N \le 40$, he is charged for 35 messages.

The number of text messages, Y, that Peter is charged for each month has the following probability distribution:

y	5	15	25	35
$P(Y = y)$	0.1	0.2	0.3	0.4

(i) Calculate the mean and the standard deviation of Y.

(4)

(ii) The Goodtime phone company makes a total charge for text messages, C pence, each month given by

$$C = 10Y + 5$$

Calculate $E(C)$.

(1)

(b) The number of text messages, X, sent by Joanne each month on her mobile phone is such that

$$E(X) = 8.35 \quad \text{and} \quad E(X^2) = 75.25$$

The Newtime phone company makes a total charge for text messages, T pence, each month given by

$$T = 0.4X + 250$$

Calculate $\text{Var}(T)$.

(4)
(Total 9 marks)

5 As part of her breakfast duties, Fatima, a newly-appointed trainee chef, fries eggs.

It may be assumed that, for a newly-appointed trainee chef, the proportion of egg yolks broken during frying is 0.15, and that the breaking of yolks is independent from egg to egg.

(a) Determine the probability that, on a day when Fatima fries 50 eggs for breakfast, the number of yolks that she breaks is:

(i) exactly 6;

(3)

(ii) more than 6 but at most 12.

(3)

(b) Calculate the mean and the variance for the number of yolks broken on a day when Fatima fries **80** eggs.

(2)
(Total 8 marks)

6 The records at a passport office show that, on average, 15 per cent of photographs that accompany applications for passport renewals are unusable.

Assume that exactly one photograph accompanies each application.

(a) Determine the probability that in a random sample of 40 applications:

(i) exactly 6 photographs are unusable;

(ii) at most 5 photographs are unusable;

(iii) more than 5 but fewer than 10 photographs are unusable.

(7)

(b) Calculate the mean and the standard deviation for the number of photographs that are unusable in a random sample of **32** applications.

(3)
(Total 10 marks)

Mark schemes

1

P(C) = 0.6   P($C \cap B$) = 0.25
{P(C only) = 0.35   P(B only) = 0.4}

In (a), ratios (eg 4 : 10) are only penalised by 1 mark at first correct answer

(a) (i) P(C') = 1 – P(C) = 1 – 0.6 = 0.4

CAO; or equivalent

B1

1

(ii) P($C \cap B'$) = 0.6 – 0.25
= 1 – (0.4 + 0.25)

Can be implied by correct answer

M1

= 0.35

CAO; or equivalent

A1

2

(iii) P(B) = (i) + p with p < 0.6

Can be implied by correct answer

M1

= (i) + 0.25

Can be implied by correct answer

A1

= 0.65

CAO; or equivalent

A1

OR

P(B) = 1 – (ii)

Can be implied by correct answer

(M2)

= 0.65

(A1)

OR

$1 = P(C) + P(B) - P(C \cap B)$

Can be implied by correct answer

(M1)

Thus $P(B) = 1 - (0.6 - 0.25)$

Can be implied by correct answer

(A1)

= 0.65

CAO; or equivalent

(A1)

3

(b) $P(L \mid G_C) = 0.9$ $P(L \mid G_{CB}) = 0.7$

$P(L \mid G_B) = 0.3$

$P(G \cap L) \Rightarrow$ (a)(ii) × 0.9 (0.315)

Follow through or correct

M1

0.25 × 0.7 (0.175)

M1

[(a) (iii) – 0.25] × 0.3 (0.12)

Follow through or correct

M1

Note: Each pair of multiplied probabilities must be > 0 to score the corresponding method mark

Ignore any multiplying factors
Ignore any additional terms

$\Rightarrow 0.315 + 0.175 + 0.12 = 0.61$

CAO

A1

4

[10]

2 (a) (i) P(F & C) = **<u>0.3 or 3 / 10 or 30%</u>**

Ratios (eg 3:10) are only penalised by
1 accuracy mark at first correct answer

CAO *(0.3)*

B1

(1)

(ii) P(G or S) = **<u>0.45 or 45 / 100 or 45%</u>**

CAO *(0.45)*

B1

(1)

(iii) P(C | F) = $\frac{0.3 \text{ or } (i)}{0.55}$ =

M1

<u>30 / 55 or 6 / 11</u>

CAO *(6 / 11)*

or

<u>(0.54 to 0.55) or (54% to 55%)</u>

AWFW *(0.54545)*

A1

(2)

(iv) P(R′ | D) = $\frac{0.25 \text{ or } (0.30 - 0.05)}{0.30}$

Correct numerator

M1

Correct denominator

M1

<u>25 / 30 or 5 / 6</u>

CAO *(5 / 6)*

or

<u>(0.83 to 0.834) or (83% to 83.4%)</u>

AWFW *(0.83333)*

A1

(3)

(v) P(F | C′) = $\frac{0.25 \text{ or } (0.60 - 0.35)}{0.60}$

Correct expression

M1

<u>25 / 60 or 5 / 12</u>

CAO *(5 / 12)*

or

<u>(0.416 to 0.42) or (41.6% to 42%)</u>

AWRT *(0.41667)*

A1

(2,3)

9

(b) P = $[P(F \& C)]^2 + [P(F \& G)]^2$

Attempt at ***sum of at least 2 squared terms; 0 < term < 1****; not* $(a + b)^2$
May be implied by a ***correct*** *expression or a* ***correct*** *answer*

M1

$0.30^2 + 0.25^2$ **or** 0.09 + 0.0625 =

OE
Ignore additional terms or integer multipliers
May be implied by a ***correct*** *answer*

A1

1525 / 10000 or 305 / 2000 or 61 / 400

CAO

or

(0.1525)

(0.152 to 0.153) or (15.2% to 15.3%)

AWFW

A1

3

[12]

3 (a) (i) $1 - (^1/_3 + {^1/_4} + {^1/_5} + {^1/_6})$

OE

M1

= $^1/_{20}$ = 0.05

AG

A1

2

(ii) E(X) =

$1 \times {^1/_3} + 2 \times {^1/_4} + 3 \times {^1/_5} + 4 \times {^1/_6} + 5 \times {^1/_{20}}$

At least 2 terms

M1

= 2.35

OE: give B2 for only 2.35 seen

A1

2

(iii) $E(X^2) =$

$1\times{}^1/_3 + 4\times{}^1/_4 + 9\times{}^1/_5 + 16\times{}^1/_6 + 25\times{}^1/_{20}$
(= 7.05)

All 5 terms

$E(X^2)$ = 7.05 with no working scores M0

Correct working but labelled Var(X) and then no more done also scores M0

M1

$\text{Var}(X) = E(X^2) - E(X)^2$

Applied to this problem

m1

= 1.5275

AG

A1

3

(iv) $1 - ({}^1/_3 + {}^1/_4)$ **or** $({}^1/_5 + {}^1/_6 + {}^1/_{20})$

M1

= ${}^5/_{12}$ or 0.417

AWRT Accept answer only for B2

A1

2

(b) ‘2.35’ × 100 − 50

Their value of mean

M1

= 185

*FT from **(a)(ii)***
Give B2 for only 185 seen

A1F

$100^2 \times 1.5275$ **or** $100 \times \sqrt{1.5275}$

M1

SD = $\sqrt{15275} = 5\sqrt{611} = 124$

AWFW 123.5 to 124 or 5√611

Give B2 for only 123.5 to 124 or 5√611 seen

A1

4

[13]

4 (a) (i) $E(Y) = \Sigma y\, P(Y = y)$

$= 5 \times 0.1 + 15 \times 0.2 + 25 \times 0.3 + 35 \times 0.4$

$= 25$

B1

$\text{Var}(Y) = E(Y^2) - [E(Y)]^2$

$= 725 - 25^2$

M1

$= 100$

CAO

A1

Standard deviation = 10

ft on Var(Y) > 0

A1ft

(ii) $C = 10Y + 5$

$E(C) = 10E(Y) + 5$

$= 10 \times 25 + 5$

= 255 pence

OE

B1

1

(b) $\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$
$= 75.25 - 8.35^2$

M1

$= 75.25 - 69.7225$
$= 5.5275$

AWFW 5.52 to 5.53

A1

$T = 0.4X + 250$
$\text{Var}(T) = \text{Var}(0.4X + 250)$
$= 0.4^2 \times \text{Var}(X)$

Var(X) > 0

M1

$= 0.16 \times 5.5275$
$= 0.8844$

AWFW 0.884 to 0.885

A1

4

[9]

5 (a) B (50, 0.15)

Used at least once in (a)

M1

(i) P(E = 6) = 0.3613 – 0.2194

Accept 3 dp accuracy

or

$= \binom{50}{6}(0.15)^6(0.85)^{44}$

Either expression

M1

= 0.142

AWRT (0.1419)

A1

3

(ii) $P(6 < E \le 12) = 0.9699$ or 0.9372

Accept 3 dp accuracy / truncation

M1

minus 0.3613 or 0.2194

Accept 3 dp accuracy

M1

= 0.608 to 0.609

AWFW (0.6086)

A1

OR

B(50, 0.15) expressions stated for at **least 3 terms** within $6 \le E \le 12$ gives probability

Or implied by a correct answer

(M1)

= 0.608 to 0.609

AWFW

(A2)

3

(b) Mean, $\mu = np = 80 \times 0.15 = 12$

CAO

B1

Variance, $\sigma^2 = np(1 - p)$
$= 80 \times 0.15 \times 0.85 = 10.2$

CAO; or equivalent

B1

2

[8]

6 (a) $\underline{U \sim \mathrm{B}(40, 0.15)}$

Used somewhere in (a)

M1

(i) $P(U = 6) = 0.6067 - 0.4325$

or

$= \binom{40}{6}(0.15)^6 (0.85)^{34}$

Accept 3 dp rounding or truncation

M1

= 0.174

Can be implied by a correct answer

AWRT (0.1742)

A1

3

(ii) $P(U \leq 5) = 0.432$ to 0.433

AWFW (0.4325)

B1

1

(iii) $P(5 < U < 10) = 0.9328$ or 0.9701 (p_1)

Accept 3 dp rounding or truncation but allow 0.97

M1

$p_2 - p_1 \Rightarrow$ M0 M0 A0

$(1 - p_2) - p_1 \Rightarrow$ M0 M0 A0

$p_1 - (1 - p_2) \Rightarrow$ M0 M0 A0

$(1 - p_2) - (1 - p_1) \Rightarrow$ M1 M1 (A1)

only providing result > 0

MINUS 0.4325 or 0.2633 (p_2)

Accept 3 dp rounding or truncation

M1

= 0.5(00) to 0.501

AWFW (0.5003)

A1

3

Alternative solution

B(40, 0.15) expressions stated for at least 3 terms within
$5 \le U \le 10$ gives probability

Can be implied by a correct answer

(M2)

= 0.5(00) to 0.501

AWFW (0.5003)

(A1)

u	(5)	6	7	8	9	(10)
$P(U = u)$	(0.1692)	0.1742	0.1492	0.1087	0.0682	(0.0373)

(3)

(b) Mean or $\mu = 32 \times 0.15 = 4.8$

CAO

B1

(V or σ^2 =) $\underline{32 \times 0.15 \times 0.85}$
or
(SD or σ =) $\underline{\sqrt{32 \times 0.15 \times 0.85}}$

Either numerical expression; ignore terminology

May be implied by 4.08 CAO seen or 2.02 AWRT seen

M1

(SD or σ) = 2.02

AWRT (2.0199)

Do not award if labelled V or σ^2

A1

3

[10]